NORTH-HOLLAND

Pricing in Electricity Markets

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The technical characteristics of electricity generation and transmission have implications for the way in which economic principles are adapted to evaluate pricing and regulation issues in electricity markets. In particular, there is an externality associated with the way in which electricity flows in networks because of Kirchoff's laws. In this paper, a mathematical programming model is presented that simulates a *competitive electricity market*, based on the spatial-intertemporal equilibrium models pioneered by Takayama and Judge (1971). The model is used to simulate the operation of a hypothetical electricity market, illustrating some of the issues arising from the network externality.

1. INTRODUCTION

Australia is in the process of establishing a national electricity market. The objective is to develop a market that operates as close as possible to the concept of economic efficiency by creating competition in those components that are contestable. However, the technical nature of electricity production and transmission still requires a significant amount of regulation to create and run an electricity market that produces market outcomes consistent with economic efficiency.

The way in which the market is regulated can affect the degree to which the operation of the market is economically efficient. In addition to short-term efficiency considerations, the operation of the market has important consequences for the way in which the

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augmentation of both transmission and generation takes place. One important issue is whether the design of the electricity market leads to an efficient system of transmission and generation in the long run.

Recently, there has been an increase in research relating to regulation of the industry and pricing issues, particularly on the pricing of electricity transmission in networks (Bushnell and Stoft, 1996; Chao and Peck, 1996; Wu et al., 1996). A theme emerging from these articles is that some of the principles underlying proposals on the regulation of electricity markets are so-called *folk theorems.* That is, they are commonly accepted assertions about the economic principles that, in fact, do not apply to the electricity market. Wu et al. (1996) claimed that these assertions arise because the economic principles being applied to the electricity market are borrowed from other applications of economics, such as transport economics. However, the technology of electricity production and transmission is such that the use of principles from other applications is inappropriate.

This paper aims to illustrate the economic principles embodied in an economically efficient electricity market through the use of a mathematical programming model.

In the model presented here, the power flow equations and variables for a network are included, based on the methodology outlined in Chao and Peck (1996). This paper extends their work by introducing time-specific demand and supply for electricity and allowing the transmission network structure to be varied. A theoretical model is used to analyze how an economically efficient market would price electricity in these circumstances.

A numerical version of this model is then solved to illustrate some of the economic principles.

The structure of the paper is as follows. The mathematical programming methodology is outlined in Section 2. Section 3 formally describes the model used here, and derives the pricing rules necessary for long-run economic efficiency. To set the scene for the numerical example, Section 4 presents the structure of a hypothetical electrical market. Sections 5, 6, and 7 outline the methods relating to demand, transmission, and production of electricity. The results are discussed in Section 8, and Section 9 is the conclusion and suggestions for further research.

2. MATHEMATICAL PROGRAMMING METHODOLOGY

Samuelson (1952) showed that it was possible to construct a maximization problem that guarantees fulfilment of the conditions of perfectly competitive equilibria among spatially separated markets. This provided the opportunity to use mathematical programming to simulate market behaviour. Later, Takayama and Judge (1971) significantly extended the applicability of the technique by showing that the competitive and monopoly models could be formulated as quadratic programming models.

They also showed that two alternative formulations—the quantity formulation (primal), and the price formulation (the purified dual of the primal)—could be used to compute market equilibrium.[1] Takayama and Woodland (1970) proved the equivalence between these two formulations. Takayama and Judge (1971) also showed that the quantity and price formulations could be combined to form another maximization problem where both quantity and price are explicit variables in the model. This is the *general* formulation referred to by MacAulay (1992), and is sometimes referred to as the *self-dual* or *primal-dual* formulation. Takayama and Judge (1971) also refer to it as the *net social revenue* formulation. The model presented here has been solved for three formulations—quantity (primal), the dual to the nonlinear primal as described in Balinski and Baumol (1968), and the primal-dual.

The general formulation (primal-dual) has wider applicability than either the quantity formulation (primal) or price formulation (dual). For example, it applies where interdependent demand functions do not satisfy the intergrability condition (that is there is no unique solution to their integration), or where policy imposes constraints on both prices and quantities.

