Estimation of Flexibility Coefficients for Recursive Programming Models—Alternative Approaches*

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This paper presents a critical evaluation of some methods commonly used in estimating flexibility coefficients for recursive programming models. It also suggests an alternative procedure for estimating coefficients that vary from year to year depending upon economic and noneconomic conditions. This method was found empirically superior to some previous approaches.

Key words: **recursive programming; flexibility coefficient; agricultural production response; Canada.**

SEVERAL METHODS have been used in agricultural production response studies to estimate flexibility coefficients for recursive programming models[1] [3, 6, 13, 15]. In these studies, recursive programming has not been overly successful in the empirical estimation of supply response. The relatively poor performance of these models has been in part due to some major weaknesses in the estimation procedures for the flexibility coefficients. Bawden [1], Day [3], Doll [4], Miller [9], Schaller [12, 13], and others have recognized that the crux of recursive programming models lies in the estimation of the flexibility coefficients, and they have correspondingly recommended improvements in this direction. The purpose of this paper is to evaluate the theoretical underpinnings and empirical performance of four methods for estimating flexibility coefficients and to present an alternative approach.

Recursive Programming and Flexibility Restraints

Recursive programming may be described as an attempted synthesis of ordinary linear programming[2] and regression analysis of time series data. In terms of solution procedures, recursive programming and linear programming are similar. Both are employed to optimize a linear objective function subject to linear constraints. The difference between the models is of a conceptual nature. Recursive programming is capable of predicting the actual behavior of firms, whereas linear programming can only estimate an optimal behavior [6, pp. 242–43]. This characteristic of the recursive model is obtained through the use of flexibility constraints which are often estimated through a regression analysis of time-series observations.

Maximum and minimum flexibility restraints are utilized to represent upper and lower bounds on the allowable changes in the level of each enterprise in the programming solution from one time period to another [6, pp. 242–43]. Assuming an annual time period, these restraints relate the production pattern of one year with that of the preceding year and as such are based on the assumption that farmers' current year production decisions are deviations from the allocation pattern in the preceding year. A conglomeration of factors is responsible for farmers' inability or unwillingness to make large changes in an established production pattern. Some of these are risk and uncertainty associated with marketing and weather, insufficient knowledge, institutional restrictions, and personal preferences including goals other than short-run profit maximization [13, pp. 7–8]. Since these forces are usually not directly measurable, some indirect measurement needs to be employed. The use of flexibility restraints is such a method.

These restraints can be expressed mathematically as follows:

Upper flexibility restraints

$$X_{it} \leqslant (1 + \overline{\beta}_{it}) \quad X_{it-1} \tag{1}$$

Lower flexibility restraints

$$X_{it} \geqslant (1 - \underline{\beta}_{it}) \quad X_{it-1} \tag{2}$$

$$i = 1, \ldots, n$$

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[1] A definition of "recursive programming" is given below, along with a discussion of the role of flexibility coefficients.

[2] Hereafter referred to as "linear programming."

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where

X_{it} = level of the ith activity to be determined in t year,

X_{it-1} = level of the ith activity in $t-1$ year,

$\overline{\beta}_{it}, \underline{\beta}_{it}$ = maximum allowable proportionate increase and decrease, respectively, in the level of the ith activity from the $t-1$ year to the t year; these are known as upper and lower flexibility coefficients,

and

n = total number of activities.

The first equation indicates that the level of the activity in period t will not exceed the previous year's level plus some proportion of it as determined by the upper flexibility coefficient, $\overline{\beta}_{it}$. The second equation suggests that the current level of the activity must not be less than an amount determined by the lower flexibility coefficient, $\underline{\beta}_{it}$, and the preceding year's level. Thus, the level of the ith activity in the solution for t year is constrained within these limits.

