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Theory and Methodology

Optimal cropping patterns under water deficits

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Abstract

A mathematical programming model is proposed for optimal cropping patterns under water deficits in dry regions. Crops may be deliberately under-irrigated in order to increase the total irrigated area and possibly the profit. An operating policy will identify both the total area and the irrigation level allocated to a given selected crop taking into account the possible successors and predecessors of this crop. Both annual and seasonal crops are examined in the same study. The model starts by identifying the optimal operating policy for each grower in the region having a given stock of irrigation water. Then, in order to allocate water efficiently among growers, the model determines the global optimal cropping plan of the entire region. To solve efficiently the problem, a decomposition algorithm is developed. Also some useful economic interpretations are given. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Agriculture; Linear programming; Water allocation

1. Introduction

There is an increasing awareness observed in recent times to make the best use of water, a scarce and valuable resource for all economic activities. This is emphasized particularly in regions characterized by arid and semi-arid climates, where irrigation water is constantly suffering from deficit.

In the context of reservoir management for irrigation planning, a large number of models have been developed in order to identify optimal operating policies for a given time horizon. An operating policy consists of mainly a schedule of water releases over different periods of the planning horizon and allocation of water among competing crops at each release. One could mention models by Dudley (1970, 1972, 1988), Dudley and Burt (1973), Loucks et al. (1981), Stedinger et al. (1984) and Vedula and Mujumdar (1992), Vedula and Nagesh Kumar (1996) to name only a few. Yeh (1985) offers a state-of-the-art review on reservoir management and operation models.

In the context of cropping patterns, however, only a limited number of optimization models have been developed. In addition, most works in the literature use the land to be allocated to some

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fixed crops as the only control variables. In this case, the irrigation levels assigned to each crop are taken to satisfy full demand (for instance, Eckert and Wang, 1992). This approach may maximize the yield per unit cropped area but not necessarily the total profit under water deficits, as the total cropped area may be too small to warrant optimal profit. It may be more beneficial to expand the cropped area at the expense of reducing the irrigation level to be attributed to each crop. Therefore, some works allow crops to be deliberately under-irrigated in order to increase the irrigated area and possibly the profit (English and Nuss, 1982; Hargraves and Samani, 1984; English, 1990). Mannocchi and Mecarelli (1994) use deficit irrigation in cropping patterns and provide a procedure for estimating the production function at different irrigation levels of a given crop. The model developed by Vedula and Mujumdar (1992) as well as its improved version developed by Vedula and Nagesh Kumar (1996) integrate reservoir operations with irrigation planning under water deficit. However, in all models mentioned above, the decisions to be made concern only land and irrigation levels to be allocated to some pre-selected crops. Exceptions can be found in Ziari et al. (1995), and in Loucks et al. (1981). Ziari et al. (1995) developed a multi-stage decision model to evaluate the economic feasibility of on-farm impoundment-based supplemental irrigation under uncertainty, in the Blacklands region of Texas. The model includes in particular crop mix selection. However, the model limits consideration to a single farm rather than to a region. The problem is formulated as a nonlinear mixed integer program. In the chapter on irrigation planning and operation by Loucks et al. (1981), a number of important input factors are discussed. The operating policy, for each of the models developed, identifies in particular the crops to be selected among all the candidate crops, the area to be allocated to each selected crop and the location to be chosen to each crop in order to benefit from the existing types of soil. In addition, some suggested generalizations include deficit irrigation in which case the irrigation levels also become control variables.

2. Shortcomings of existing models

Whether the mathematical procedure to select a given irrigation level to a given crop was explicitly specified or not in the literature, no references consider the possibility of choosing the same crop more than once, each time with a different irrigation level. For instance, a given crop may yield the best profit over the competing crops, if irrigated at its full demand. It may still provide the best profit, if the amount of water left is only sufficient to apply a lower irrigation level. In this case, the crop should be selected twice, each time with the appropriate irrigation level.

In addition, the models that have been developed so far in optimal cropping patterns deal only with one-stage decision plans by considering a single season (possibly a year) and a set of candidate crops that grow in that particular season. However, for real applications, growers are rather concerned with planning policies in which both seasonal and annual crops need to be incorporated in the decision process in the same farm for the same year. This leads to a multi-stage decision problem.

Another important factor neglected in the literature consists of the dependence (quantitative and qualitative) of crop yield on crop predecessors. This dependence may considerably affect the optimal operating policy and therefore should be explicitly incorporated in the cropping patterns.

3. Scope and objectives

The present study develops a linear programming model for optimal cropping patterns in regions suffering from water deficits. First, the model optimizes cropping patterns, for a given grower in a region of interest, and for a given stock of water available. Then, a global optimal cropping plan for the entire region is determined in order to permit optimal water allocation among growers of the region. This global problem is found to be of large scale. Therefore, an adequate decomposition algorithm is developed to reduce the computational effort.

The model developed in the present study attempts to cope with the above weaknesses of earlier models developed in the literature. In particular, each crop will be associated to a selected range of different irrigation levels and the productivity for each irrigation level will be identified. Also, the model will consider both seasonal and annual crops in the planning policy giving rise to a multi-stage decision problem. Moreover, the model will include an explicit dependence of crop yields on crop predecessors. This will be achieved by considering conditional unit profits generated by the given crops on their predecessors. Thus, for a given grower, an optimal planning policy of the suggested model will identify:

- Which crops to choose among seasonal and annual crops for the year of interest in order to maximize the total profit.
- How much land to allocate to each selected crop.
- How much water to allocate to each selected crop (possibly the same crop may be chosen more than once with different irrigation levels).
- Taking into consideration the initial state, where to grow each selected crop to benefit the most from the crop yield dependence on crop predecessors. This dependence will also affect the choice of the sequences of winter–summer crops to be selected.

