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Ecological Economics 48 (2004) 451–467
ECOLOGICAL ECONOMICS
www.elsevier.com/locate/ecolecon

ANALYSIS

Functions, commodities and environmental impacts in an ecological–economic model

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Received 12 May 2003; received in revised form 20 October 2003; accepted 23 October 2003

Abstract

In contrast to macroscopic tools, life cycle assessment (LCA) starts from the microstructure of an economic system: the production and consumption of functional flows. Due to the level of resolution required for function-level details, the model used for LCA has relied on process-specific data and has treated the product system as a stand-alone system instead of a system embedded within a broader economic system. This separation causes various problems, including incompleteness of the system and loss of applicability for a variety of analytical tools developed for LCA or economic models. This study aims to link the functional flow-based, micro-level LCA system to its embedding, commodity-based, meso- or macro-level economic system represented by input–output accounts, resulting in a comprehensive ecological–economic model within a consistent and flexible mathematical framework. For this purpose, the LCA computational structure is reformulated into a functional flow by process framework and reintroduced in the context of the input–output tradition. It is argued that the model presented here overcomes the problem of incompleteness of the system and enables various analytical tools developed for LCA or input–output analysis (IOA) to be utilised for further analysis. The applicability of the model for cleaner production and supply chain management is demonstrated using a simplified product system and structural path analysis as an example.

Keywords: Life cycle assessment (LCA); Input–output analysis (IOA); Hybrid analysis; System boundary; Comprehensive ecological–economic model; Functional flow

1. Introduction

Since the beginning of the last century, ecologists have been borrowing ideas and concepts from economics. Entities in an ecosystem are viewed as economic agents that process materials and energy, terms like producers and consumers have been widely adopted in ecology, and productivity and efficiency have become the main interests of ecologists (Worster, 1994).

Recently, there has been a movement in the opposite direction as well, in that principles of ecology have come to be utilised for industrial and economic systems. In the field of industrial ecology, for instance, an industrial system is viewed as a self-organising system, with interest focusing on its metabolism, which describes how materials and energy are processed, used and disposed of (see e.g. Ayres and Ayres, 1996; Graedel and Allenby, 1995). One pillar of the industrial metabolism discourse is the role of commodities. An obvious role of commodities and their supply network is the circulation of materials and

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doi:10.1016/j.ecolecon.2003.10.013

energy in an economy, which, in turn, generates pollutants and wastes and causes environmental impacts. The circulation of materials and energy and the generation of pollutants and wastes characterise the physical terms of commodities in an economic system, while what is more interesting from the economics side is the utilities or functions of commodities, which actually lead consumers to demand the commodity (Sen, 1999). Thus, it is not only the physical implications but also the functions of commodities that are essential in describing the metabolic structure of an economic system.

The present paper concerns the connection between the physical and the functional terms of commodities in a model which is called here ecological–economic, and which describes materials and energy exchanges within and between the economic system and the environment. The model is structured on the basis of an extended framework of life cycle assessment (LCA).

LCA describes the microstructure of an ecological–economic system, its main focus being the production and consumption of a functional flow and its environmental consequences (Guinée et al., 2002). This bottom-up approach concerns prevention of pollution at the level of production and consumption of a specific product, or more precisely, a specific function of a product, through eco-labelling, process redesign, cleaner production, supply chain management, etc. Thus, the model needed by LCA should, on the one hand, be able to describe individual processes and their interrelations in detail and, on the other hand, be system-encompassing. In practice, however, the two objectives, i.e. level of detail and system completeness, are difficult to attain at the same time. As the number of inputs increases through upstream processes, system analysts have to stop compiling upstream data at a certain stage, or they have to use more aggregated data, thus losing process specificity. Most LCA studies opt for process specificity, rather than for completeness of the system.

Attempts to overcome the incompleteness of a process analysis by using input–output analysis (IOA) are generally referred to as hybrid analysis (see e.g. Bullard et al., 1978). However, the model structures of process analysis and IOA have not been fully integrated in hybrid analyses so far. Hybrid analysis, including hybrid energy analysis, utilises matrix representation only for the input–output part, while process analysis is dealt with separately by using a process flow diagram approach. This separation in the computational structure imposes several constraints on hybrid models.

The main questions addressed by the present paper are 'how can we better link the microstructure of an ecological–economic system dealt with in LCA to its embedding economic system?', and 'what are the relevant forms and structures of the LCA and IOA that are to be integrated?'. To answer these questions, I present a model that integrates the computational structures of IOA and LCA within a consistent framework, enabling various analytical tools to be applied to the model.

2. Survey of hybrid models

The general framework of hybrid analysis was introduced as early as the 1970s in the context of energy analysis. The discipline of energy analysis has used process analysis—or vertical analysis—and input–output based energy analysis in parallel for slightly different purposes (see IFIAS, 1974). It was Bullard and Pillati (1976); Bullard et al. (1978) who calculated the net energy requirements of a product by combining the results of process analysis and IOA. This allowed the incomplete system of process-based energy analyses to be significantly improved. In the field of input–output energy analysis, the approach developed by Bullard and his colleagues has become common practice, and many empirical studies are available (see e.g. Engelenburg et al., 1994; Wilting, 1996).

Input–output techniques have been studied as a tool for LCA since the early 1990s. Moriguchi et al. (1993) were the first to analyse the life-cycle CO_2 emissions of an automobile, using both the Japanese input–output table and process analysis. Since the study by Moriguchi et al. (1993), there have been many LCA studies and software tools using input–output techniques, including Lave et al. (1995); Treloar (1997); Marheineke et al. (1998); Hendrickson et al. (1998); Joshi (2000); Suh and Huppes (2002) (see Suh et al., (2004) for detail).

Although the area of application, the level of aggregation and the number of pollutants covered in these studies vary, the result of a hybrid analysis has been the simple sum of process analysis and input–output based analysis. In other words, the computational structure of LCA has not been fully integrated with that of IOA, which creates several difficulties.

One problem is that the commodity flows described in the process-based system are, in principle, also described in the input–output system, which leads to misspecification through double counting.[1] Furthermore, current hybrid techniques are unable to systematically model the interactive relationship between process-based system and input–output system through both inputs and outputs. For example, in analysing different options of reusing or recycling wastes from the disposal phase of a product system, each option simultaneously changes the input structure not only of the process-based system but also of the input–output based system. It is important to note that the relationship between the process-based system and the input–output based system, representing the microstructure of the commodity flows web and the wider, embedding economy, respectively, is interactive, and that an integrated model is required to represent this interactive relation.

In addition, there are also practical difficulties in using analytical tools consistently. Various analytical tools have been developed for LCA or IOA, including structural decomposition analysis, structural path analysis, field of influence analysis, Monte Carlo simulation, perturbation analysis, linear programming, sensitivity analysis etc. In implementing the computations for these analyses, each system has to be treated differently, due to the difference in computational structure, resulting in loss of consistency.