Methods for Estimating Flexibility Coefficients

Previous methods

A number of methods have been proposed for estimating flexibility coefficients [13, pp. 8 and 66]. In summary form, some of these are:

(1) Calculate year-to-year proportionate change in the level of an enterprise in the past, and select maximum positive proportionate change as $\overline{\beta}$, and maximum negative proportionate change (ignoring sign) as $\underline{\beta}$,

(2) Determine the flexibility coefficient as the average of absolute proportionate changes which have occurred in the past and use the same average both as the upper and lower coefficients,

(3) Stratify year-to-year proportionate changes into two groups on the basis of direction of change, and then compute the average of positive changes to obtain upper flexibility coefficient and average of negative changes to estimate lower flexibility coefficient, and

(4) Determine upper and lower flexibility coefficients by estimating the following simple regression equations[3] using observations stratified into two groups on the basis of positive or negative year-to-year changes:

$$(3)\quad X_{it} = (1+\overline{\beta}_i)X_{it-1} \qquad X_{it} > X_{it-1}$$

$$(4)\quad X_{it} = (1-\underline{\beta}_i)X_{it-1} \qquad X_{it} \leqslant X_{it-1}.$$

All four of these methods estimate a pair of flexibility coefficients ($\overline{\beta}_i, \underline{\beta}_i$) which are used to calculate upper and lower flexibility restraints for each year. There are two major limitations of coefficients estimated in these ways. First, they are based on an unreasonable assumption that the maximum proportion by which farmers would like to increase (or decrease) the level of an enterprise is the *same* in all years regardless of the level of that enterprise in the preceding year. In actual practice, producers probably vary their production patterns at different rates in different years. Bawden [1, pp. 1549–1551] also criticized these coefficients for being independent of the production level of the previous year. Coefficient estimates based solely on proportional changes imply that the larger the base year acreage of one particular crop, the greater is the potential for absolute expansion. But in reality this situation is unlikely to happen due to a producer's desire for diversity, his reaction to the uncertainty attached to specialization, and limited total land. Generally, a farmer who devotes a higher proportion of one year's acreage to a given crop will increase the acreage proportionately less than a farmer who devotes a smaller proportion of his acreage to that crop [6, p. 244]. If this is the case, the flexibility coefficient should be estimated such that it varies with changes in the preceding year acreage.

Doll [4, p. 126] and King [8, pp. 1536–1538] have outlined a second limitation of the above methods for estimating flexibility coefficients. Since the coefficients do not vary from year to year in response to changes in economic and noneconomic conditions, they inadequately explain dynamic production response. In reality, the maximum rates by which farmers change production patterns from one year to the next vary, even though rates are based on factors such as personal preferences, risk, and uncertainty.

Glenn Johnson's criticism [7, pp. 25–28] of Nerlove's distributed lag model is also applicable to these methods of estimating flexibility coefficients. He states the following:

[3] In order to avoid redundancy, two separate equations, one for $\overline{\beta}$ and another for $\underline{\beta}$, will not be presented hereafter. Rather, an equation will be specified for β (without upper and lower bars) because basically the same equation is applied in both cases. The difference is the application to two different data sets, one consisting of positive year-to-year changes in the level of an enterprise and the second of negative changes.

> Nerlove's coefficient of adjustment, γ, seems somewhat inadequate. A boy appears to have been sent to do a man's job. Do we really expect the difference between next year's production and this year's production to be some portion γ, where $0 < \gamma \leqslant 1$, of the difference between this year's 'long-run equilibrium output' and this year's actual output *regardless* of whether product prices have just reversed, have been going up for a number of years or have been going down for a number of years [italics ours]?

As flexibility coefficients are similar in concept to Nerlove's coefficient of adjustment [2, p. 110], they are amenable to the same criticisms.

This weakness of the above methods has also been revealed in several recursive programming studies. Schaller and Dean [13, p. 68] found that flexibility coefficients estimated in these ways were not adaptable to the conditions of each new year. The flexibility restraints in some years allowed too much flexibility in the solution and thereby caused overestimation of the crop acreages. In other years, the restraints were too narrow. Therefore, they recommended that the bounds should be estimated using more information than just the preceding year acreages. Richard Day [3, pp. 87–88] suggested that flexibility coefficients should be related to changes in prices and weather conditions.

