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On the Ricardian rent and the allocation of land under joint price and yield uncertainty

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Summary

A result on the comparative statics of land rent under joint price and yield uncertainty and expected utility maximisation is provided. An increase in the strength of the negative price–yield dependence is modelled using the concordance stochastic order. The effect of a greater price–yield dependence on equilibrium rent depends on the degree of producer risk aversion. Specifically, rent increases (decreases) under an increase in negative dependence between price and yield if the relative risk aversion is greater (less) than one and non-farm income is less (greater) than production costs.

Keywords: land rent, uncertainty, concordance order

JEL classification: D21, D81, Q12

1. Introduction

In recent articles, Chavas (1993) and Hennessy (1997) analysed the determinants of equilibrium land rent in the context of Ricardo's classic model of rent for fixed factors under output price uncertainty and expected utility maximisation. Chavas (1993) and Hennessy (1997) studied the effects of stochastic price shifts, increases in input prices, and cost function shifts on equilibrium rent. In both papers, output is a deterministic function of production inputs and land resource. However, agricultural producers are typically confronted with both price and yield uncertainty when land rent is negotiated. Although the effects of a single source of uncertainty on economic decisions have received much attention, the focus here is on the interaction between price and yield uncertainty. The purpose of this note is to study the effects of an increase in the strength of the negative price–yield correlation ('natural hedge') on equilibrium land rent. The issue is interesting, as the level of price–yield correlation varies across production regions. Because of common weather and soil conditions the correlation is stronger in regions where crop production is geographically concentrated relative to regions where crop production is dispersed across different areas (Harwood *et al*., 1999).

2. Ricardian model of Chavas with joint price and output uncertainty

Following notation in the Chavas paper, $\pi = pf(x,L,e) - r'x - sL$, where π is profit, s is Ricardian land rent, L is the land resource level chosen, r is a row vector of input prices, x is the input choice vector, and p represents uncertain output price. In contrast with the Chavas formulation, farm production technology f depends not only on the levels of the inputs, x and L, but also on the realisation of the random variable e, where, for concreteness, $\partial f/\partial e \geq 0$. We hold that p and e have the joint probability distribution $G^a(p,e)$, $p, e \in [\underline{p}, \bar{p}] \times [\underline{e}, \bar{e}]$. The producer chooses x and L to maximise

$$V[I, s, G^a] = \max_{x,L} EU[I + pf(x, L, e) - r'x - sL] \qquad (1)$$

where U is a von Neumann–Morgenstern utility function with $U_I > 0$ and $U_{II} \leq 0$, I is exogenous non-farm income and E is the expectations operator over the joint probability distribution of p and e, $G^a(p,e)$. The fixed supply of land and competition with free entry in rental markets ensure that land rent, s, is bid up or down until

$$V[I, s, G^a] = U[I]. \qquad (2)$$

Equilibrium levels of input use and land rent, $\{s^a, x^a, L^a\}$, are solutions to the expected utility maximisation problem (1) and equation (2). Under the assumption that e is non-random (i.e. $\Pr\{e = e^o\} = 1$), Chavas (1993) and Hennessy (1997) showed that the rent increases under any first- and second-degree stochastically dominating shifts in the distribution of product price, a decrease in input prices, and cost-saving technological improvements. Next, we introduce the dependence concept that is used to study the effect of a stronger price–yield dependence on equilibrium rent.

3. Concordance order

To model the strength of price–yield dependence, we will use the concept of positive dependence among multivariate random variables known as the concordance order (e.g. see Joe, 1990).[1] This stochastic order formalises the qualitative notion of greater covariability (concordance) between random variables X and Y: 'how likely it is that big (small) values of X go with big (small) values of Y'. A bivariate probability distribution G' is said to be more concordant than G (denoted $G \prec_c G'$) if there exists a sequence of

1 Hennessy and Lapan (2003) provide a fundamental treatment of the concept of 'more systematic risk' and its formalisations in a broad economic setting with some applications. The concordance and related orders are widely used in insurance and financial management literatures.

probability distributions $G = G_1, \ldots, G_n = G'$ such that G_i is obtained from G_{i-1} by adding mass $\varepsilon_i > 0$ at some points (x', y') and (x'', y'') while subtracting mass ε_i at the points (x', y'') and (x'', y') where $x' < x''$, $y' < y''$. Redistributing the probability mass in such a manner preserves the marginal distributions of X and Y while affecting the degree of their interdependence. In the case of random variables with two-point marginal distributions a concordance-increasing transformation is illustrated in Figure 1, where $\varepsilon > 0$. It should be noted that the realisations when both x and y take 'high' or 'low' values are more likely whereas the realisations when x and y 'mismatch' are less likely under the transformed distribution G' compared with G.