The optimal cropping policy takes into account constraints on the total land and irrigation water stock available. The rest of the resource inputs are taken at their optimal level. In fact, the model does not consider such resource inputs as limiting factors. This assumption is widely adopted in the literature (e.g., Mannocchi and Mecarelli, 1994; Vedula and Mujumdar, 1992; Vedula and Nagesh Kumar, 1996). The chapter on irrigation planning and operation by Loucks et al. (1981), however, provides a thorough discussion on resource inputs.

For the decision-makers of the region (local authorities), the model also determines how to make the best water allocation among the different growers of the region. Furthermore, from the marginal value of water at optimum exploitation, the decision-makers will have insight as to the impact of some perturbations of the model (such as more water deficits, new water pricing, new crops to be considered, etc.) on the optimal planning strategies.

As the present study is primarily concerned with cropping patterns under water deficits, it is important to know the yield per unit area as a function of each crop and each level of irrigation considered. In the absence of related data, many formulae are suggested to express analytically the crop yield as a function of the irrigation water applied (for instance, Jensen, 1968; Stewart and Haggan, 1973; Sudar et al. 1981). Mannocchi and Micarelli (1994), Vedula and Mujumdar (1992), and Vedula and Nagesh Kumar (1996) provide detailed procedures for estimating the expected yield per unit area for a given level of applied water as a fraction of the maximum yield that would be obtained at an optimal irrigation level.

In the next section, the model is explicitly exposed. In addition, an illustrative example based on hypothetical data is provided to explain the applicability of the model. Then, a decomposition algorithm is developed in order to reduce the computational burden of the global problem of the entire region. Then, an economic interpretation of the model is provided. This interpretation uses the marginal value of water at optimal exploitation to discuss the impact of some parameter perturbations. This includes more water deficits and/or higher water prices on the optimal cropping strategies as well as the required minimum profit for a given new crop to be considered among the selected crops, for a given season or year. Finally, some concluding remarks are provided along with the directions for future work.

4. The model

The cropping problem is concerned with selecting crops, and allocating land and water to them. Therefore, there are initially two types of decision variables, land and water. However, the dependence of crop yield on applied irrigation water suggests a nonlinear profit function of these decision variables. Thus, some adjustment as to the definition of the decision variables is needed in order to obtain a linear program. For this purpose, irrigation water is discretized for each crop by carefully selecting levels that range from 0 to the full irrigation demand of that particular crop. It is worth noting that the model considers the *same* crop associated with different irrigation levels as *distinct* crops. Consequently, a first tentative to define the decision variables of the model would be to consider the land to be allocated to a given crop with which a given level of irrigation is associated. In this case, the profit function would be additive.

Another difficulty in modeling the decision problem as a linear program arises from the precedence order that needs to be respected (as some crops may not be grown right after others, or may give the best yield only if grown over some specific crops). Furthermore, the winter/summer-crop combinations to be selected (again taking into account the precedence order) along with the annual crops suggest multi-stage decision policies as the selection, say of a winter crop, will depend both on its predecessors (initial state of the land) and its successors (summer crops that may follow). Therefore, another adjustment regarding the definition of the decision variables needs to be made. This can be achieved by further breaking down the adjusted decision variables according to their predecessors/successors, and by appropriately setting balance constraints.

The region of interest, Nabeul in Tunisia, receives imported water from the north of the country through a canal (Medjerda-Cap Bon canal). Authorities in charge of water (at the ministry of agriculture) decide on yearly water allocation to the region before hand. This imported water represents practically the only source of water supply as most of the underground stock is contaminated because of seawater intrusion. Moreover, rainfall is very rare and limited. This justifies a deterministic supply for this particular study. However, it is important to notice that a probabilistic approach should be used in general, as the supply is likely to be stochastic in most other applications.

Local water authorities in the region decide on water allocation among growers. The region suffers from a constant water deficit. The planning horizon starts at the end of a given summer. The purpose is to identify the cropping plan for the subsequent year in the region. This problem may concern the entire region (local authorities) as well as individual growers. For an individual grower, the objective may be to maximize his profit by selecting the most rewarding combination of crops under land and water constraints. Decision-makers at the region level may have different strategic objectives. This may give rise to a bi-level programming formulation of the problem, where upper decision-makers (local authorities) will have to provide incentives and/or disincentives to push lower decision-makers (the growers) to act according to upper decision-makers' optimal planning policies.

For a survey of applications and algorithms of bi-level linear programming (BLP), one may refer to Ben-Ayed (1993). Bialas and Karwan (1982) present a model called Bi-level resource control model, in which the upper decision-maker controls the resources available to the lower one. In the area of agriculture, one application of BLP is found in the work by Erickson, where the upper decision-maker (the government) sets agricultural policies, and the lower decision-makers (the farmers) react to government policies and prices, (Harker, 1985).

There are other approaches in the literature dealing with situations where decisions may be taken at different levels. For instance, Dudley (1988) provides a basis for comparing results obtainable by a single decision-maker (a farmer) with results obtainable with multiple decision-makers, namely, a reservoir manager and individual irrigation farmers.