In summary, flexibility coefficient estimates based on methods used in the previous studies would be immune to year-to-year changes in economic and noneconomic conditions. An alternative framework is developed below that allows the coefficients to vary from year to year depending upon the levels of exogenous variables.

An alternative method for estimated flexibility coefficients

It is assumed that there is a true flexibility coefficient for acreage of a crop,[4] β_t, upon which production decisions are made, but it is not observable. Instead, it is observed with error through the observable variable $(X_t/X_{t-1}) - 1$. This assumption can be stated in the following stochastic equation:

$$\text{(5)} \quad \frac{X_t}{X_{t-1}} = \beta'_t + \epsilon_t \quad \text{where} \quad \beta'_t = 1 + \beta_t,$$

and it is assumed $E\ (\epsilon_t) = 0$, Var $(\epsilon_t) = \sigma^2$, and Cov $(\beta_s, \epsilon_t) = 0$ for $s \leqslant t$.

Two hypotheses can be formulated regarding factors affecting flexibility coefficients. One is that the flexibility coefficient[5] for acreage of a crop depends upon the preceding year acreage of the crop in question. This hypothesis can be expressed as:

$$\text{(6)} \quad \beta'_{it} = \gamma_{io} + \gamma_{i1} X_{it-1} \qquad i = 1, \ldots, n.$$

Substitution of equation (6) into (5) results in

$$\text{(7)} \quad \frac{X_{it}}{X_{it-1}} = \gamma_{io} + \gamma_{i1} X_{it-1} + \epsilon_{it}.$$

If flexibility coefficients are estimated from equation (7), they would not be the same in each time period. Rather, they would vary depending upon the level of the preceding year's acreage. Since it is hypothesized that $\gamma_{i1} < 0$, a large flexibility coefficient would be estimated if the preceding year acreage of a crop was small. Likewise, a small flexibility coefficient would result if the base year acreage were large. In this study, equation (7) was estimated in a preliminary stage of the analysis, and the flexibility coefficients were used to compute flexibility restraints for the programming model. However, results based on these coefficients were inferior to those based on coefficients estimated from equation (10). Therefore, this method was not pursued further.

The second hypothesis is that the flexibility coefficient for acreage of a crop varies from year to year depending upon the expected levels of several exogenous variables. For example, the coefficient for a crop might depend upon the farmers' expected prices, inventories, and exports for the crop in question and its major competitor. Preceding year acreage and technological and weather conditions (e.g., precipitation) could be considered as other relevant explanatory variables. The relationship can be expressed as follows:

$$\text{(8)} \quad \beta'_{it} = f(X_{it-1}, P^*_{it}, P^*_{jt}, S^*_{it}, S^*_{jt}, E^*_{it}, E^*_{jt}, M_t, T_t) \qquad i = 1, \ldots, n$$

where

$i =$ crop in question,

[4] To avoid ambiguity in discussion, land allocation decisions have been used as the example in this paper.

[5] Hereafter, flexibility coefficient is referred to as β' $(= 1 + \beta)$ rather than β. Conceptually, it is immaterial whether the coefficient is defined as $1 + \beta$ or β because both have the same basic economic meaning. The differences are in the mathematical interpretation and the mechanics of computing flexibility restraints. In the case of $1 + \beta$, the flexibility restraint is the product of $1 + \beta$ and preceding year acreage, whereas in the case of β, 1 is added to it before multiplying with preceding year's acreage.

$j =$ main competitive crop,

$X_{it-1} =$ acreage of the ith crop in $t-1$ year,

$P^*_{it}, P^*_{jt} =$ expected prices of the ith and jth crops, respectively, in t year,

$S^*_{it}, S^*_{jt} =$ expected inventories for the ith and jth crops, respectively, in t year,

$E^*_{it}, E^*_{jt} =$ expected exports for the ith and jth crops, respectively, in t year,

$M_t =$ springtime moisture conditions,

and

$T_t =$ time trend variable to account for technological change.

In order to transform the expected variables of equation (8) into observable variables, simple expectation models can be employed: the expected exports, prices, and inventories could be taken as the preceding year values. Because precipitation before seeding creates a gap between actual and intended land use, total rainfall during April and May could be treated as the relevant observation for the moisture variable.