[1] Suppose a simplified case that an industry sector in an input–output table, for instance, automobile manufacturing includes passenger cars and trucks, which shares 80% and 20% of the total sales, respectively. A process-based LCA study compiled process-specific data for production of passenger cars within the process-based system. There are, however, some missing inputs of trucks, and they are to be linked to a relevant input–output sector, which is, in this case, the automobile manufacturing, through a hybrid model. However, the sector in an input–output based system still includes the passenger car manufacturing processes, so that those inputs are misspecified as 80% of passenger cars and only 20% trucks.

3. Computational structures of IOA and LCA

3.1. Input–output analysis (IOA)

The basic computational structure of input–output models is briefly discussed here, based on Leontief (1936); Leontief (1941). Leontief's model starts with transaction records between industries within a national economy.[2] Let us define the transaction matrix $\mathbf{Z}$ such that $(\mathbf{Z})_{ij}$ indicates the amount of domestic industry output purchased by industry j from domestic industry i in monetary terms. By assuming that each industry produces only one distinct output, we obtain a square transaction matrix $\mathbf{Z}$. It is a convention in input–output economics that the transaction matrix is converted into a coefficient matrix, which is generally called the direct requirement matrix. Let $\mathbf{g}$ be the total industry output vector such that $(\mathbf{g})_i$ shows the amount of the total output by industry i, which is the sum of the total output of the industry that is consumed by domestic industries, households and export. An industry-by-industry direct requirements matrix, $\mathbf{A}$, is then defined by

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{g}}^{-1}. \tag{1}$$

The hat (ˆ) in (1) makes a diagonal matrix out of a vector, such that $(\mathbf{g})_i$ is located at $(\hat{\mathbf{g}})_{ii}$ and $(\hat{\mathbf{g}})_{ij} = 0$, where $i \neq j$. An element of the direct requirements matrix $(\mathbf{A})_{ij}$ shows the amount of industry output i required by industry j to produce a unit of its output. An equality

$$\mathbf{g} - \mathbf{A}\mathbf{g} = \mathbf{f} \tag{2}$$

holds in a national economy where the total amount of domestic industry output produced ($\mathbf{g}$) minus the total industry output consumed by domestic industries ($\mathbf{Ag}$) equals the amount of industry output consumed by final consumers and export ($\mathbf{f}$) (Leontief, 1941). Rearranging (2) gives

$$\mathbf{g} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} \tag{3}$$

for non-singular $(\mathbf{I} - \mathbf{A})$. Assuming further that the input structure of each industry does not change when

[2] Imports and capital investment have been omitted here for the sake of simplicity, but are dealt with in a later section.

it changes its scale, meaning that input coefficients are scale-insensitive, the total amount of industry output $\mathbf{x}$ required by an arbitrary final demand for industry output $\mathbf{y}$ is calculated by:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}. \tag{4}$$

The amount of industry-wide environmental intervention generated by an arbitrary final demand for industry output $\mathbf{y}$ is then calculated by:

$$\mathbf{q} = \mathbf{B}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}, \tag{5}$$

where $\mathbf{B}$ is the environmental intervention by industries matrix, in which an element $(\mathbf{B})_{ij}$ denotes the amount of environmental intervention i generated in producing a unit of output by industry j (cf. Ayres and Kneese, 1969, p. 288; Isard, 1968; Leontief, 1970).

The basic computation of IOA described above is based on the assumption of one distinct output by each industry. In practice, however, each industry produces primary products and secondary products as well as scrap. Furthermore, the output by each industry does not have to be unique to that industry, so that the commodity produced by an industry may also be produced by another industry. This problem has led to theoretical improvements of IOA toward commodity-based accounting. Input–output accounts based on commodity instead of industry output have been developed by improving the basic accounting scheme known as supply and use framework (Stone et al., 1963). This supply and use framework then enables the creation of a commodity-by-commodity-based input–output model (see e.g. Konijn, 1994; Kop Jansen and ten Raa, 1990; Londero, 1999; Steenge, 1990; ten Raa, 1988; ten Raa et al., 1984).

3.2. Life cycle assessment (LCA)

A basic question in life cycle inventory (LCI) is 'how much of environmental intervention is generated to fulfil a particular function?'. Therefore, LCA basically deals with physical, function-based systems with much higher resolution in terms of process interdependence than IOA. As in energy analysis, the computation of the total environmental intervention in LCI started with a process flow diagram approach, and this approach has remained the most common practice in LCA study and software tools (see e.g. Consoli et al., 1993; Fava et al., 1991; US EPA, 1993).

Process flow diagrams show how the processes of a product system are interconnected through commodity flows. In process flow diagrams, boxes generally represent processes, while arrows indicate the commodity flows. Using plain algebra, the amount of commodities required to supply a certain functional unit is obtained, and an LCI is calculated by multiplying by the amount of environmental intervention required to produce them. Although the process flow diagram approach is attractively simple, it has its limitations when dealing with a complex system, where both inputs and outputs are interconnected between processes, thus establishing many internal loops. Although a few techniques, including iterative methods and infinite geometric progression, can be used to solve this problem, the process flow diagram approach is generally not adequate for a complex system (see Suh and Huppes, in press for an overview).

A more flexible mathematical expression of process interrelations and its use has been introduced to LCI computation by Heijungs (1994). Heijungs' approach uses a commodity-by-process model based on physical flows between processes. The computational structure of LCA in Heijungs (1994) is further illustrated here by an example. A hypothetical system of toaster use is shown in Fig. 1.

The quantities of the commodity flows of each process are shown in Table 1.

In the product system shown in Fig. 1 and Table 1, one toaster is produced using 2 kg of

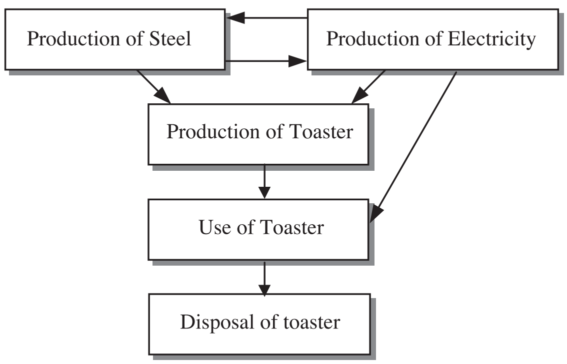

Fig. 1. Process flow diagram with internal commodity flow loops.

Table 1
Inputs and outputs of physical flows by processes (inputs are indicated by negative sign, outputs by positive)

	Production of steel	Production of electricity	Production of toaster	Use of toaster	Disposal of toaster
Steel (kg)	1	− 0.5	− 2	0	0
Electricity (kW h)	− 0.5	1	− 0.1	− 1	0
Toaster (unit)	0	0	1	− 1	0
Toast (piece)	0	0	0	1000	0
Waste (kg waste)	0	0	0	1	− 1

steel and 0.1 kWh of electricity, after which it is used and then disposed of. Let us assume that such a toaster produces 1000 pieces of toast during its lifetime.