In this particular study, the quantity formulation has advantages over the general formulation. First, it reduces the number of variables and equations, which is important when dealing with large-scale models. Second, it is easier to explain the technique and develop and implement the model. This is important, when the time to complete the study is short. For similar reasons, the quantity formulation has advantages over other related techniques used to compute economic equilibria, such as nonlinear complementary programming and computable general equilibrium models.

In electricity markets, cost and demand conditions vary by time and from place to place. For example, electricity demand can be met by generation from a range of technologies (gas and coal) and

[1] A useful reference on the dual in nonlinear programming is Balinski and Baumol (1968).

a range of plant sizes, all connected by a network of transmission. Therefore, spatial-temporal models, which include elements of networks, are particularly useful to capture this complexity.

Theoretical developments and the application of this methodology to study pricing and deregulation in spatial energy markets increased during the 1980s. Some examples include Salerian (1992), Kolstad (1989), Provenzano (1989), Uri (1983, 1989), Hobbs and Schuler (1985), and Sohl (1985).

In the mathematical programming model developed here, the supply of electricity is represented by economic-engineering models of power stations and transmission lines. Mathematical programming has been widely applied in modeling electricity supply, primarily to evaluate the least-cost options to meet forecast demand. That is, demand is exogenously specified for these models. Examples include Munasinghe (1990), Scherer (1977), and Turvey and Anderson (1977). Two Australian applications of note are ABARE's version of the MENSA model (Dalziell, Noble, and Ofei-Mensah, 1993) and CSIRO's earlier version of the MENSA model (Stocks and Musgrove, 1984).

An advantage of the model presented here is that the quantity demanded (and implicitly price) is endogenous to the model.

3. MATHEMATICAL MODEL

This section formally describes the model used in this study. As mentioned in the previous section, the model presented here simulates the long-run market equilibrium. The model represents a hypothetical electricity market that could involve 12 generators distributed around a possible network consisting of four nodes and five links. Demand for electricity takes place at two of the nodes, and generation can take place at three of the nodes. The model represents an annual market consisting of 34 time periods—that is, the 8760 hours in the year have been allocated to 34 time periods (load blocks).

The model has nonlinear variables in both the objective function and constraints. It also includes variables that may have negative or positive values (that is, they are unconstrained in sign).

3A. Notation

The notation used to present the model is divided into sets, parameters, and variables. The notation is consistent with the

GAMS[2] source code used to generate the model, which is available from the authors on request.

Sets

b	1, . . . , 34	Time blocks in which electricity is demanded.
n	1, . . . , 4	Nodes on network.
np	1, . . . , 4	Nodes on network.
p	1, . . . , 12	Power stations.
s	1, . . . , 5	Transmission corridors.

Primal variables

NSW Net social welfare.

$QD_{b,n}$ Demand for power at node n in each block b (PWh).

QGC_p Installed capacity of each plant p (GW).

$QGO_{b,p}$ Output of plant p in load block b (GW).

QTC_s Number of lines of a given capacity in each transmission corridor s.

$QS_{b,n}$ Electricity generated at node n in load block b from generators located at the node (GW).

$QP_{b,n,np}$ Quantity of power flow in load block b on an individual line, leaving (arriving at) node n for (from) node np (GW).

$\delta_{b,n,np}$ Difference between phase angles over a transmission corridor linking nodes n and np in load block b.

$\theta_{b,n}$ Voltage phase angle at node n in load bock b, for all nodes except node 1.

Lagrangian variables

$S_{b,n,np}$ Shadow price of the equation defining the difference in phase angles between nodes n and np (Equation 5).

T_s Shadow price of constraint on the maximum number of power lines that can be erected between two nodes along transmission corridor s (Equation 9).

$U_{b,n}$ Shadow price of balancing demand and supply at node n in block b (Equation 4).

$V_{b,p}$ Shadow price of balancing the generation of power station p in block b with its installed capacity (Equation 2).

W_p Shadow price of the limit on installed capacity of power station p (Equation 3).

[2] For more information on the GAMS computer software package see: Brooke, Kendrick and Meeraus (1992); Meeraus (1983); and Bisschop and Meeraus (1982).

$X_{b,n,np}$ Shadow price of power flows between nodes n and np (in block b) obeying Kirchoff's laws (Equation 6).
$Y_{b,n}$ Shadow price of electricity at node n in block b (Equation 7).
$Z_{b,n,np}$ Shadow price in block b of the capacity limit of each power line connecting node n to node np (Equation 8).