After using the expectation models and assuming linear relationships between dependent and independent variables, equation (8) can be expressed as:

$$(9) \quad \beta'_{it} = \alpha_{0i} + \alpha_{1i}X_{it-1} + \alpha_{2i}P_{it-1} + \alpha_{3i}P_{jt-1} + \alpha_{4i}S_{it-1} + \alpha_{5i}S_{jt-1} + \alpha_{6i}E_{it-1} + \alpha_{7i}E_{jt-1} + \alpha_{8i}M_t + \alpha_{9i}T_t.$$

The variables of this function are not defined here because basically the same variables have been described in equation (8); a star indicates expected values and lack of a star represents actual values.

Substitution of equation (9) into (5) produces the following equation:[6]

$$(10) \quad \frac{X_{it}}{X_{it-1}} = \alpha_{0i} + \alpha_{1i}X_{it-1} + \alpha_{2i}P_{it-1} + \alpha_{3i}P_{jt-1} + \alpha_{4i}S_{it-1} + \alpha_{5i}S_{jt-1} + \alpha_{6i}E_{it-1} + \alpha_{7i}E_{jt-1} + \alpha_{8i}M_t + \alpha_{9i}T_t + \epsilon_{it}.$$

Equation (10) shows that sources of year-to-year variations in flexibility coefficients are changes in the base year acreage and variations in the levels of other exogenous variables. Since the coefficient estimates from the above method vary in response to changes in the explanatory variables, this method can be applied to estimate the effects of some policy changes on land use. However, in the event of major structural change, the adequacy of this method is greatly reduced because it is based on historical time-series data. Farm surveys could be employed to estimate the coefficients in this situation.

Due to insufficient degrees of freedom, it was possible to include only a few variables in each regression equation. The selection of variables was influenced by: (1) multicollinearity between independent variables; (2) inconsistent signs of coefficients with *a priori* expectations; (3) the "t" values of the coefficients; and (4) the R^2. In some equations, it became necessary to transform price variables in such a way that price ratios were used in the place of individual prices.

Analytical Models for Canadian Prairie Land Utilization Patterns

Constraints

In order to compare the empirical performance of the method developed in this study with some of the approaches used in previous studies,[7] three recursive programming models for prairie land utilization patterns were developed. These models were identical in every respect, except in the estimation of the flexibility restraints. The three methods used to estimate flexibility coefficients are:

(1) The maximum of positive proportionate changes which occurred during 1953 to 1967 was selected as $\overline{\beta}$, and the maximum negative proportionate change (ignoring sign) was treated as $\underline{\beta}$. The programming model based on this method is referred to as Model M-1.

(2) Simple regression equations such as (3) and (4) were utilized to estimate coefficients. The

[6] Inclusion of X_{it-1} on both sides of the equation clearly violates the assumption for least-square estimators that disturbance term and explanatory variables should be independent. Under this situation least-square estimators are biased [5, pp. 273–77]. Using an alternative specification, this problem might be overcome. However, other problems could arise. For example, by multiplying X_{it-1} to both sides of equation (10), we will eliminate the variable from the left side of the equation, but a very serious case of multicollinearity results. The second limitation of this formulation is that the estimates of flexibility coefficients cannot be obtained directly; rather, some further calculations are necessary. For computational simplicity, therefore, equation (10) was used.

[7] Only two of the four "Previous Methods" discussed above were selected to estimate flexibility coefficients. Methods 2 and 3 were not used because their estimates are likely to be inferior or equal to those of method 4.

programming model using these flexibility coefficients is designated as Model M-2.

(3) The method developed in this study [equation (10)] was utilized to estimate coefficients and to construct Model M-3.

For estimating upper and lower flexibility coefficients separately, observations over the period 1953 through 1967 were stratified into two groups on the basis of positive or negative changes in year-to-year acreages of a crop. All three methods were then used to estimate upper flexibility coefficient for each crop using positive changes in year-to-year acreages. Likewise, the lower coefficient for each crop was estimated using negative changes in crop acreages. *Ex ante* flexibility coefficients for 1968 and 1969 were estimated in a similar way. The coefficients were used to compute flexibility restraints for each crop.