However, if the objectives of individual farmers and local authorities converge in the sense that the only concern of decision-makers, at the region level, is to maximize the total profit of the entire region, then a grower problem will be a subproblem of the global problem of the entire region. In this case, optimal solutions will agree in the sense that the optimal total profit of the entire region will simply be the sum of optimal profits of individual growers. Note that the amount of water to be available for a given grower will be a constraint of the grower problem. It will be, however, a decision variable in the global program (i.e., water authorities will be concerned with allocating water to the different growers most efficiently in order to maximize the total profit of the region). This study deals only with the latter case (i.e., when both objectives agree).

Let L denote the total land that can be cropped and LEV the total stock of water available. Also, let A be the set of crops that can grow on an annual basis in the region, where the *same* crop is considered *several times*, each time with a *different* level of irrigation per unit area. Similarly, let S and W be the sets of crops that can grow in that region during summer and winter, respectively (again, each crop is associated with several irrigation levels that are carefully selected). In addition, assume there are K growers in the region.

In order to estimate the unit profit of a given crop with a given level of irrigation, while all other production factors are taken at their optimum, and in the absence of suitable data, it is possible to use, for instance, the procedure developed by Mannocchi and Mecarelli (1994). This procedure is based on Stewart's multiplicative formula (Stewart and Haggan, 1973), and provides the expected yield of the applied water as a fraction of the maximum yield which occurs if the crop were to be irrigated at its full demand. More details are provided in the example below.

Now, define E as the area of the entire region cropped at the end of the given summer ($E \leqslant L$) and D as the set of the corresponding crops ($D \subseteq A \cup S$). Also, define N as the area of the entire region that was uncropped during that summer.

For convenience, denote a generic grower by k, and a generic crop of A by a, of W by w of S by s, and of D by d. Also, denote the area that was cropped with some crop d in D by E_d.

Then, consider a crop a in A, and define the set of predecessors of a by

$$\Omega_D^-(a) = \{d \text{ in } D \text{ that can be followed by } a \text{ on the same land}\}. \tag{1}$$

Define similarly the predecessors of a crop w in W. Also, define the set of successors of a crop w in W by

$$\Omega_S^+(w) = \{s \text{ in } S \text{ that can be cropped after } w \text{ on the same land}\}. \tag{2}$$

The set of predecessors of a crop s in S is given by

$$\Omega_W^-(s) = \{w \text{ in } W \text{ that can be followed by } s \text{ on the same land}\}. \tag{3}$$

In order to specify that a given crop will grow on a land that was not cropped during the previous season, set arbitrarily its predecessor to be 0. Similarly, to point out that a land that was cropped with a given crop will not be cropped for the following season, set arbitrarily its successor to be 0.

Decision variables

- x_{da} area of the entire region that was cropped with d and that is to be cropped for the year of interest with a, for all a in A and d in D
- y_{dw} area of the entire region that was cropped with d and that is to be cropped with w, for the year of interest, for all w in W and d in D
- z_{ws} area of the entire region that will be cropped with w during the winter of interest and that is to be cropped with s, for the following summer, for all w in W and s in S
- x_{0a} area of the entire region that was not cropped, and that is to be cropped with a, for the year of interest, for all a in A
- y_{0w} area of the entire region that was not cropped, and that is to be cropped with w, for the winter of interest, for all w in W
- z_{0s} area of the entire region that will not be cropped during the winter of interest, and that is to be cropped with s for the following summer, for all s in S

To provide more clarity for the model, introduce the dependent variables:

- x_a area of the entire region that is to be cropped with a, for all a in A
- y_w area of the entire region that is to be cropped with w, for all w in W
- z_s area of the entire region that is to be cropped with s, for all s in S

Define similarly, decision variables relative to grower k, $k = 1, 2, \ldots, K$, by adding the superscript k to each variable introduced above. For instance,

x^k_{da} area of the land of grower k that was cropped with d and that is to be cropped with a, for the year of interest, for all a in A and d in D

Also, define E^k, D^k, E^k_d, and N^k as the total cropped area, the corresponding set of crops, the area that was cropped with d during the previous summer, and the uncropped area relative to grower k, respectively, in the same way as E, D, E_d, and N are defined above.

Parameters of the problem

p_{da} profit per unit area obtained from cropping a on the land that was cropped with d, for all a in A and d in D

p_{dw} profit per unit area obtained from cropping w on the land that was cropped with d for all w in W and d in D

p_{ws} profit per unit area obtained from cropping s during the summer of interest on the land that is to be cropped with w, for the winter of interest, for all s in S and w in W

I_a irrigation level associated with crop a in A

I_w irrigation level associated with crop w in W

I_s irrigation level associated with crop s in S

L total area that can be cropped in the entire region

L^k total area that can be cropped by grower k, $k = 1, 2, \ldots, K$

LEV total stock of water for the entire region

LEV^k total stock of water allocated to grower k, $k = 1, 2, \ldots, K$

Note here that LEV^k is a parameter of the local problem that concerns the cropping plan of grower k. However, for the global problem, LEV^k will be a decision variable that upper decision-makers will allocate to grower k in order to maximize the total profit of the region. Note also that LEV, which is considered in this particular study as deterministic, is most often stochastic. Therefore, future extensions to this work that incorporate the stochasticity of water supply would suggest transforming the linear program of the current model to a stochastic linear program.