Parameters

$k_{n,np}$ Maximum capacity of each line connecting nodes n and np (GW).
j_s Maximum number of lines along transmission corridor s.
$a_{b,n}$ Constants for inverse demand curve in block b at node n.
$c_{b,n}$ Slope of inverse demand for electricity at node n in block b.
$m_{b,p}$ Variable costs ($m/MW) of station p in block b.
d_p Fixed cost ($m/MW) of plant p.
t_s Annualized fixed cost of constructing one transmission line on link$s$.
u 1000. Scalar for converting GW to MW.
e_p Generation unit availability factor (1 = 100% availability of all units at station).
f_p Maximum generation capacity of station p.
$aa_{n,np}$ Parameters on the power flow equations.
$bb_{n,np}$
$cc_{n,np}$
r_b 1000 divided by hours of duration in block b, converts PWh to GW.

3B. The Primal Model

Objective function ($ million):

$$NSW = \sum_b\sum_n(a_{b,n}QD_{b,n} + \frac{1}{2}c_{b,n}QD_{b,n}^2) - \sum_b\sum_p m_{b,p}QGO_{b,p} - \sum_p d_pQGC_p - \sum_s t_sQTC_s \quad (1)$$

The objective function maximises net social welfare (measured as consumer plus producer surplus—the area under the demand curve minus the sum of the variable costs). The first right-hand side term in the equation is the area under the demand curve (integral of the demand curve). The second and third terms are the total costs of operating the power stations and the total costs (annualized) of the installed generating capacity for power stations. The fourth term is the total cost of constructing power lines along the transmission corridors. The model is long run in nature, so that the location and capacity of both generation and transmission are to be optimized.

Energy generation installed capacity balance (GW):

$$QGO_{b,p} \leqslant e_p \, QGC_p \qquad \text{for } b, p \tag{2}$$

This equation limits the output of each power station in any time period to the amount of capacity installed.

Maximum plant generation capacity (GW):

$$QGC_p \leqslant f_p \qquad \text{for } p \tag{3}$$

The amount of capacity installed for each plant is limited by f_p the maximum number of units allowed times the rated generation capacity of each unit.

Nodal electrical generation (MW):

$$QS_{b,n} - \sum_p QGO_{b,p} \leqslant 0 \qquad \text{for } b,n \tag{4}$$

The amount of electricity generated at each node in each load block is limited by the output of all power stations generating electricity at that node in the load block.

Phase-angle difference (radians):

$$\delta_{b,n,np} - \theta_{b,n} + \theta_{b,np} = 0 \qquad \text{for } b, n, np \tag{5}$$

This is the difference in the phase angles between two nodes, n and np, which form a link. The phase angles and the difference in phase angles are free variables, being unconstrained in sign. There are only $n - 1$ independent phase angles, and so the phase angle at node one is set to zero.

Equation 5 and the following real power-flow equation encapsulate Kirchoff's laws on the flow of electricity in networks (see Section 6 for further explanation).

Real power flow (MW):

$$uQP_{b,n,np} - bb_{n,np} \, \delta_{b,n,np} - cc_{n,np} \, \delta^2_{b,n,np} = aa_{n,np} \qquad \text{for } b, n, np \tag{6}$$

The real power flow on an individual line is a quadratic function of the difference between the voltage phase angles at each end of the line. The power flow variable is a free variable, and there is one variable at each end of the link between two nodes. By convention, a negative value means power is being imported to the node, and a positive value means that power is being exported from the node along the line. The sum of the two power flow variables at each end of the line is the total transmission loss along the line.

Supply and demand (GW):

$$r_b QD_{b,n} - QS_{b,n} + \sum_{np} QP_{b,n,np}\, QTC_s \leq 0 \qquad \text{for } b, n \tag{7}$$

The quantity of electricity demanded at the node in any time period is limited to the quantity of electricity generated at the node and the net sum of electricity imported and exported by lines connected to the node.

Transmission line capacity (GW):

$$uQP_{b,n,np} \leq k_{n,np} \qquad \text{for } b, n, np \tag{8}$$

In any load block, the total amount of power flowing along an individual line connecting two nodes is limited to the maximum (thermal) capacity of that line.