Only two physical resource restraints were included in the model—the total arable acreage for the current year and the summerfallow acreage of the preceding year. Grain marketing quota restraints were used in the model since quotas influence farmers' production decisions by affecting the marginal return of using land for one crop relative to other crops (cf. [10] for greater detail).

Activities

The six major crops of the prairies were included in this study. These crops are: wheat, oats, barley, rye, flax, and rapeseed. In addition, summerfallow was included in the model as an activity. Two producing activities were employed for each crop, one for crops seeded on summerfallow and one for crops grown on stubble. To account for Canadian Wheat Board restrictions on sales of crops, selling alternatives were included for each crop.

Table 1. Turning point errors in estimates of the crop acreages in the prairies, 1958 to 1967

Crops	Total Number of Turning Points in Each Model	Number of Turning Point Errors: Model M-1	Model M-2	Model M-3
Wheat	10	2	2	2
Oats	10	4	4	5
Barley	10	4	5	2
Rye	10	7	7	1
Flaxseed	10	5	6	4
Rapeseed	10	3	3	1
Summerfallow	10	5	5	2
Total	70	30	32	17

Results

To determine the relative performance of the three methods of estimating flexibility coefficients, all three recursive programming models were solved for each individual year from 1958 through 1967. Estimates of the models were compared with actual crop acreages treated as a norm to determine their relative ability to explain land utilization patterns. Each model was also utilized to predict acreages for 1968 and 1969. These estimates were then compared with the actual acreages to evaluate the predictive ability of the models.

The explanatory test

The relative accuracy of alternative models in explaining prairie land utilization patterns was judged on the basis of three criteria:

(1) Direction of change in crop acreages (turning points),

(2) Theil's inequality coefficient,[8] U, and

(3) The weighted average of absolute percent deviations between actual and estimated acreages.

Based on all three criteria, the prairie land use was explained more adequately by Model M-3 than Model M-1 or M-2 during the period 1958 through 1967. The number of turning point errors for each crop except oats was the smallest with Model M-3 (Table 1). For all land use, Model M-3 had half as many turning errors as Model M-1 or M-2.

Theil's inequality coefficient, U, for entire land use during 1958 to 1967 was .054 for Model M-1, .029 for Model M-2, and .019 for Model M-3 (Table 2). The weighted average of absolute deviations was 12.03 percent for Model M-1, 6.52 percent for M-2, and 3.76 percent for M-3. On an individual year basis, Model M-3 had smaller errors than Models M-1 and M-2 in each year. For five years out of 10, the errors for Model M-3 were about half the size of the errors

[8] The coefficient was estimated using the following formula:

$$U = \frac{\sqrt{\frac{1}{n}\Sigma (P_i - A_i)^2}}{\sqrt{\frac{1}{n}\Sigma P_i^2} + \sqrt{\frac{1}{n}\Sigma A_i^2}}$$

where A_i is actual acreage, P_i is predicted acreage, and n is the number of observations. The coefficient U varies between zero and unity; zero indicates complete semblance between estimated and actual values, and unity reflects a negative proportionality [14, pp. 31–42].

Table 2. Deviations of estimated from actual prairie crop acreages for three selected models, 1958–1967[a]