Note that the product system shown in Fig. 1 and Table 1 is already difficult to analyse using the process flow diagram approach, since it has internal loops between the processes of 'steel production' and 'electricity production'. The model by Heijungs (1994) can solve the LCI problem based on the physical flow relations between processes. The physical flows between the processes in Table 1 can be summarised in matrix form by

$$\tilde{\mathbf{A}} = \begin{bmatrix} 1 & -0.5 & -2 & 0 & 0 \\ -0.5 & 1 & -0.1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}. \qquad (6)$$

From left to right, the columns indicate steel production, electricity production, toaster production, the use of the toaster and the disposal of the toaster, while the rows are assigned from top to bottom to steel (kg), electricity (kWh), toaster (unit), toast (piece) and waste (kg).

Suppose that these processes generate CO_2 gas to produce their outputs, which can be described by a row vector in kg,

$$\tilde{\mathbf{B}} = [\,1 \quad 4 \quad 2 \quad 1 \quad 0.5\,]. \qquad (7)$$

The functional unit of this product system is given by 1000 pieces of toast, which can be shown by a column vector,

$$\tilde{\mathbf{y}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1000 \\ 0 \end{bmatrix}. \qquad (8)$$

The total life cycle CO_2 emission by this product system is then calculated by

$$\tilde{\mathbf{q}} = \tilde{\mathbf{B}}\tilde{\mathbf{A}}^{-1}\tilde{\mathbf{y}} = 18.1. \qquad (9)$$

The contribution to the total CO_2 emission is distributed over the processes as shown in Fig. 2.

Since the study by Heijungs (1994), a number of analytical tools and improvements have been developed for LCA computation, including the use of pseudo-inverse, perturbation analysis etc. and a few

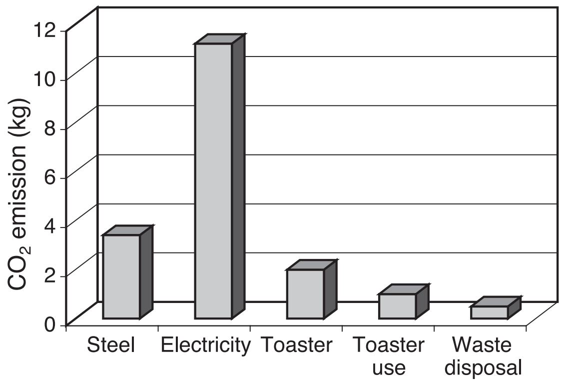

Fig. 2. Result of process-based LCA-CO_2 contributions by processes.

software programs and databases have been created using the Eq. (9) (see Heijungs and Suh, 2002).

4. IOA and LCA for integrated hybrid models

It is important to note that both environmental IOA and LCA on their own run into serious problems as ecological–economic models for detailed environmental systems analysis. Although the input–output model covers a wider system, including all interactions between industries within a national economy, the result is an average value of a set of processes, while LCA also has a fundamental problem of truncation. Therefore, a model is required that reveals the microstructure of the important parts of a product system and, at the same time, covers the entire economic system.

As shown in the previous section, the systems that IOA and LCA deal with have a lot in common. Despite the similarities, however, the systems that LCA deals with also have more than a few important differences: there are no annual 'transaction' records available, quantities are in physical units, it concerns the direction of physical flows instead of that of money flows, it contains use and end-of-life stages, it primarily concerns the function of a system etc. These differences provided enough reasons for LCA researchers to independently develop slightly different computational structures from that of IOA (see e.g. Projektgemeinschaft Lebenswegbilanzen, 1991; Heijungs, 1994; Heijungs and Suh, 2002).[3] Those unique features indeed make it difficult for an LCA to be directly integrated with IOA in current form (see Section 6). The rest of this section discusses the relevant formats and adaptations required to integrate IOA and LCA, for which I tried to reformulate the LCA model structure in the context of input–output economics' tradition but only with a different set-up namely, functional flow-by-process accounts.

The format of the input–output table used here is the commodity-by-commodity format derived from make and use matrices. The industry-by-industry format is less applicable in the current model, due to the aggregation of commodities in an industry output. Moreover, the industry that produces an input material for a process is generally less fully known to the LCA practitioners than the commodity itself. In order to distinguish our format from the industry-by-industry matrix—$\mathbf{A}$ in (4)—we use $\mathbf{A}'$ for the commodity-by-commodity direct requirements matrix.

Furthermore, the input–output technology coefficient matrix should include domestic and imported capital goods, as well as domestic and imported current products, such that

$$\mathbf{A}'_{*} = \mathbf{A}'_{\mathrm{DC}} + \mathbf{A}'_{\mathrm{DK}} + \mathbf{A}'_{\mathrm{IC}} + \mathbf{A}'_{\mathrm{IK}}. \qquad (10)$$

Matrices $\mathbf{A}'_{\mathrm{DC}}$, $\mathbf{A}'_{\mathrm{DK}}$, $\mathbf{A}'_{\mathrm{IC}}$, and $\mathbf{A}'_{\mathrm{IK}}$, are the commodity-by-commodity direct requirements matrices for domestic current products, domestic capital goods, imported current products and imported capital goods, respectively, with imported current and capital goods assigned to the relevant domestic indices. An assumption here is that the technology and the economic structure used to produce these imported products and capital goods are exactly the same as the domestic ones (see Lenzen, 2001 for a comprehensive treatment).

Since the available input–output table is generally several years old, the prices should be rescaled to current price levels as well. The input–output technical coefficient matrix $\mathbf{A}'_{*}$, in (10) is rescaled to a coefficient matrix $\mathbf{A}'_{**}$, with current prices by

$$\mathbf{A}'_{**} = \hat{\mathbf{p}}\mathbf{A}'_{*}\hat{\mathbf{p}}^{-1}, \qquad (11)$$

where vector $\mathbf{p}$ shows the price ratio of each commodity between current and the year on which the input–output table is based. The input–output technology coefficient matrix used below refers to the matrix with price rescaling by (11). The environmental intervention-by-industry matrix $\mathbf{B}$ in (5) should also be adjusted to an environmental intervention-by-commodity matrix, $\mathbf{B}'_{**}$, by applying an allocation model and reflecting price differences in accordance with the procedure outlined above.

The LCA model shown in the previous section cannot be used directly for integration either. Here we introduce a computational structure of LCI, based on

[3] Many LCA case studies, databases and software tools developed and used are, however, based on Heijungs (1994); Heijungs and Suh (2002).

the supply–demand relationship of functional flows that will be fully integrated with IOA in the next section.

LCA deals with the production and consumption of functions by processes (ISO, 1998). In this context, a *function* refers to a useful trait of a *commodity*, and a commodity may have multiple functions (cf. Lancaster, 1966). A shampoo, for instance, may have multiple functions, such as 'cleaning', 'conditioning', 'moisturising', 'protein supply' etc. In order to refer to a quantitative function flow, we will use the term *functional flow*. Since a function is the basis of computation in LCA, it means that if two shampoos are studied, they should be compared on the basis of equivalent function(s) by subtracting or adding additional function(s). A set of functions can also be collectively referred as a function if individual functions need not be distinguished. The level of resolution used in defining a function depends on the objective of the study. If it is unnecessary to distinguish all functions of a commodity, these functional flows can be represented by the flow of the commodity. In this respect, commodity flows can be good surrogates for functional flows in many cases, though not in all.