Y / X	x_{low}	x_{high}
y_{low}	$p_{low,low}$	$p_{low,high}$
y_{high}	$p_{high,low}$	$p_{high,high}$

$G(x, y)$

$\Rightarrow$

Y' / X'	x_{low}	x_{high}
y_{low}	$p_{low,low} + \varepsilon$	$p_{low,high} - \varepsilon$
y_{high}	$p_{high,low} - \varepsilon$	$p_{high,high} + \varepsilon$

$G'(x, y)$

Figure 1. Increase in positive dependence.

Tchen (1980) and Epstein and Tanny (1980) proved the following theorem showing that an increase in positive dependence based on the procedure described above can be equivalently generated using the class of supermodular (also known as correlation-affine or super-additive) functions.

Theorem. (The Concordance Order). The following three statements are equivalent:

(i) $G \prec_c G'$;

(ii) $G(x,y) \leq G'(x,y)$ for all x and y, where $G(x, \infty) = G'(x, \infty)$ and $G(\infty,y) = G'(\infty,y)$;

(iii) $\iint \phi(x, y) \mathrm{d}G \leq \iint \phi(x, y) \mathrm{d}G'$ for all supermodular functions ϕ for which the expectations exist.

A function ϕ is called supermodular if for any evaluations $x' < x''$ and $y' < y''$, we have

$$\phi(x', y') + \phi(x'', y'') \geq \phi(x', y'') + \phi(x'', y'). \tag{3}$$

It should be noted that the points of evaluation on the left-hand side of (3) are ordered in the sense that $(x', y') < (x'', y'')$, whereas the points of evaluation on the right-hand side are not. This property of supermodular functions to assign higher values to evaluations at points with matched components (both are 'big' or both are 'small') is used to capture an increase in stochastic interdependence.[2] We observe that any concordance-increasing transformation of the probability distribution $G_{i-1} \to G_i$ increases the expected value of any supermodular function $\phi(x,y)$:

$$\iint \phi(x,y)\mathrm{d}G_{i-1} \leq \iint \phi(x,y)\mathrm{d}G_{i-1} + \epsilon_i[\phi(x',y') + \phi(x'',y'') - \phi(x',y'') - \phi(x'',y')] = \iint \phi(x,y)\mathrm{d}G_i$$

so that we obtain $\iint \phi(x,y)\mathrm{d}G \leq \iint \phi(x,y)\mathrm{d}G'$.[3] For example, in Figure 1, the expected value of supermodular function $\phi = xy$ is greater under G' than under G because $x_{low}y_{low} + x_{high}y_{high} \geq x_{low}y_{high} + x_{high}y_{low}$.

By rearranging (3), the supermodularity can be stated as the 'increasing differences' property: $\phi(x',y') + \phi(x'',y'') - \phi(x',y'') - \phi(x'',y') = \Delta_x^\tau \Delta_y^\delta \phi(x',y') \geq 0$ where $\Delta_x^\tau \phi(x,y) = \phi(x+\tau,y) - \phi(x,y), \tau = x'' - x'$, and $\delta = y'' - y'$. If function ϕ is twice differentiable, this is equivalent to $\partial^2\phi/\partial x\partial y \geq 0$. In words, the value of a supermodular function increases more with x when y takes on high values (the same holds when y increases holding x fixed). This implies that an agent maximising a supermodular objective function prefers to match 'small' ('big') values of y with 'small' ('big') values of x (Topkis, 1998). For example, we observe that function $U(ax + by)$ is submodular in (x,y) for positive a and b, so that risk averters ($U_{II} \leq 0$) prefer joint distributions such that X and Y are 'less aligned', and therefore, allow for more effective risk diversification (think of X and Y as uncertain asset returns).[4]

Another attractive feature of the concordance order is its immediate connection with a more familiar notion of correlation. Dhaene and Goovaerts (1996) demonstrated that $G \prec_c G'$ if and only if $\mathrm{Cov}[f(X),g(Y)] \leq \mathrm{Cov}[f(X'),g(Y')]$ for any functions f and g monotonic in the same direction given that the covariances exist.

4. Increase in the strength of negative price–yield dependence

Let $A(w) = -U_{II}(w)/U_I(w)$ denote the Arrow–Pratt measure of risk aversion.

2 Suppose that condition (c) in the statement of Theorem holds. Because indicator functions $\phi(x,y) = 1_{x\leq a, y\leq b}$ and $\phi(x,y) = 1_{x\geq a, y\geq b}$ are supermodular, it follows that $\Pr(X \leq x, Y \leq y) \leq (\Pr(X' \leq x, Y' \leq y)$ and $\Pr(X \geq x, Y \geq y) \leq \Pr(X' \geq x, Y' \geq y)$, i.e. the realisations when both X and Y are 'small' ('large') are more likely under G' than under G. Furthermore, by taking $a = \pm\infty$ ($b = \pm\infty$), it is clear that distributions G and G' are possessed of the same marginals.