Year	Wheat M-1	Wheat M-2	Wheat M-3	Oats M-1	Oats M-2	Oats M-3	Barley M-1	Barley M-2	Barley M-3	Rye M-1	Rye M-2	Rye M-3	Flaxseed M-1	Flaxseed M-2	Flaxseed M-3
								percent							
1958	—8.39	—1.49	—3.13	26.51	15.16	14.94	18.87	—4.76	—1.00	48.96	26.91	.46	—121.62	—72.29	—37.05
1959	.08	6.45	4.26	21.72	—5.19	1.44	5.18	—4.40	—4.86	54.59	34.93	3.93	—107.35	—56.71	—35.34
1960	—11.82	—4.71	—2.67	26.32	3.91	9.27	7.56	—1.78	—.42	54.90	35.31	.41	—35.79	—2.66	—2.98
1961	—8.20	—1.31	—.65	5.47	—11.62	—31.08	.07	—10.02	2.91	51.93	31.24	5.88	35.88	9.17	15.36
1962	—4.67	2.00	—.52	45.72	37.33	14.92	—33.88	—20.11	—9.16	57.19	38.67	7.37	22.13	—59.38	—29.87
1963	—8.36	—1.46	—.97	13.40	.03	.94	1.76	4.98	4.36	54.03	33.96	1.20	54.57	35.67	14.97
1964	—3.51	3.08	1.06	6.11	—8.39	1.11	—13.92	—24.06	—14.45	54.52	35.00	2.58	54.96	36.17	37.16
1965	—16.68	—9.25	—6.07	32.13	21.67	13.59	—11.81	11.43	5.43	56.73	37.92	1.74	55.14	36.47	35.28
1966	—3.37	.52	.09	21.49	9.38	—4.15	34.32	14.24	11.84	50.22	28.76	1.34	36.27	9.67	3.45
1967	—.22	—1.75	1.78	18.84	6.31	—10.96	26.03	18.55	2.28	48.41	26.11	—2.23	0.00	—41.68	—13.03
Wt. Av. of Abs. Per. Devs.[b]	6.41	3.18	1.99	22.72	12.45	10.19	15.66	10.13	5.26	53.19	32.95	2.67	57.66	35.24	22.99
Theil's *U*	.039	.021	.013	.150	.092	.066	.102	.057	.032	.363	.199	.017	.305	.197	.133

Year	Rapeseed M-1	Rapeseed M-2	Rapeseed M-3	Summerfallow M-1	Summerfallow M-2	Summerfallow M-3	Wt. Av. of Abs. Per. Devs.[b] M-1	Wt. Av. of Abs. Per. Devs.[b] M-2	Wt. Av. of Abs. Per. Devs.[b] M-3	Theil's *U* M-1	Theil's *U* M-2	Theil's *U* M-3
						percent						
1958	—254.15	—77.64	—100.00	11.35	7.83	5.52	19.48	9.24	7.01	.077	.042	.030
1959	—788.73	—428.17	—212.21	7.39	3.71	1.59	11.58	8.09	4.63	.050	.033	.020
1960	0.00	49.80	—6.29	4.72	1.89	.74	11.00	3.88	2.34	.049	.018	.012
1961	—284.93	—93.10	—7.46	9.94	6.37	4.69	12.21	6.39	5.45	.054	.028	.028
1962	—253.10	—243.94	—25.61	—.03	—2.36	.03	11.41	9.98	3.26	.051	.041	.016
1963	—178.03	—39.54	—11.92	.86	—1.81	—.91	7.95	3.12	1.64	.034	.012	.006
1964	—116.43	—8.60	15.68	3.74	—.22	—1.75	7.75	5.23	3.52	.027	.022	.015
1965	—97.42	.98	—1.53	.96	—1.55	—.68	14.85	8.50	5.39	.066	.038	.026
1966	—237.05	—69.11	—6.23	.00	—3.96	—2.45	13.04	5.78	2.64	.059	.023	.013
1967	—237.16	—69.14	5.86	3.34	.61	.32	11.22	5.75	2.21	.056	.024	.010
Wt. Av. of Abs. Per. Devs.[b]	197.61	67.85	19.54	4.26	3.04	1.87	12.03	6.52	3.76			
Theil's *U*	.520	.278	.125	.030	.019	.013				.054	.029	.019

[a] A positive deviation indicates an underestimate (i.e., estimated acreage less than actual), and a negative error represents an overestimate.

[b] Weighted average of absolute percent deviations.

of Model M-2 and less than one-third of the errors of Model M-1.

For each crop, Model M-3 was superior, on average, to Models M-1 and M-2 in explaining the 1958 to 1967 acreages. Model M-3 estimated rye acreages more closely than Models M-1 and M-2 for all years and estimated acreages of all other crops, except oats, more closely for eight years out of 10.