The optimal cropping plan of grower k (for some $k = 1, 2, \ldots, K$) is formulated as follows:

Objective function

$$\max \sum_{a\in A} \sum_{d\in\Omega^-_D(a)\cup\{0\}} p_{da} x^k_{da} + \sum_{w\in W} \sum_{d\in\Omega^-_D(w)\cup\{0\}} p_{dw} y^k_{dw} + \sum_{s\in S} \sum_{w\in\Omega^-_W(s)\cup\{0\}} p_{ws} z^k_{ws}. \tag{4}$$

The objective is then to maximize the total profit obtained from annual and seasonal crops for the whole year of the plan.

Constraints

(i) Surface constraints

$$\sum_{a\in A} x^k_a + \sum_{w\in W} y^k_w \leqslant L^k, \quad \sum_{a\in A} x^k_a + \sum_{s\in S} z^k_s \leqslant L^k. \tag{5}$$

These constraints say that, what will be cropped on an annual basis and for winter (respectively summer) cannot exceed the total land of grower k.

(ii) *Balance constraints*

$$\begin{aligned} x^k_a &= \sum_{d\in\Omega^-_D(a)\cup\{0\}} x^k_{da} \quad \forall a \in A, \\ y^k_w &= \sum_{d\in\Omega^-_D(w)\cup\{0\}} y^k_{dw} \quad \forall w \in W, \\ z^k_s &= \sum_{w\in\Omega^-_W(s)\cup\{0\}} z^k_{ws} \quad \forall s \in S. \end{aligned} \tag{6}$$

These constraints say that, what is to be cropped with a, w or s was either noncropped or some predecessor of the given crop.

$$\begin{aligned} &\sum_{a\in\Omega^+_S(d)} x^k_{da} + \sum_{w\in\Omega^+_W(d)} y^k_{dw} \leqslant E^k_d \quad \forall d \in D^k, \\ &\sum_{s\in\Omega^+_S(w)} z^k_{ws} \leqslant y^k_w \quad \forall w \in W. \end{aligned} \tag{7}$$

The first constraint says that, what is to be cropped with all a and was cropped with a same d can not exceed the land cropped by d. The other two constraints have similar interpretations with the corresponding crops.

$$\sum_{a\in A} x_{0a}^k + \sum_{w\in W} y_{0w}^k \leqslant N^k,$$
$$\sum_{s\in S} z_{0s}^k + \sum_{a\in A} x_a^k + \sum_{w\in W} y_w^k \leqslant L^k. \tag{8}$$

The first constraint says that, what was not cropped and is to be cropped with some a or w cannot exceed the total area that was not cropped during the previous summer. The second says that the crops for the following summer that are to be grown on an uncropped land cannot exceed the total area of the uncropped land.

(iii) *Water constraint*

$$\sum_{a\in A} I_a x_a^k + \sum_{w\in W} I_w y_w^k + \sum_{s\in S} I_s z_s^k \leqslant \mathrm{LEV}^k. \tag{9}$$

(iv) *Nonnegativity constraints*

$$x_a^k, x_{da}^k, x_{0a}^k, y_w^k, y_{dw}^k, y_{0w}^k, z_s^k, z_{ws}^k, z_{0s}^k \geqslant 0 \quad \forall d, a, w, s. \tag{10}$$

5. Example

This example aims only at illustrating the suggested method rather than solving a real problem. Consequently, most of the data is hypothetical. However, the authors are preparing for a case study based on real data (Haouari and Azaiez, 2000).

Consider a grower having 80 ha of land. Assume that the only crops that may grow are: wheat as an annual crop, sorghum and maize as winter crops, and sorghum and safflower as summer crops. The total stock of water to be available will take a number of levels varying from 60,000 (high deficit) to 240,000 m^3 (satisfying full demand at optimal exploitation). The last exploitation of the land (of annual and summer crops) is assumed to be as follows: 20 ha were cultivated with wheat, 30 ha were cultivated with safflower, 10 ha were cultivated with sorghum, and 20 ha were uncultivated.

For each of the crops above, four levels of irrigation are considered, consisting of the full irrigation requirement as well as 80%, 60%, and 40% of this full level, respectively. In addition, all input parameters (other than water) are kept at their optimum level. The reduction in yield due to deficit irrigation can be expressed from the ratio of the actual to the maximum yield given by (see for instance Mannocchi and Mecarelli, 1994)

$$\frac{Y_\mathrm{a}}{Y_\mathrm{m}} = \prod_{i=1}^{n} \left[1 - K_{y_i}\left(1 - \frac{ET_\mathrm{a}}{ET_\mathrm{m}}\right)\right], \tag{11}$$

where Y_a is the actual yield, Y_m the maximum yield, i a generic growth stage, n the number of growth stages considered (here $n = 5$: establishment stage, vegetative stage, flowering stage, yield formation stage, and ripening stage), K_y the yield response factor at growth stage i, ET_a and ET_m are the actual and maximum evapotranspiration, respectively.

The yield response factors can be obtained from the literature (Doorembos and Kassam, 1981). The ratio of actual to maximum evapotranspiration can be estimated using a number of parameters including applied water, root depth, initial moisture, and moisture levels at field capacity and at permanent wilting point (Vedula and Mujumdar, 1992). For simplicity, however, the ratios of actual to maximum evapotranspiration for the different crops and different irrigation levels are given arbitrarily in this example. Full irrigation requirements, maximum yields and profits are also given arbitrarily. (Interested readers are referred, for instance, to Doorembos and Pruitt (1984) for crop water requirements.)