Maximum number of lines:

$$QTC_s \leq j_s \qquad \text{for } s \tag{9}$$

The number of transmission lines along a corridor connecting two nodes is limited to the maximum number of lines the corridor can accommodate.

Non-negative and free variables:

$$QTC, QD, QGO, QGC, QS \geq 0; \qquad QP, \theta, \delta \text{ are free variables.}$$

3C. Economic Interpretation of the Kuhn-Tucker Conditions

The Kuhn-Tucker conditions for the existence of a solution show the economic principles embodied in the model. They contain information on the pricing of electricity at all nodes of the network, the economic dispatch of power stations, the effects of transmission losses on the operation of the system, and the manner in which the capital costs of generation and transmission are recovered. The Kuhn-Tucker conditions are derived from the following Langrangian model. To avoid repetition, the Kuhn-Tucker conditions in the form of the original primal constraints (Equations 2 to 9) are not repeated here.

Lagrangian of the primal problem:

$$\text{Max L} = \sum_b \sum_n a_{b,n} QD_{b,n} + \tfrac{1}{2} c_{b,n} QD^2_{b,n}$$

$$- \sum_b \sum_p m_{b,p} QGO_{b,p} - \sum_p d_p QGC_p - \sum_s t_s QTC_s$$

$$+ \sum_b \sum_p V_{b,p}\left(e_p QGC_p - QGO_{b,p}\right)$$
$$+ \sum_p W_p\left(f_p - QGC_p\right) + \sum_b \sum_n U_{b,n}\left(\sum_p QGO_{b,p} - QS_{b,n}\right)$$
$$+ \sum_b \sum_n \sum_{np} S_{b,n,np}\left(\theta_{b,n} - \theta_{b,\mathrm{np}} - \delta_{b,n,np}\right)$$
$$+ \sum_b \sum_n \sum_{np} X_{b,n,np}\left(aa_{n,np} + bb_{n,np}\delta_{b,n,np} + cc_{n,np}\delta^2_{b,n,np} - uQP_{b,n,np}\right)$$
$$+ \sum_b \sum_n Y_{b,n}\left(QS_{b,n} - \sum_{np}(QP_{b,n,np}QTC_s) - r_b QD_{b,n}\right)$$
$$+ \sum_b \sum_n \sum_{np} Z_{n,np}(k_{n,np} - uQP_{b,n,np}) + \sum_s T_s(j_s - QTC_s)$$

$$QTC, QD, QGO, QGC, QS, T, U, V, W, Y, Z \geqslant 0; \qquad QP, \theta, \delta, X, S \text{ free} \tag{10}$$

$$\frac{\partial L}{\partial QD_{b,n}} = a_{b,n} + c_{b,n}QD_{b,n} - r_b Y_{b,n} \leqslant 0 \qquad \text{for } b,n \text{ and}$$
$$\left(\frac{\partial L}{\partial QD_{b,n}}\right)QD_{b,n} = (a_{b,n} + c_{b,n}QD_{b,n} - r_b Y_{b,n})QD_{b,n} = 0 \qquad \text{for } b,n \tag{11}$$

Equation 11 states that when the quantity of electricity demanded is positive for a load block, the nodal price ($Y_{b,n}r_b$) is equal to the price that consumers are willing to pay for a unit of energy at that node as given by the linear demand function.

$$\frac{\partial L}{\partial QS_{b,n}} = -U_{b,n} + Y_{b,n} \leqslant 0 \qquad \text{for } b,n \text{ and}$$
$$\left(\frac{\partial L}{\partial QS_{b,n}}\right)QS_{b,n} = (-U_{b,n} + Y_{b,n})QS_{b,n} = 0 \qquad \text{for } b,n \tag{12}$$

Equation 12 states that when power is generated at a node in a load block, the nodal price ($Y_{b,n}$) is equal to the (shadow) price of supply ($U_{b,n}$) at that node.[3] If there is no local generation at a node, then the nodal price can be less than the supply price. This equation also means that if generation takes place at the node, then the total revenue received for electricity generated at the node ($Y_{b,n}QS_{b,n}$) equals the revenue paid to the generators at the node ($U_{b,n}QS_{b,n}$).