A crop-wise comparison of errors estimated by Model M-3 reveals that acreages of wheat, barley, and rye were estimated better (in percentage terms) than acreages of flax and rapeseed. The reason was that year-to-year fluctuations in acreages of the latter crops were much sharper (in percentage terms) than in acreages of the former crops.

The predictive test

This test is more rigorous than the explanatory one because the ability of the model to predict *outside* the period used for its construction is examined, and permanence and completeness of the structure are thereby evaluated. The models which were developed on the basis of the 1958 through 1967 data were used to make predictions for 1968 and 1969. Comparisons were then made among estimates of all models again treating actual acreage as a norm.

The Model M-3 predicted prairie land use more closely than Models M-1 and M-2 (Table 3). The coefficient *U* averaged over 1968 and 1969 was .078 for Model M-1, .065 for Model M-2, and .041 for M-3. The weighted averages of the absolute deviations for the same years were 18.88 percent for Model M-1, 13.15 percent for Model M-2, and 7.24 percent for M-3. In each year, Model M-2 predicted acreages more accurately than Model M-1, but less accurately than Model M-3.

A crop-wise comparison indicates that Model M-3 predicted acreages of all crops except flax more accurately than Models M-1 and M-2. Thus it can be concluded that among the models considered, Model M-3 performed the best and M-2 predicted acreages better than M-1.

Table 3. Deviations of predicted from actual prairie crop acreages for three selected models, 1968–1969[a]

Crops	1968 M-1	1968 M-2	1968 M-3	1969 M-1	1969 M-2	1969 M-3	Wt. Av. of Abs. Per. Devs.[b] M-1	Wt. Av. of Abs. Per. Devs.[b] M-2	Wt. Av. of Abs. Per. Devs.[b] M-3	Theil's *U* M-1	Theil's *U* M-2	Theil's *U* M-3
						percent						
Wheat	—9.43	—6.97	—1.69	—9.52	—23.48	—16.90	9.47	14.53	8.66	.045	.075	.053
Oats	27.75	10.56	4.36	28.10	15.56	—11.28	27.93	13.13	7.91	.162	.072	.043
Barley	26.83	19.44	1.62	25.77	—5.70	4.51	26.28	12.30	3.12	.151	.071	.018
Rye	51.05	29.73	—5.33	65.19	50.17	23.98	59.27	41.61	16.17	.433	.281	.105
Flaxseed	64.78	17.50	10.46	—3.22	21.98	39.71	185.15	20.25	28.43	.603	.146	.201
Rapeseed	—451.62	—176.71	—53.99	—87.28	6.06	29.22	212.37	64.65	37.73	.551	.324	.185
Summerfallow	9.19	5.58	2.11	13.63	10.21	4.96	11.49	7.98	3.59	.063	.044	.020
Wt. Av. of Abs. Per. Devs.[b]	20.63	11.04	3.01	17.16	15.24	11.42	18.88	13.15	7.24			
Theil's *U*	.083	.044	.012	.074	.081	.057				.078	.065	.041

[a] A positive deviation indicates an underestimate (i.e., predicted acreage less than actual), and a negative error represents an overestimate.

[b] Weighted average of absolute percent deviations.

Summary

In previous recursive programming studies, several methods were used to compute flexibility coefficients. The coefficients were estimated such that they were immune to year-to-year changes in economic and noneconomic variables. In reality, the maximum rates by which farmers are likely to increase (or decrease) the level of an enterprise vary from year to year in response to changes in these variables.

In this study, an alternative approach for estimating flexibility coefficients was developed. The coefficients were estimated so that they were dependent upon the levels of economic and noneconomic forces prevailing in that year. The performance of the new approach was compared with two methods selected from the previous studies. The recursive programming model based on the new method explained the 1958 to 1969 changes in prairie crop acreages with substantially more accuracy than models based on previous methods for estimating flexibility coefficients. It is therefore concluded that the new method is superior to methods previously utilized.

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