To take into consideration the yield dependence on crop predecessors, hypothetical discounting factors are chosen to express the reduction in yield due to inappropriate selections of predecessors. This approach can be generalized to extended planning horizons in order to allow policies for explicitly incorporating crop rotation.

Table 1 specifies the full irrigation demand, maximum yield and profits for the different crops. Table 2 provides the yield response factors for the different growing stages for each of the crops. Table 3 contains ratios of actual to maximum

Table 1
Data at optimal level of exploitation

	Water demand (m^3/ha)	Y_m (tons/ha)	Profit (TD/ton)[a]
Wheat (A)[b]	1000	7	200
Sorghum (W)[c]	700	16	150
Maize (W)[c]	1200	10	350
Sorghum (S)[d]	1200	14	180
Safflower (S)[d]	1600	12	300

[a] Stands for Tunisian Dinars.
[b] Stands for annual.
[c] Stands for winter.
[d] Stands for summer.

Table 2
Yield response factors

	Yield response factors				
	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Wheat (A)	0.2	0.2	0.65	0.55	0.2
Sorghum (W)	0.2	0.2	0.55	0.45	0.2
Maize (W)	0.2	0.4	1.5	0.5	0.2
Sorghum (S)	0.2	0.2	0.55	0.45	0.2
Safflower (S)	0.2	0.3	0.55	0.6	0.2

Table 3
Ratios of actual to maximum evapotranspiration

	ET_a/ET_m		
	80% Irrig.	60% Irrig.	40% Irrig.
Wheat (A)	0.9	0.8	0.75
Sorghum (W)	0.9	0.85	0.8
Maize (W)	0.8	0.6	0.5
Sorghum (S)	0.9	0.8	0.7
Safflower (S)	0.8	0.5	0.3

Table 4
Ratios of actual to maximum yield

	Y_a/Y_m		
	80% Irrig.	60% Irrig.	40% Irrig.
Wheat (A)	0.83	0.69	0.62
Sorghum (W)	0.83	0.75	0.68
Maize (W)	0.54	0.23	0.12
Sorghum (S)	0.85	0.72	0.60
Safflower (S)	0.67	0.35	0.21

evapotranspiration, for the different irrigation levels considered for each crop. Table 4 specifies ratios of actual to maximum yield for the different crops under the different irrigation levels, based on the above formula. Table 5 displays hypothetical discounting factors expressing the dependence of yield on crop predecessors. Finally, Table 6 provides the optimal cropping patterns.

In Table 6, optimal exploitation is given for a variety of total stocks of water available. The first row specifies the last exploitation. For each level of water provided, two rows (W and S standing for winter and summer, respectively) specify optimal exploitation. The crops are denoted by Wh, Mz, Sg, and Sf for wheat, maize, sorghum, and safflower, respectively. To distinguish between sorghum cropped for winter and for summer, w and s are added as subscripts, respectively to Sg. Also, to specify the irrigation level(s) to be attributed to a given crop, a number has been added to the symbol of each crop ranging from 1 to 4, where 1, 2, 3, and 4 designate, respectively, 100%, 80%, 60% and 40% of the full demand level. If more than one crop are occupying a specific piece of land, then the allocation in hectares will be given between square brackets in the respective order.

Table 5
Discounting factors on yield with respect to crop predecessors

	Predecessors					
	None	Wheat	Sorgum (W)	Maize	Sorgum (S)	Safflower
Wheat	1	0.5	–	–	0.9	1
Sorgum (W)	1	1	–	–	0.8	0.9
Maize	0.95	1	–	–	0.9	1
Sorgum (S)	1	–	0.8	0.9	–	–
Safflower	1	–	1	0.9	–	–

Table 6
Optimal exploitation for various levels of water availability

Initial state (m^3)		None (20 ha)	Wh1 (20 ha)	Sf1 (30 ha)	Sgs1 (10 ha)
NIV = 60,000	W	Sg_w4	Sg_w4	Sg_w4	Sg_w4
	S	Sg_s4	Sg_s4	Sg_s4	Sg_s4 $[8^{1/3}]$
NIV = 70,000	W	Sg_w4	Sg_w4/Mz1: [10,10]	Sg_w4	Sg_w4
	S	Sg_s4	Sg_s4	Sg_s4	Sg_s4
NIV = 100,000	W	Sg_w4	Mz1	Sg_w4	Mz1
	S	Sg_s4/Sf1: [9.64, 10.36]	Sg_s4	Sg_s4	Sg_s4
NIV = 224,000	W	Mz1	Mz1	Mz1	Mz1
	S	Sf1	Sf1	Sf1	Sf1

The optimal cropping policies do not suggest any cultivation of wheat regardless of the level of total water available. Sorghum is always selected at 40% of its full demand level. The other seasonal crops are only selected at full demand levels. When there is a high deficit of water, only sorghum is to be cropped. For the summer season, the land need not be entirely cropped. As more water becomes available, the optimal cropping policy suggests cropping all the land in both seasons and gradually introducing maize (to be cropped for winter) to partially substitute sorghum. As the level of water increases, safflower is also gradually introduced. When no water deficit occurs, then the optimal cropping policy suggests cropping the entire land with maize for winter and safflower for summer, both with full irrigation levels. Table 6 also specifies the precedence order in which crops are to be cultivated. The size of the linear program (LP) of the grower was 92 decision variables and 31 constraints.