Let us define a *process* as a unit activity that produces function(s). In other words, each process produces at least one functional flow. A process exists because there is a *demand* for its functional flow from outside the process. We shall use the demand as the basis of the *imputation* of environmental intervention and input requirements. In this context, a process may refer to an industrial process as well as a household activity or a post-consumer activity (cf. Sen, 1999).

A process also demands functional flows produced by other processes for its operation. Production and consumption of functions by a product system can be expressed by a matrix $\tilde{\mathbf{Z}}*$, of which a column, $(\tilde{\mathbf{Z}}*)_{\bullet j}$ represents the amount of functional flows consumed and produced over a certain period of operation by process j being given in the relevant physical unit. Thus, a household process like the 'use of TV' may have 'hour of TV watching' as its functional flow output and 'kWh of electricity' as an input. One clear advantage of using physical units is that functional flow relations between processes are not distorted by price fluctuations over time or across consuming processes.

In compiling the vector of functional flows of a process j, $(\tilde{\mathbf{Z}}*)_{\bullet j}$, the production of functional flow is shown as a positive value, while consumption is given a negative sign. Note here that, although these flows are expressed in physical units, the direction of a flow may differ from that of the physical flow.[4] The direction of a waste flow, for example, is from industrial processes to waste treatment processes in terms of physical waste flows, while the direction may be the opposite in terms of functional flows, which may be 'kg waste treatment service' (cf. Heijungs, 1994). Nevertheless, it is also possible that the waste treatment facility purchases the waste to produce other commodities such as heat or recycled products. In this case the waste become a functional output of the industrial process.

In most cases, except for household processes, the direction of a functional flow between two processes is clearly indicated by monetary transaction flows—if the waste treatment facility purchased the waste for, e.g. its heat content, then the waste is no longer waste, but a functional output that has lower economic value. There are some cases in which the direction of a functional flow may be unclear from the monetary transaction flow. Suppose, for example, that a waste recycling process receives waste materials from a demolition process for free. In this case, there is no transaction flow between the two processes. The functional flows, however, can be understood to have both directions: the waste recycling process purchases the waste materials from the demolition process, and the demolition process purchases the waste treatment services from the waste recycling process at exactly the same price that they would have to pay to one another. We will refer to the relationship between processes in producing and consuming functional flows as a supply–demand relationship between the processes, and we will use this relationship in imputing functional flow inputs and environmental interventions by a process, i.e. the demand for a functional flow by a process will get not only the functional flow but also part of the inputs used and environmental interventions caused by the process in producing the functional flow.

We also make a *steady-state assumption* in compiling the vector of the functional flows of a process. We assume that processes are operated under complete steady-state conditions. In reality, of course, hardly any industrial or consumer process is operated under complete steady-state conditions—processes

[4] LCA case studies and databases, so far, are based on the direction of physical flows.

may be subject to changes in production volume and degradation in performance over time. The steady-state condition here, however, means that we look at a period of process operation that is long enough to cover all the abnormalities and short enough to represent current operating conditions, and that we distribute all these abnormalities homogeneously over a given period of time, resulting in an averaged typical input–output ratio for each process. In contrast to the input–output table, the absolute value of the time period chosen for each process may differ between processes. We define a vector called *basis period of steady-state approximation* $\mathbf{t}$, of which an element $(\mathbf{t})_i$ shows the size of the temporal window used for the steady-state approximation for process i.

Let $\tilde{\mathbf{Z}}*$ be a functional flows by processes matrix such that $(\tilde{\mathbf{Z}}*)_{ij}$ is the amount of functional flow i used or produced by process j during the period of time that has been determined as the basis of steady-state approximation for process j. Note that the matrix $\tilde{\mathbf{Z}}*$ may have more than one positive value in each column, and may also be rectangular. A rectangular $\tilde{\mathbf{Z}}*$ should be further treated to make it square. The rectangularity problem is generally caused by the difference between the row and column indices. The matrix $\tilde{\mathbf{Z}}*$, for instance, has functional flows as its row indices and processes as its columns. The procedure to transform a functional flow-by-process matrix into a functional flow-by-production of a functional flow matrix is referred to as *allocation* here (cf. Heijungs and Frischknecht, 1998; Weidema, 2001). Details of the allocation procedure in this context will not be dealt with here, but can be found elsewhere (Suh, 2001a). For the sake of simplicity, we assume that $\tilde{\mathbf{Z}}*$ is square.

Since we have compiled $\tilde{\mathbf{Z}}*$ by assuming that each process is operated under complete steady-state conditions, the choice of a temporal window of process operation that is smaller than the basis period of steady-state approximation will not make any difference for the ratio between each input and output. However, it is convenient to define a unit operation time for each process. The absolute value of the unit operation time for a process may vary across processes. A vector called unit operation time $\mathbf{u}$ is defined such that $(\mathbf{u})_i$ shows the unit operation time chosen for process i, where

$$\mathbf{t} \geq \mathbf{u}. \tag{12}$$

For instance, the unit operation time can be chosen in such a way that the functional flow output by each process becomes 1 (see e.g. Heijungs, 1994). The basis period of steady-state approximation can then be expressed in terms of the unit operation time, such that

$$\mathbf{t} = \hat{\mathbf{u}}\tilde{\mathbf{g}}, \tag{13}$$

where $\tilde{\mathbf{g}}$ is a vector of time ratio by processes such that $(\tilde{\mathbf{g}})_i$ shows the unit operation time, $(\mathbf{u})_i$ as a ratio of basis period of steady-state approximation for process i. Rearranging (13) gives

$$\tilde{\mathbf{g}} = \hat{\mathbf{u}}^{-1}\mathbf{t}. \tag{14}$$

We can now define the functional flow-by-process LCA technology coefficient matrix $\tilde{\mathbf{A}}*$ by

$$\tilde{\mathbf{A}}* = \tilde{\mathbf{Z}}*(\hat{\tilde{\mathbf{g}}})^{-1}, \tag{15}$$

where $(\tilde{\mathbf{A}}*)_{ij}$ is the physical amount of functional flow i used or produced by process j during the unit operation time chosen. Again, a negative sign is assigned to the use of functional flow and a positive value to production. Note further that, unlike the input–output technology coefficient matrix, the LCA coefficient matrix has no values for self-consumption, which is located on the main diagonal of the intermediate part of an input–output table. The idea of self-consumption is in fact a statistical artefact, due to the level of aggregation in industry classification, which is not generally the case for LCA.[5] The amount of functional flow delivered outside the system during the basis period of steady state approximation is then calculated by

$$\tilde{\mathbf{A}}*\tilde{\mathbf{g}} = \tilde{\mathbf{f}}. \tag{16}$$

Note that an identity

$$\tilde{\mathbf{f}} = \tilde{\mathbf{Z}}*\mathbf{i} \tag{17}$$

[5] It also conflicts the assumption of only one homogeneous output per each sector, since it is not well imaginable for an industry to buy the same product that the industry produces (see Georgescu-Roegen, 1971; Lenzen, 2001).