3 Note the analogy between the characterisations of the notion of 'greater variability (riskiness)' in the univariate case (Rothschild and Stiglitz, 1970) and the notion of 'greater covariability (systematic risk)'. Whereas the former notion is formalised using mean-preserving spreads and the class of concave utility functions, the latter one is based on concordance-increasing transformations and the class of supermodular functions.

4 If a function $-\phi$ is supermodular then function ϕ is said to be submodular.

Proposition. Land rent increases (decreases) under an increase in the strength of the negative price–yield dependence depending on whether $A(\pi + I) \geq (<)1/pf(x,L,e)$ for all $p, e \in [\underline{p}, \bar{p}] \times [\underline{e}, \bar{e}]$.

Proof. The proof proceeds in two steps. First, we ascertain that $U[\pi(p,e) + I]$ is supermodular (submodular) in p,e depending on the direction of the stated inequality. The second step is completely analogous to the argument used by Hennessy (1997). It is based on the observation that re-optimisation can never decrease the value of the objective function, and that farm income and expected utility decrease with rent.

Step 1. Differentiating $U[\pi(p,e) + I]$ twice with respect to p and e yields $\partial^2 U[.]/\partial p \partial e = [1 - pf(x,e)A]U_I \partial f/\partial e$. And so, $\partial^2 U[.]/\partial p \partial e$ inherits the sign of $1 - pf(x,e)A$.

Step 2. For concreteness, we assume that $G^b \prec_c G^a$ (i.e. negative dependence between price and yield is stronger under distribution G^b relative to that under G^a) and $1 - pf(x,e)A \leq 0$ so that $U[\pi(p,e) + I]$ is submodular, $\partial^2 U[.]/\partial p \partial e \leq 0$. This means that the utility of farming increases with price, p, less (more) when yield, $f(x,e)$, is high (low).

Then we have

$$U[I] = V[I, s^a, G^a] = \int\int U[I + pf(x^a, L^a, e) - r'x^a - s^a L^a]\mathrm{d}G^a(p, e)$$

$$\leq \int\int U[I + pf(x^a, L^a, e) - r'x^a - s^a L^a]\mathrm{d}G^b(p, e)$$

$$\leq \int\int U[I + pf(x^b, L^b, e) - r'x^b - s^a L^b]\mathrm{d}G^b(p, e) = V[I, s^a, G^b].$$

The first inequality is due to the submodularity of $U[\pi(p,e) + I]$ in p,e and $G^b \prec_c G^a$. The second inequality is because the input vector x^a,L^a may no longer be optimal under the probability distribution G^b and any re-optimisation over x,L (weakly) increases the expected utility. Suppose that $s^b < s^a$. Then we have $V[I,s^b,G^b] > V[I,s^a,G^b] \geq V[I,s^a,G^a] = U(I)$ because $U[\pi(p,e; x,L,r,s) + I]$ decreases in s and the adjusted choice of the input levels x,L is optimal. But this is impossible in the equilibrium represented by equation (2). To restore equilibrium, the Ricardian rent must increase, i.e. $s^b \geq s^a$.

An increase in the strength of negative price–yield dependence has, in general, an ambiguous effect on the expected utility of farming because not only does it stabilise the gross farm revenue but it also lowers its expected value, $Epf(x,L,e)$. The arising trade-off between farm revenue risk and return depends on attitudes toward risk. If the producer has a high degree of risk aversion, the effect of the negative price–yield dependence on farm revenue risk dominates, and the willingness to bid for land increases. The converse is true if the producer has a low degree of risk aversion. If the producer is risk neutral ($A = 0$), land rent falls under an increase in the negative price–yield dependence because the farm income π is supermodular in p and e.

It is instructive to rewrite the condition in the proposition in the form $1 - A^r[pf(x^a, L^a, \varepsilon) + w^a] + w^a A[pf(x^a, L^a, \epsilon) + w^a] \leq (>) 0$, where $w^a = I - r'x^a - s^a L^a$ and $A^r(w) = wA(w)$ denotes the Arrow–Pratt measure of relative risk aversion. If non-farm income happens to equal production costs, $I = r'x^a + s^a L^a$, the condition in the proposition reduces to $A^r \geq (<)\ 1$. In words, when the degree of relative risk aversion is greater (less) than one, an increase in the strength of the negative price–yield dependence raises (lowers) the Ricardian rent. The same relationship pertains if the producer has a small (large) non-farm income relative to production costs ($I \leq (>)\ r'x^a + s^a L^a$) and relative risk aversion that is greater (less) than one.

5. Summary

This note demonstrates that, in a standard equilibrium model of land rent under price and yield uncertainty, the rent increases with the strength of negative price–yield dependence if the producer is sufficiently risk averse. Specifically, this relationship holds if the degree of relative risk aversion exceeds one and non-farm income is less than the production expenses inclusive of cash rent. On the other hand, if the degree of relative risk aversion is less than one and non-farm income is large relative to farm production costs, then rent decreases under a greater negative price–yield dependence. This is because a less risk-averse producer places a smaller weight on risk and a greater weight on the expected return to farming.

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