While the LP of the example is relatively small, a real case may lead to a large LP to solve for each grower k. Consider, for instance, 20 annual competing crops, 50 winter competing crops and another 50 summer competing crops. Also, assume that 10 (summer and annual) crops are initially cropped just before the planning horizon. Then, the number of variables x^k_{da}, y^k_{da}, and z^s_w will be, respectively, 800, 2000, and 10,000, leading to an LP with more than 12,800 decision variables. This LP can be solved using the available software. However, if there are 200 growers in the entire region (in Nabeul, there are more than 1000 growers), then the global problem of the entire region will have more than 2,560,000 decision variables, which is by far beyond the capacity of existing LP solvers. However, this global problem will have a special structure that can sharply reduce the computational effort if a suitable decomposition is applied, as will be explicitly elaborated below. Before that, the formulation of the global problem is outlined: The objective function will be the sum of individual growers' objective functions, given by

$$\max \sum_{k=1}^{K} \left(\sum_{a\in A} \sum_{d\in\Omega^-_{AS}(a)} p_{da} x^k_{da} + \sum_{w\in W} \sum_{d\in\Omega^-_{AS}(w)} p_{dw} y^k_{dw} + \sum_{s\in S} \sum_{w\in\Omega^-_{W}(s)} p_{ws} z^k_{ws} \right). \tag{12}$$

The surface and balance constraints are those for all growers k ($1 \leqslant k \leqslant K$). The water constraint will be

$$\sum_{k=1}^{K}\left(\sum_{a\in A} I_a x_a^k + \sum_{w\in W} I_w y_w^k + \sum_{s\in S} I_s z_s^k\right) \leqslant \mathrm{LEV}. \qquad (13)$$

Note here that we have $\forall k = 1, \ldots, K$,

$$\sum_{a\in A} I_a x_a^k + \sum_{w\in W} I_w y_w^k + \sum_{s\in S} I_s z_s^k = \mathrm{LEV}^k. \qquad (14)$$

In other words, the amount of water the decision-makers will allocate to grower k will be that which makes the grower obtain the optimal profit according to this program. The nonnegativity constraints are considered for all k ($1 \leqslant k \leqslant K$).

This global program has a special structure of multi-divisional problems that can be exploited to decompose it into a set of subproblems of relatively moderate sizes. In fact, the global problem is almost decomposable into independent problems (each representing a given grower problem) with one more coupling constraint that coordinates water allocations among growers.

The work referenced above by Ziari et al. (1995) uses Benders decomposition for nonlinear subproblems to solve the formulated nonlinear mixed integer program. The original problem (of a single grower) is decomposed into two separate problems, the master problem and a subproblem. The master problem consists of a pure IP, while the subproblem consists of an NLP (with only continuous variables and constraints). The problem, however, does not have a multi-divisional structure, as in the case of the present study.

6. Dichotomous decomposition algorithm

The global problem of the entire region is frequently very large. In addition, it has a multi-divisional structure with one coupling constraint, and K independent subproblems, each corresponding to a particular grower. Therefore, it is reasonable to solve it through some decomposition technique. Clearly, Dantzig–Wolfe decomposition, (Dantzig and Wolfe, 1963) is one alternative procedure. The algorithm iteratively alternates between solving a master program containing $(K+1)$ constraints and K subproblems of moderate sizes. In the case of the current model, even solving the master program will require to solve, at each iteration, an LP containing typically several thousands of constraints, which is not an easy task. Instead of using a standard decomposition technique, a more efficient algorithm (dichotomous decomposition algorithm (DDA)) is developed. It is based on a Lagrangian relaxation and it uses a constraint generation procedure (CGP) leading to an equivalent of Benders decomposition (Benders, 1962) algorithm, but applied to the dual global problem. A key step of the CGP will use a dichotomous search (see, for instance, Taha, 1992) that will reduce to finding the intersection of two lines, rather than solving LPs of relatively large sizes. This makes the basic difference between this DDA developed below and the standard techniques.

Consider the global program. It is possible to write it in the following form:

$$\begin{array}{ll} \text{(GLP)} & \\ \max & \sum_k c_k^{\mathrm{T}} x_k, \\ \text{s.t.} & A_k x_k \leqslant b_k, \quad k = 1, 2, \ldots, K, \\ & \sum_k d_k^{\mathrm{T}} x_k \leqslant b, \quad x_k \geqslant 0, \; k = 1, 2, \ldots, K. \end{array}$$

In this case, b is the total stock of water available for the year of interest (i.e., $b = \mathrm{LEV}$). The corresponding constraint is the only coupling constraint of the global problem.