[3] Nodal price here is not scaled for the hours of duration of the load.

$$\frac{\partial L}{\partial QGO_{b,p}} = -m_{b,p} - V_{b,p} + U_{b,n} \leqslant 0 \qquad \text{for } b,p \text{ and}$$

$$\left(\frac{\partial L}{\partial QGO_{b,p}}\right)QGO_{b,p} = (-m_{b,p} - V_{b,p} + U_{b,n})QGO_{b,p} = 0 \qquad \text{for } b,p \quad (13)$$

Equation 13 states that when a plant p (located at node n) is generating electricity, the supply price ($U_{b,n}$) at the node is equal to the operating cost of the power station ($m_{b,n}$) plus the rent ($V_{b,p}$) earned because its installed capacity is fully utilized in this period. The rent occurs when the power station is operating but it is not the marginal plant being dispatched on the network. This means the price received exceeds the operating cost of production (short run marginal cost). The rents, V, represent a contribution to capacity cost. When the plant is the system marginal plant dispatched, the nodal price equals the plant's short-run marginal cost of production. This condition also shows that the revenue paid to an individual power station is equal to the total operating cost plus the rent accrued, because its generating capacity is limiting in that time period.

$$\frac{\partial L}{\partial QGC_{p}} = -d_p + \sum_b e_p V_{b,p} - W_p \leqslant 0 \qquad \text{for } p \text{ and}$$

$$\left(\frac{\partial L}{\partial QGC_{p}}\right)QGC_{p} = \left(-d_p + \sum_b e_p V_{b,p} - W_p\right)QGC_p = 0 \qquad \text{for } p \qquad (14)$$

Equation 14 states that the sum of the profits or rents [that is, the extent to which the nodal (supply) price exceeds the plant marginal cost] for all load blocks, should equal the unit capital cost of the power station plus any rent it accrues because its installed capacity is limited by the maximum allowed. Plant capacity is installed only if the cost of the capacity can be recovered. However, if there is a restriction on installed capacity that is binding, then the imputed value or opportunity cost of capacity can exceed its unit construction cost. Equations 14 and 13 form the typical peak-load pricing approach used to determine the optimal mix of power stations on the system.

$$\frac{\partial L}{\partial \theta_{b,n}} = \sum_{np}(S_{b,n,np} - S_{b,n,np}) = 0 \qquad \text{for } b,n \neq 1 \qquad (15)$$

Equation 15 states that the sum of the differences in (shadow) prices of changing the phase angles at each end of links connected to the node must sum to zero. The (shadow) price of changing a phase angle is defined in the next equation. Equation 15 illustrates the network externality. It can be thought of as a zero profit condition, so that there are no gains to be made by adjusting phase angles and changing power flows (arbitrage), taking into account that adjusting one power flow causes simultaneous adjustments in other power flows.[4]

$$\frac{\partial L}{\partial \delta_{b,np}} = -S_{b,n,np} + (bb_{n,np} - 2cc_{n,np}\delta_{b,n,np})X_{b,n,np} = 0 \qquad \text{for } b,n,np \tag{16}$$

Equation 16 states that the (shadow) price of a difference in phase angle over a line is equal to the marginal power flow (with respect to the difference in the phase angle) times the (shadow) price of power flow along a single line.

$$\frac{\partial L}{\partial QP_{b,n,np}} = -X_{b,n,np}u - Y_{b,n}QTC_s - Z_{n,np}u = 0 \qquad \text{for } b,n,np \tag{17}$$

Equation 17 implies that the nodal price of electricity must equal the sum of the (shadow) price of power flow along an individual line divided by the number of lines and the shadow price on the maximum thermal capacity of an individual line divided by the number of lines. If individual transmission lines are operating at less than maximum capacity, then the nodal price is equal to the shadow price of power flow along the line. If capacity is limiting, then the nodal price is equal to the shadow price of power flow plus a rent.

$$\frac{\partial L}{\partial QTC_s} = -t_s - \sum_b\sum_n\sum_{np} Y_{b,n}QP_{b,n,np} - T_s \leqslant 0 \qquad \text{for } s \text{ and}$$

$$\left(\frac{\partial L}{\partial QTC_s}\right)QTC_s = \left(-t_s - \sum_b\sum_n\sum_{np} Y_{b,n}QP_{b,n,np} - T\right)QTC_s = 0 \qquad \text{for } s \tag{18}$$

Equation 18 states that if an individual line is built, then the net revenue received from purchasing and selling power over the line in all time periods must be equal to the unit capacity cost of the line plus any imputed rent it receives because line capacity is limiting. This represents a zero profit condition.