holds, where $\mathbf{i}$ is a summation vector with only one in a relevant dimension, and $\tilde{\mathbf{f}}$ is the total production of functional flow. Rearranging (16) gives

$$\tilde{\mathbf{g}} = \tilde{\mathbf{A}}_*^{-1}\tilde{\mathbf{f}} \tag{18}$$

for non-singular $\tilde{\mathbf{A}}_*$. Assuming that the coefficients of technology coefficient matrix in (18) do not change as the amount of functional flow delivered outside the system changes, the amount of unit operation time $\tilde{\mathbf{x}}$ required to produce an arbitrary final demand for functional flow $\tilde{\mathbf{y}}$ is calculated by

$$\tilde{\mathbf{x}} = \tilde{\mathbf{A}}_*^{-1}\tilde{\mathbf{y}}. \tag{19}$$

The total environmental intervention due to an arbitrary final demand is then given by

$$\tilde{\mathbf{q}} = \tilde{\mathbf{B}}\tilde{\mathbf{A}}_*^{-1}\tilde{\mathbf{y}}, \tag{20}$$

where $\tilde{\mathbf{B}}$ is the environmental intervention by process matrix, of which an element $(\tilde{\mathbf{B}})_{ij}$ shows the amount of environmental intervention i generated by process j during its unit operation time. Eq. (20) returns the amount of environmental intervention caused by the external demand for a particular functional flow of a product system using the imputation algorithm based on the supply–demand relationship.

5. Integrated hybrid LCA

5.1. The model

In the previous section we have prepared formats and computational structures for the further integration of IOA and LCA. In this section I present the framework of a hybrid model that fully integrates the input–output and LCA computational structures.

I start by defining upstream and downstream cut-off matrices. The *upstream cut-off by processes* matrix is derived by dividing the total bill of goods for the inputs that are not covered by a processes in a process-based system during the period of steady-state approximation by the total unit operation time of each process. The *downstream cut-off by functional flow* matrix is derived by dividing the annual sales of functional flow—in physical units that are relevant to each functional flow—by the production of each total commodity. In matrix notation this becomes

$$\mathbf{C}^{\mathrm{u}} = \tilde{\mathbf{Z}}_*^{\mathrm{u}\hat{\tilde{\mathbf{g}}}}-1 \tag{21}$$

and

$$\mathbf{C}^{\mathrm{d}} = \tilde{\mathbf{Z}}_*^{\mathrm{d}}(\hat{\mathbf{g}}_{***})^{-1}, \tag{22}$$

where $\tilde{\mathbf{Z}}_*^{\mathrm{u}}$ denotes the total amount of the cut-off commodity flows by processes during the period of steady-state approximation in monetary terms, and $\tilde{\mathbf{Z}}_*^{\mathrm{d}}$ denotes the amount of annual sales of functional flows from processes to input–output industries in relevant physical units (cf. Eqs. (1) and (15)). The vector $\mathbf{g}_{***}$ shows total domestically produced and imported current and capital goods, with price levels updated for the difference with the base year, and with the portion of commodity flows represented by the process-based system subtracted (see Appendix A). Note that the derivation of cut-off matrices has to be done in accordance with the type of basic price with which the transaction table has been compiled. If the basic transaction table is compiled on the basis of consumer's prices, then the bill of goods for each LCA process can be directly used to compile the upstream cut-off matrix. If the basic price type is the producer's price, the information from the bill of goods should be converted to producer's prices by subtracting the cost of transportation and the wholesale margin from the amount paid.[6] Skipping this procedure can introduce considerable levels of underestimation or overestimation in the final results.

The resulting upstream cut-off matrix $\mathbf{C}^{\mathrm{u}}$ is presented in such a way that $(\mathbf{C}^{\mathrm{u}})_{ij}$ shows the amount of

[6] Default values for the transportation cost and wholesale margin can be found from a use table of input–output accounts.

cut-off of input–output commodity i to process j during the unit operation time, in monetary terms. Similarly, the downstream cut-off matrix $\mathbf{C}^{\mathrm{d}}$ is presented in such a way that $(\mathbf{C}^{\mathrm{d}})_{ij}$ shows the amount of cut-off flows of functional flow i to input–output commodity j per unit of monetary value of its output, in relevant physical units.

Now we are ready to present the basic balancing equation for integrated hybrid analysis:

$$\begin{bmatrix} \tilde{\mathbf{A}}_{*} & -\mathbf{C}^{\mathrm{d}} \\ -\mathbf{C}^{\mathrm{u}} & \mathbf{I}-\mathbf{A}'_{***} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{g}} \\ \mathbf{g}_{***} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{f}} \\ \mathbf{f}_{***} \end{bmatrix}, \tag{23}$$

where $\mathbf{A}'_{***}$ denotes the commodity-by-commodity input–output technology coefficient matrix that includes domestic and imported current products and capital, with prices updated to current levels, and excluding the portion of commodity flows already covered by the process-based system (see Appendix A for the subtraction procedure) and $\mathbf{g}_{***}$ and $\mathbf{f}_{***}$ stand for the total production and the final demand for domestic and imported current products and capital, respectively, with prices updated and with commodity flows already covered by the process-based system subtracted. Eq. (23) shows that the amount of functional flow and input–output commodity produced, minus the amount used in the process-based system and in the input–output based system is equal to the amount delivered to the final consumers. Attention must be paid to the units of the coefficient matrix shown in (23), since all the submatrices differ from each other in terms of units. The LCA technical coefficient matrix $\tilde{\mathbf{A}}_{*}$ is expressed in various physical units per unit operation time for each process, while the input–output technical coefficient matrix $\mathbf{A}'_{***}$ is in monetary units per unit output for each input–output commodity in monetary terms, $\mathbf{C}^{\mathrm{u}}$ is in monetary units per unit operation time for each process, and $\mathbf{C}^{\mathrm{d}}$ is in various physical units per unit of output for each input–output commodity in monetary terms. Rearranging (23) gives

$$\begin{bmatrix} \tilde{\mathbf{g}} \\ \mathbf{g}_{***} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}}_{*} & -\mathbf{C}^{\mathrm{d}} \\ -\mathbf{C}^{\mathrm{u}} & \mathbf{I}-\mathbf{A}'_{***} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{f}} \\ \mathbf{f}_{***} \end{bmatrix} \tag{24}$$

for a non-singular square matrix

$$\begin{bmatrix} \tilde{\mathbf{A}}_{*} & -\mathbf{C}^{\mathrm{d}} \\ -\mathbf{C}^{\mathrm{u}} & \mathbf{I}-\mathbf{A}'_{***} \end{bmatrix}.$$

Based on the linearity assumption we can further write

$$\begin{bmatrix} \tilde{\mathbf{x}} \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}}_{*} & -\mathbf{C}^{\mathrm{d}} \\ -\mathbf{C}^{\mathrm{u}} & \mathbf{I}-\mathbf{A}'_{***} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{y}} \\ \mathbf{0} \end{bmatrix}, \tag{25}$$

which gives the amount of unit operation time by processes and the amount of commodities by input–output based system for an arbitrary final demand for functional flow $\tilde{\mathbf{y}}$. The value of $\tilde{\mathbf{y}}$ shows the functional unit of an LCA study.