Each subproblem k ($1 \leqslant k \leqslant K$) will be the kth grower problem (without the water constraint). Such a subproblem can be written in the following form:

$$\begin{array}{ll} (\mathrm{SUB}_k) & \\ \max & c_k^{\mathrm{T}} x_k, \\ \text{s.t.} & A_k x_k \leqslant b_k, \quad x_k \geqslant 0, \; k = 1, 2, \ldots, K. \end{array}$$

$\{x^j\}_{1 \leqslant j \leqslant J}$ denotes the set of vertices of the polyhedron P given by

$$P = \{x = (x_1, x_2, \ldots, x_k)^{\mathrm{T}} : \\ A_k x_k \leqslant b_k, \quad x_k \geqslant 0, \; 1 \leqslant k \leqslant K\}. \qquad (15)$$

Also, for all nonnegative real number u, denote by $c(u)$ the vector

$$c(u) = (c_1 - ud_1, c_2 - ud_2, \ldots, c_K - ud_K)^{\mathrm{T}}. \qquad (16)$$

Dichotomous decomposition algorithm (*DDA*)

Step 1. Dualize in (GLP) the coupling constraint in a Lagrangian fashion to obtain the following equivalent program:

(DLP)

$$\min_{u \geqslant 0} \max_{x_k} \sum_k c_k^{\mathrm{T}} x_k + u\left(b - \sum_k d_k^{\mathrm{T}} x_k\right) \big| x = (x_k)_k \in P.$$

Step 2. Reformulate (DLP) as

(L)

$$\min_{u,z} \quad z$$

$$\text{s.t.} \quad z \geqslant ub + c(u)^{\mathrm{T}} x_j, \quad 1 \leqslant j \leqslant J \text{ (constr)},$$

$$u \geqslant 0.$$

Step 3. Constraint Generation Procedure (CGP)

!H^s will be a set of constraints:

3.1. Set $s = 0$. Initialize H^0 as $H^0 = \{z \geqslant c^{\mathrm{T}} x^0 + u(b - d^{\mathrm{T}} x^0),\ z \geqslant c^{\mathrm{T}} x^\infty + u(b - d^{\mathrm{T}} x^\infty)\}$.

3.2. Set $s = s + 1$. Let (L^s) denote the LP defined by $\{\min_{u \geqslant 0, z} z$ under $H^{s-1}\}$, where H^{s-1} is the set of explicitly considered constraints.

3.3. Find the optimal solution of (L^s).

3.4. If the pair (u^s, z^s) obtained in Step 3.3 satisfies all the constraints (constr), then it is an optimal solution to (L). Otherwise, obtain H^s by augmenting H^{s-1} by a violated constraint (see details below). Go to Step 3.2.

End.

Note that the algorithm will perform a dichotomous type of search to generate a sequence of approximations of u^*, the optimal solution to (L) and will stop as soon as equality is reached.

Let

$$w^s = \mathrm{Max}_{j=1,\ldots,J}\{u^s b + c(u^s)^{\mathrm{T}} x^j\}.$$

Clearly, the test of Step 3.4 is equivalent to comparing z^s to w^s. Finding w^s requires solving the LP defined by

$$(\mathrm{LP}^s) \qquad \mathrm{Max} \sum_k (c_k^{\mathrm{T}} - u^s d_k^{\mathrm{T}}) x_k + u^s b | A_k x_k \leqslant b_k, \quad x_k \geqslant 0,\ 1 \leqslant k \leqslant K.$$

Obviously, (LP^s) decomposes into K independent LPs of moderate sizes. x^s denotes an optimal solution of the (LP^s). Then, two cases may occur

(i) If $w^s > z^s$, then the constraint $z \geqslant ub + c(u)^{\mathrm{T}} x^s$ is violated, and the current solution is infeasible for (L).

(ii) If $w^s = z^s$, then (u^s, z^s) satisfies all constraints (constr) of (L), and is therefore optimal for (L).

Note that Step 3.4 is similar to finding a column of maximal marginal profit in the Dantzig–Wolfe decomposition algorithm. It is worth noting that Step 3.3 of DDA *does not* require explicitly solving an LP; rather, it can be carried out very simply. In fact, at any iteration s, H^s will contain only two constraints of opposite slopes, and the new optimal dual solution is simply obtained as the intersection of two lines. The process will continue until global optimality is reached. This is the key difference between the DDA suggested in this study and the standard Dantzig–Wolfe decomposition algorithm. Now, details on performing Step 3.3 of DDA are provided.

Consider the two particular vertices x^0 and x^∞, optimal solutions of LP^0 and LP^∞, where

(LP^0)

$$\mathrm{Max} \sum_k c_k^{\mathrm{T}} x_k | A_k x_k \leqslant b_k, \quad x_k \geqslant 0,\ 1 \leqslant k \leqslant K,$$

(LP^∞)

$$\mathrm{Min} \sum_k d_k^{\mathrm{T}} x_k | A_k x_k \leqslant b_k, \quad x_k \geqslant 0,\ 1 \leqslant k \leqslant K.$$

H^0 is defined by

$$H^0 = \{z \geqslant c^{\mathrm{T}} x^0 + u(b - d^{\mathrm{T}} x^0), z \geqslant c^{\mathrm{T}} x^\infty + u(b - d^{\mathrm{T}} x^\infty)\}.$$

Note that if $b - d^{\mathrm{T}} x^0 \geqslant 0$, then x^0 is feasible and thus optimal for the global program, and the marginal value of water will be 0. This is usually not the case as the paper deals with situations of water deficit. If not, then the line defined by $L^- = c^T x^0 + u(b - d^T x^0)$ will have a negative slope. Note also that if $b - d^T x^\infty \leqslant 0$, then the global program is infeasible (which is not the case). If not, then the line defined by $L^+ = c^T x^\infty + u(b - d^T x^\infty)$ will have a positive slope.