[4] It is analogous to the concept of marginal value product used in economics, whereby the price for an input is equated with the marginal physical product times the output price.

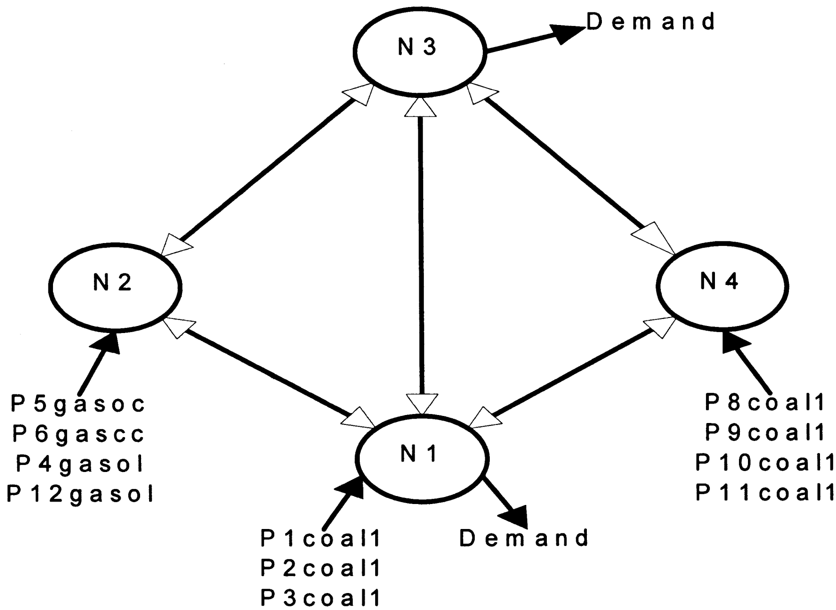

Figure 1. Possible structure of the hypothetical electricity market.

4. STRUCTURE OF AN ELECTRICITY MARKET

A hypothetical market is used in this study. It has the possibility of four nodes, which can be connected as shown in Figure 1.

Demand for electricity takes place at nodes N1 and N3. Generation can take place at nodes N1, N2, and N4.

The aim is to determine the quantities of electricity consumed at nodes N1 and N3 and the location and capacity of transmission lines and power stations, which result in an economically efficient market, representing a long-run competitive equilibrium.

5. DEMAND

For each node where demand takes place, the load duration curve provides a useful description of demand across the year (see Figure 2).

With a single node, the load duration curve is obtained by arranging the hourly loads at the node during the year into descending order (see Scherer 1977; Turvey and Anderson 1977). However, in a network with more than one demand node, there

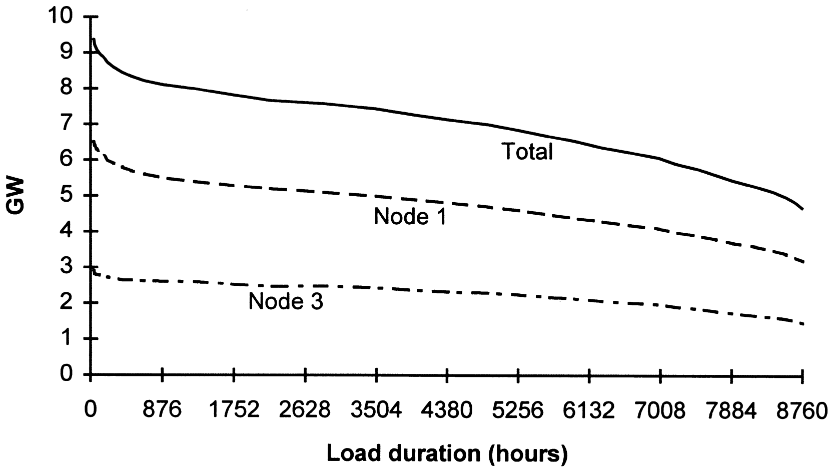

Figure 2. Load duration curve for nodes N1 and N3.

is an additional complexity introduced because the demands at each node must be for the same points in time.