The amount of environmental intervention produced during the required unit operation time and the production of input–output commodities is calculated by

$$\bar{\mathbf{q}} = [\,\tilde{\mathbf{B}} \quad \mathbf{B}'_{***}\,] \begin{bmatrix} \tilde{\mathbf{x}} \\ \mathbf{x} \end{bmatrix}, \tag{26}$$

where $\bar{\mathbf{q}}$ is the environmental intervention produced by the hybrid system, $\tilde{\mathbf{B}}$ is the environmental intervention by processes matrix and $\mathbf{B}'_{***}$ is the environmental intervention by input–output commodities matrix (see Appendix A).

The overall computation of the integrated hybrid model is obtained by combining (25) and (26) as

$$\begin{aligned} \bar{\mathbf{q}} &= [\,\tilde{\mathbf{B}} \quad \mathbf{B}'_{***}\,] \begin{bmatrix} \tilde{\mathbf{A}}_{*} & -\mathbf{C}^{\mathrm{d}} \\ -\mathbf{C}^{\mathrm{u}} & \mathbf{I}-\mathbf{A}'_{***} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{y}} \\ \mathbf{0} \end{bmatrix}, \\ &= \bar{\mathbf{B}}\bar{\mathbf{A}}^{-1}\bar{\mathbf{y}} \end{aligned} \tag{27}$$

which represents a comprehensive ecological–economic model that integrates a functional flow-based system with a commodity-based system. The bar

($\overline{\ }$) indicates integrated hybrid matrices and vectors. Eq. (27) gives the total amount of environmental intervention resulting from the interaction between the functional flow-based system and the commodity-based system in both directions, in one consistent mathematical structure.

6. Application

In this section we apply the model developed in the previous sections to a simplified example to show how the model can provide information that can be used for cleaner production and supply chain management. We start with the same example shown in Fig. 1 and Table 1. The new technology coefficient matrix is given by

$$\tilde{\mathbf{A}}_{*} = \begin{bmatrix} 1 & -0.5 & -2 & 0 & 0 \\ -0.5 & 1 & -0.1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \tag{28}$$

Note that the technology matrix in Table 2 is exactly the same as that in Table 1 except for the changes in sign in the fourth and fifth column of the fifth row. That is due to the reformulation of the relationships between processes based on the supply–demand relations (see Section 4). Although the differences between the two systems in Tables 1 and 2 seem to be negligible, these small changes not only allow a consistency in imputation mechanism over the whole system but also prevent an arbitrary result when integrated with an input–output table. Now suppose that the product system under study has the following upstream cut-offs.

The economic system in which the product system in (28) is embedded is aggregated into six categories for simplicity: agricultural products, mining products, manufactured products, construction, financial services and other products and services. Table 3 shows the incoming commodity flows from this embedding economy to the processes that were previously neglected. For example, the production of 1 kg of steel uses \$0.1 worth of products that belong to the input–output commodities of mining products and manufacturing products in producer's prices. The monetary value of these cut-offs can be derived by dividing the total purchases during the basis period of steady-state approximation by the total unit operation time chosen for each process and subtracting the transportation cost and wholesale margin (Table 4).

Suppose further that the product system also has downstream cut-offs.

Downstream cut-off shows that the functional flows produced by the processes are supplied not only within the process-based system but also outside the system. For example, the disposal process supplies its services not only to the toaster use process within the LCA system boundary but also to manufacturing products and other products and services.

Table 2
Inputs and outputs of functional flows by processes

	Production of steel	Production of electricity	Production of toaster	Use of toaster	Disposal of toaster
Steel (kg)	1	− 0.5	− 2	0	0
Electricity (kWh)	− 0.5	1	− 0.1	− 1	0
Toaster (unit)	0	0	1	− 1	0
Toast (piece)	0	0	0	1000	0
Waste disposal service (kg disposal)	0	0	0	− 1	1

Table 3
Upstream cut-offs (in monetary units per unit operation time)

	Production of steel	Production of electricity	Production of toaster	Use of toaster	Disposal of toaster
Agricultural products ($)	0	0	0	0	0
Mining products ($)	0.1	0.01	0	0	0
Manufacturing products ($)	0.1	0.1	0	0	0
Construction ($)	0	0	0.1	0	0
Financial services ($)	0	0	0	0	0
Other products and services ($)	0	0	0.1	0	0.1

$\mathbf{C}^{u}$ and $\mathbf{C}^{d}$ are then given by

$$\mathbf{C}^{u} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.01 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0.1 \end{bmatrix} \tag{29}$$

and

$$\mathbf{C}^{d} = \begin{bmatrix} 0 & 0.015 & 0.01 & 0.05 & 0 & 0 \\ 0 & 0.05 & 0.08 & 0 & 0 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.05 & 0 & 0 & 0.03 \end{bmatrix} \tag{30}$$

The matrix in (31) shows the input–output technology coefficient matrix in consumer's prices, with relevant pre-treatments, including the addition of domestic capital goods and imported current and capital goods, updated prices and subtraction of the flow covered by the process-based system.

$$\mathbf{A}'_{***} = \begin{bmatrix} 0.3 & 0.1 & 0 & 0 & 0 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.1 & 0 & 0.2 \\ 0.2 & 0.2 & 0.3 & 0.2 & 0.1 & 0 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0.1 & 0 & 0 & 0.2 & 0.2 & 0.2 \\ 0.1 & 0.2 & 0.1 & 0.2 & 0.2 & 0.1 \end{bmatrix} \tag{31}$$

The matrix in (31) is a commodity-by-commodity aggregated technical coefficient matrix, its row and column indices being (1) agricultural products, (2) mining products, (3) manufactured products, (4) construction, (5) financial services, (6) other products and services (left to right and top to bottom). For instance, $(\mathbf{A}'_{***})_{23}$ is 0.2, which means that \$0.2 worth of mining products are required to produce \$1 worth of manufacturing product. The environmental intervention matrix showing the amount of CO_2 emission in kg per unit of input–output commodity output, adjusted for price differences and excluding those envi-

Table 4
Downstream cut-offs (in physical units per unit of production in monetary value)

	Agricultural products	Mining products	Manufacturing products	Construction	Financial services	Other products and services
Steel (kg)	0	0.015	0.01	0.05	0	0
Electricity (kWh)	0	0.05	0.08	0	0	0.01
Toaster (unit)	0	0	0	0	0	0
Toast (piece)	0	0	0	0	0	0
Waste disposal service (kg)	0	0	0.05	0	0	0.03

ronmental interventions already covered by processes, is given by

$$\mathbf{B}'_{***} = [0.5 \quad 3 \quad 2 \quad 0.1 \quad 0.1 \quad 1]. \tag{32}$$

Now the process-based LCA system is ready to be integrated with the input–output table through upstream and downstream cut-offs. Eq. (27) delivers the result, using the integrated hybrid system and taking into consideration the interactions between the process-based system and the input–output based system by

$$\bar{\mathbf{q}} = [\tilde{\mathbf{B}} \quad \mathbf{B}'_{***}] \begin{bmatrix} \tilde{\mathbf{A}}_{*} & -\mathbf{C}^{\mathrm{d}} \\ -\mathbf{C}^{\mathrm{u}} & \mathbf{I} - \mathbf{A}'_{***} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{y}} \\ 0 \end{bmatrix} = 30.015, \tag{33}$$

yielding a result that is about 66% higher than that of the process-based LCI in (9). The contributions to CO_2 emission by processes and input–output commodities are shown in Fig. 3.