It is easy to see that the unique optimal solution to the linear program (L^1) is $u^1 = L^- \cap L^+$. If u^1 is not optimal for (L), then the new set of constraints H^1 is obtained by adding to H^0, the most violated constraint $z \geqslant c^T x^1 + u(b - d^T x^1)$. However, if $b - d^T x^1 \geqslant 0$, then by dropping from H^1 the constraint: $z \geqslant L^+$, the feasible set defined by H^1 remains unchanged. In this case, L^+ is updated by $L^+ = c^T x^1 + u(b - d^T x^1)$. Otherwise ($b - d^T x^1 < 0$), one can drop from H^1 the constraint: $z \geqslant L^-$. In this situation, L^- is updated by $L^- = c^T x^1 + u(b - d^T x^1)$. More generally, one can observe that, at iteration s, the optimal solution of (L^s) is $u^s = L^- \cap L^+$. If u^s is not optimal for (L), then H^s is obtained by adding to H^{s-1}, the constraint: $z \geqslant c^T x^s + u(b - d^T x^s)$. If $b - d^T x^s \geqslant 0$ then one can drop from H^s the constraint: $z \geqslant L^+$, and L^+ is updated by $L^+ = c^T x^s + u(b - d^T x^s)$. Otherwise, one can drop from H^s the constraint: $z \geqslant L^-$, and L^- is updated by $L^- = c^T x^s + u(b - d^T x^s)$.

As a consequence, at any iteration s, H^s will contain only two constraints ($z \geqslant L^+$ and $z \geqslant L^-$), with slopes of L^+ and L^- of opposite signs. Therefore, the new optimal dual solution is again simply obtained as the intersection of two lines. The process will continue until global optimality is reached. At this stage, if $L^+ = c^T x^s + u(b - d^T x^s)$, and $L^- = c^T x^t + u(b - d^T x^t)$, then the optimal solution of (GLP) is given by

$$x_k^* = \frac{b + \sum_k d_k x_k^s}{\sum_k d_k x_k^t + \sum_k d_k x_k^s} x_k^{t1} + \frac{\sum_k d_k x_k^t - b}{\sum_k d_k x_k^{t1} + \sum_k d_k x_k^s} x_k^s, \quad 1 \leqslant k \leqslant K. \qquad (17)$$

7. Insight on the economic interpretation of the marginal value of water u^*

Three interesting situations will be discussed.

(1) *New water pricing*. The price of water in Tunisia is heavily subsidized, in order to encourage agricultural activities. However, such a cheap price has led to a lot of wastage and unprofitable activities. Therefore, many researchers are calling for a new pricing that reflects (at least partially) the real value of water, reduces the wastage, and preserves the resource. Now, if a new pricing is suggested, then for the optimal cropping activities of the region to remain profitable, the new price of water should not exceed the old one by more than u^*.

(2) *Introducing new crops*. The only real constraints of the model are water and land. However, land is not considered as a limiting factor. (It is often the case that the optimal cropping policy suggests having some of the land uncropped, as a result of deficit in water and probably in other input factors such as capital.) Therefore, the marginal value of the land would often be zero. In this case, it is possible to make some rough computations to evaluate whether it is worthwhile to introduce new crops (with suggested levels of irrigation) in the cropping plan by considering their respective water consumption per unit area, the estimated unit benefits and the marginal value of water.

(3) *Reconsidering irrigation water allocation for the entire region*. As many activities in different sectors compete for water, also as several neighboring regions compete for irrigation water, it is possible to reconsider the allocation of irrigation water of the region in order to improve efficiency of the overall stock of water. If u^* is high when compared with the marginal value of water in other activities/regions, then it is worthwhile to increase LEV. However, if u^* is low, it will be worth increasing the allocation of other activities at the expense of reducing LEV. These are some meaningful situations where u^* can provide tools for important decisions.

8. Conclusion

In this paper, a model on cropping patterns is developed under water deficits. The model consists of a multi-phase decision process that is formulated as a large scale LP. It allows the selection of the most profitable crops, the optimal area and the irrigation level for each selected crop, as well as the specific locations for growing these selected crops. In particular, the model helps growers benefit the most from the initial state by considering the relationship between a crop yield and its predecessors. In addition, the model includes important factors that are either completely neglected in the literature or mentioned without quantitative analyses. Considering both annual and seasonal crops in the same cropping patterns, considering a same crop more than once, with different irrigation levels, and incorporating explicitly the dependence of crop yield on crop predecessors are examples of these factors. A small example, based on hypothetical data, is offered for illustration.

The model gives rise to two types of problems: total water allocation at the region level and cropping patterns of individual growers. The model deals with the situation where upper decision-makers (at the region level) are interested only in maximizing the total profit to be generated by the entire region. In this case, the global problem will be a multi-divisional problem, where each subproblem will simply be some grower problem. Therefore, an efficient decomposition algorithm, DDA, is developed to solve the global problem. The algorithm also has the advantage of explicitly giving the marginal value of water that can be helpful in a number of important decisions, mainly concerning reallocation of water among crops, regions, and even sectors.

Work in progress by the authors Haouari and Azaiez (2000) will provide a detailed application of the model based on a real case study. Such a real application would be very helpful in proving how the suggested model could improve existing cropping policies. Future work will extend the cropping policies to a larger planning horizon (of several years). This will allow in particular to include crop rotation in the model. Another important extension would assume a stochastic supply of water (suggesting a probabilistic approach). Other possible extensions would include the minimum size for growing a given crop, the maximum number of crops a grower can manage, and the explicit incorporation of some parameters such as costs and market prices, as well as their perturbation as a result of production changes.

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