Fig. 3 shows that cut-offs on mining and manufacturing products contribute a significant amount of life-cycle CO_2 emissions, which indicates the direction for further data collection efforts. The total CO_2 contribution by the process-based system is 20.027 kg and that by the input–output system is 9.988 kg, representing 67% and 33% of the total CO_2 contributions by the system, respectively. Note that the CO_2 contribution by the functional flow-based system has also been changed relative to the result shown in Fig. 2, due to the interaction between functional flow-based system and input–output based system.

However, the difference in total CO_2 emissions between (10) and (33) is not to be regarded as a general value: adaptation of the input–output table can produce much smaller or much greater differences depending on the product group studied and system boundary initially chosen. It is not the purpose of this example to generalise the typical amount of contribution by cut-offs.

The result can be further analysed using tools developed for LCA or IOA. We have performed a structural path analysis (Defourny and Thorbecke, 1984) as an example to show how the model can be utilised for cleaner production and supply chain management (see Suh, 2003 for details on the structural path analysis of a hybrid system). The result is shown in Table 5.

Table 5 lists the paths that contribute more than 1% of the total CO_2 emission by the product system. Six of the 13 paths end with ‘electricity’, while seven start from ‘toaster’, which indicates that electricity production is an important direct polluter, while the production of the toaster is an important indirect pollution

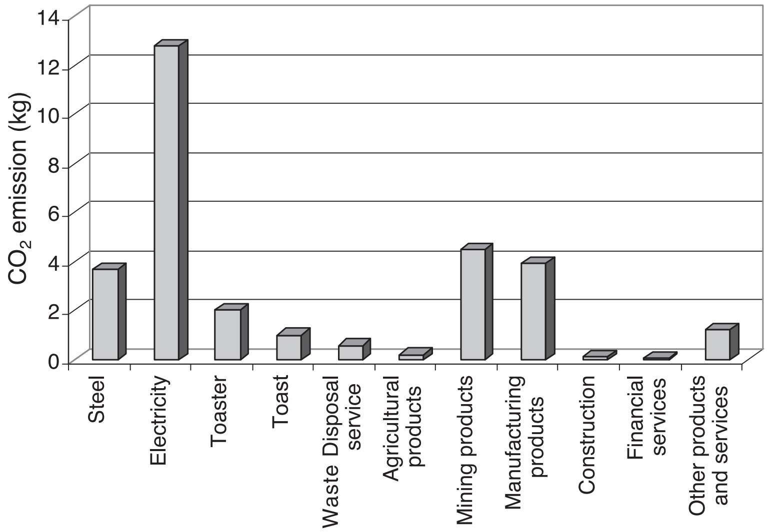

Fig. 3. Result of hybrid LCA-CO_2 contributions by functional flows and input–output commodities.

Table 5
Important paths in terms of CO_2 contribution

Rank	Path name*	Upstream order	CO_2 emission (kg)	Contribution to total (%)
1	El	1	4	13.3
2	El → St → Tr	3	4	13.3
3	Tr	1	2	6.7
4	St → Tr	2	2	6.7
5	Direct Emission	0	1	3.3
6	El → St → El	3	1	3.3
7	El → St → El → St → Tr	5	1	3.3
8	*Mn* → St → Tr	3	0.6	2.0
9	Wd	1	0.5	1.7
10	El → St	2	0.5	1.7
11	St → El → St → Tr	4	0.5	1.7
12	El → Tr	2	0.4	1.3
13	*Mf* → St → Tr	3	0.4	1.3
Sum			17.9	60

* El, electricity; St, steel; Tr, toaster; *Mn*, mining products; Wd, waste disposal; *Mf*, manufacturing products (italics for input–output commodities).

inducer. The path from 'steel' to 'toaster' in particular is identified as a strong linkage, as is also repeated in the list. The total direct and indirect contributions induced by the path from 'steel' to 'toaster' are indeed significant and are responsible for 49.4% of the total CO_2 emissions by the product system. From the perspective of cleaner production or supply chain management, the information shown in the right-hand side of the path is more important, since the controllability of the process operation or supply chain is much more limited in upstream processes. From this point of view, the result shown in Fig. 3, which shows the importance of the electricity production process, provides only limited information, and structural path analysis helps to further identify directions for effective environmental impact reduction efforts, which could for instance include the use of steel in the toaster production process.

Structural path analysis also indicates directions for further data collection efforts. The most important path from the embedding economy identified is the use of mining product to produce steel, which is used in the toaster production in the third-order upstream path (rank 8). Thus, processes requiring closer attention in terms of data collection include especially the mining activities that provide inputs for steel production (compare with Lenzen, 2002).

7. Conclusions and recommendations

LCA has been playing an increasingly important role in understanding the environmental impacts of a commodity. In doing so, the 'function' of commodities has been chosen as the basis in quantifying environmental impacts and comparing alternatives. This unique feature of LCA has not been, to my opinion, fully acknowledged in the model structure of LCAs. In this paper, I tried to reformulate the model structure of LCA as a functional flow-by-process framework and tried to bring it within the context of the input–output tradition. Based on the framework, I proposed a comprehensive ecological–economic model constructed by interconnecting a physical, functional flow-based micro-level system with a monetary, commodity-based broader economic system. The model enables full feedback loops to be modelled, including inputs from the embedding economy to the detailed functional flow-based system and vice versa, which expands the system, preserves process specificity, and is useful for various applications, including cleaner production and supply chain management.

I would like to make a number of recommendations for LCA practitioners and database builders. First, in performing an LCI study, it is recommended to document at least the prices of cut-offs and the sales pattern of functional flows, which will allow later users to adopt the integrated hybrid model. Second, the use of the integrated hybrid model is recommended especially for comparative LCA studies. Equivalence of system boundaries has been one of the main obstacles in comparative studies (see e.g. Anonymous, 1991; Hocking, 1991). The integrated hybrid model provides a fairly neutral and complete background system for LCA practitioners, enabling a comparative study on the basis of equivalent system boundaries.

Another recommended direction of research is building reliable and publicly available environmental intervention databases for the input–output table. Although a number of national and international projects are in progress to incorporate environmental variables in national accounts, such as the National Accounting Matrix including Environmental Accounts (NAMEA), System of integrated Environmental and Economic Accounting (SEEA), the number of pollutants covered and the resolution of the commodity classification are rather limited for LCA purposes. Efforts are also being made by various research groups to build up publicly available databases (see e.g. Green Design Initiative, 2002; Nansai et al., 2002; Suh, 2001b; Suh, 2004). For most countries, however, detailed, sectoral environmental statistics are still not available. Furthermore, reliable data at the national level may not be enough for many countries, due to the proportion of imports. Therefore, efforts should be made to develop a multi-national database.

The model developed here is also generally applicable to studies on broad interindustry interdependence in which some part of the system deserves special attention, including analyses of impact by consumption, the role of specific technology in connection to its embedding economic system, substance flow analysis, material flow accounting, etc.

Acknowledgements

I would like to thank Dr. Reinout Heijungs and Dr. Gjalt Huppes, both at Centre of Environmental Science (CML) in Leiden University, as well as the three reviewers, for their valuable comments.

Appendix A

The input–output technology coefficient matrix derived by the Eq. (11) in the main text includes some of the functional flows already covered by the process-based system, especially when (1) a functional flow between two processes within a process-based system involves monetary transaction and (2) both of the processes belong to industries in the intermediate part of an input–output table. Therefore, in order to avoid double counting, these portions of the functional flows have to be subtracted from the input–output based system. If a functional flow satisfies neither of the two conditions above, the subtraction procedure is not necessary for the flow.

Since the input–output framework used for the integrated hybrid model is based on a commodity-by-commodity technology coefficient matrix, subtraction of the portions counted double is done at the level of make and use matrices.

Let us start with the functional flow records matrix, $\tilde{\mathbf{Z}}_*$. If some of the basis periods used for steady-state approximation are other than 1 year, a diagonal matrix must be multiplied by the relevant values to adjust them all to 1 year periods. For our present calculations, we assume that $\tilde{\mathbf{Z}}_*$, $\tilde{\mathbf{Z}}^{\mathrm{u}}_*$ and $\tilde{\mathbf{Z}}^{\mathrm{d}}_*$ have been compiled with a basis period of 1 year. Part of the functional flow-by-process matrix $\tilde{\mathbf{Z}}_*$ is extractedtocompose$\tilde{\mathbf{Z}}^{\pi}_*$suchthat$(\tilde{\mathbf{Z}}^{\pi}_*)_{ij}$shows$(\tilde{\mathbf{Z}}_*)_{ij}(\mathbf{Z}‹*)_{ij}$ $(\tilde{\mathbf{Z}}_*)_{ij}$ if the functional flow of i to process j satisfies the above mentioned two conditions, and 0 if not. We further divide $\tilde{\mathbf{Z}}^{\pi}_*$ into two matrices, $\tilde{\mathbf{V}}^{\pi}_*$ and $\tilde{\mathbf{U}}^{\pi}_*$, such that

$$\{\tilde{\mathbf{V}}^{\pi}_* \,|\, (\tilde{\mathbf{V}}^{\pi}_*)_{ij} = (\tilde{\mathbf{Z}}^{\pi}_*)_{ji} \;\text{ if }\; (\tilde{\mathbf{Z}}^{\pi}_*)_{ji} > 0, \;\text{ or 0 otherwise}\}. \tag{A1}$$

and

$$\{\tilde{\mathbf{U}}^{\pi}_* | (\tilde{\mathbf{U}}^{\pi}_*)_{ij} = -(\tilde{\mathbf{Z}}^{\pi}_*)_{ij} \;\text{ if }\; (\tilde{\mathbf{Z}}^{\pi}_*)_{ij} < 0, \text{or 0 otherwise}\}. \tag{A2}$$

Clearly,

$$(\tilde{\mathbf{V}}^{\pi}_*)^{\mathrm{T}} - \tilde{\mathbf{U}}^{\pi}_* = \tilde{\mathbf{Z}}^{\pi}_*. \tag{A3}$$

Note that $\tilde{\mathbf{V}}^{\pi}_*$ is a process-by-functional flow matrix and $\tilde{\mathbf{U}}^{\pi}_*$ is a functional flow-by-process matrix. Let us further define an commodity-by-functional flow matrix $\mathbf{P}_{\mathrm{F}}$ such that

$$\{\mathbf{P}_{\mathrm{F}} \,|\, (\mathbf{P}_{\mathrm{F}})_{ij} = 1 \quad \text{if functional flow } j \text{ belongs to commodity } i, \text{ or 0 otherwise}\}. \tag{A4}$$

Similarly, a process by input-output industry matrix $\mathbf{P}_{\mathrm{P}}$ is defined such that

$$\{\mathbf{P}_{\mathrm{P}} \,|\, (\mathbf{P}_{\mathrm{P}})_{ij} = 1 \quad \text{if process } i \text{ belongs to industry } j, \text{ or 0 otherwise}\}. \tag{A5}$$

The matrices $\mathbf{P}_{\mathrm{F}}$ and $\mathbf{P}_{\mathrm{P}}$ are a functional flow permutation matrix and a process permutation matrix, respectively.

Let $\mathbf{U}_{**}$ be a commodity-by-industry matrix that shows the total use of domestic and imported current commodities and capital by domestic industries, with updated prices, and let $\mathbf{V}_{**}$ be an industry-by-commodity matrix that shows the total production of commodities by domestic industries, with updates prices. The portion of commodities consumed by the process-based system is then subtracted from the use matrix, $\mathbf{U}_{**}$ by

$$\mathbf{U}_{***} = \mathbf{U}_{**} - (\hat{\mathbf{m}}\mathbf{P}_{\mathrm{F}}\tilde{\mathbf{U}}^{\pi}_{*}\mathbf{P}_{\mathrm{P}} + \hat{\mathbf{m}}\mathbf{P}_{\mathrm{F}}\tilde{\mathbf{Z}}^{\mathrm{d}}_{*} + \tilde{\mathbf{Z}}^{\mathrm{u}}_{*}\mathbf{P}_{\mathrm{P}}), \quad \text{(A6)}$$

where $\mathbf{m}$ denotes the price vector. The portion of commodities produced by the process-based system is also subtracted from the make matrix $\mathbf{V}_{**}$ by

$$\mathbf{V}_{***} = \mathbf{V}_{**} - \hat{\mathbf{m}}(\mathbf{P}_{\mathrm{P}})^{\mathrm{T}}\tilde{\mathbf{V}}(\mathbf{P}_{\mathrm{F}})^{\mathrm{T}}. \quad \text{(A7)}$$

The commodity-by-commodity technology coefficient matrix derived by the reduced make and use matrix in (A6) and (A7), using a relevant model such as the industry-technology model or the commodity-technology model, shows the commodity flow relations, excluding those already covered in the process-based system. We use $\mathbf{A}'_{***}$ to denote the commodity-by-commodity input–output technology coefficient matrix that includes domestic and imported current products and capital, with prices updated to current levels, and excluding the portion of commodity flows already covered by the process-based system. Similarly, the environmental intervention-by-commodity matrix $\mathbf{B}'_{**}$ is reduced to $\mathbf{B}'_{***}$ by subtracting the environmental interventions by processes that were represented in the input–output accounts.

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