To ensure that the demands at each node are coincident in time, the following procedure is used. First, the load duration curve for one node (N1) was derived in the manner described above. Second, the load duration curve for the second demand node was determined using the chronological order of loads from the load duration curve at N1. This method was chosen for convenience. However, the method introduces some averaging issues into load duration curves of nodes other than the base node, N1. This means that the shape of the implied load duration curve for N3 may differ from that of its actual load duration curve. It would be useful to investigate the effects of any bias and consider alternative methods for determining load blocks.

Thirty-four demand periods are defined by dividing the load duration curve for the first node into 100-MW load intervals. This create load blocks of unequal duration, measured in hours.

Each of the 34 load blocks is assumed to have an independent linear demand function that relates the amount of electricity demanded to its price, and is given by:

$$\text{Price} = a + w \text{ Quantity}. \tag{19}$$

Any effects other than price (e.g., weather) are assumed exogenous and are implicit in the constant term of the demand function.

The parameters of each demand function, a and w, are calibrated using assumed prices, quantities, and an own-price elasticity of demand, E. The parameters are given by:

$$a = \text{Price } (1 - 1/E) \text{ and } w = 1/E * \text{Price/Quantity} \quad (20)$$

The price-elasticity of demand is assumed to be −0.3.

6. TRANSMISSION

The special feature of this model is the incorporation of power flow equations into a meshed network. The presentation here is based on that of Chao and Peck (1996). The real power flow in a network, based on Kirchoff's laws, is given by:

$$Q_{ij} = G_{ij}V_{ij}^2 - G_{ij}V_iV_j\cos(\theta_i - \theta_j) + Y_{ij}V_iV_j\sin(\theta_i - \theta_j) \quad (21)$$

where Q is the power flow (measured in Gigawatts) from node i to node j, G, V, and Y are parameters relating to resistance, voltage, and admittance.[5] θ is the voltage phase angle at each node (measured in radians). It is the difference in phase angles between two nodes that determines the magnitude of the power flow between two nodes (see Chao and Peck, 1996, p. 35). The voltage phase angle and the power flow can be negatively valued. When the power flow for Q_{ij} is negative, the flow is from node j to node i. The transmission loss along the line is given by $Q_{ij} + Q_{ji}$.

Under normal operating conditions, the real power flow equations can be approximated by the following quadratic function (see Chao and Peck, 1996, p. 36, pp. 41–42, and pp. 50–52):

$$Q_{ij} = G_{ij}(V_{ij}^2 - V_iV_j) + Y_{ij}V_iV_j(\theta_i - \theta_j) + 1/2G_{ij}V_iV_j(\theta_i - \theta_j)^2 \text{ for } i,j \quad (22)$$

In this power-flow equation, marginal power flow increases at a decreasing rate with respect to the phase angle and the function is convex.

Each of the nodes are connected by corridors made up of a number of transmission lines. The assumed technical properties of the transmission lines and the assumed maximum transmission capacities along the lines are described in Table 1.

The distance between nodes plays an important role in determining the overall characteristics of the line. For example,

[5] $G_{i,j} \equiv r_{i,j}/(r_{i,j}^2 + x_{i,j}^2)$ and $Y_{i,j} \equiv x_{i,j}/(r_{i,j}^2 + x_{i,j}^2)$, where $r_{i,j}$ is the product of the resistance per kilometer and distance shown in Table 1, and $x_{i,j}$ is the product of the impedance per kilometer and distance shown in Table 1.

Table 1: Characteristics of the Network

			Transmission lines					
	Link capacity (MW)	Distance (km)	No	Voltage (kV)	Resistance per km (Ω/km)	Impedance per km (Ω/km)	Construction cost ($'000/km)	Life (years)
N1–N2	2 500	25	5	33	0.03	0.3	383	50
N1–N3	1 875	150	5	50	0.025	0.25	391	50
N1–N4	3 750	150	5	33	0.03	0.3	389	50
N2–N3	3 000	160	5	33	0.03	0.3	383	50
N3–N4	5 000	400	5	33	0.03	0.3	678	50

Table 2: Plant Data

Node	Power station	Availability	Maximum allowable capacity MW	Fuel cost $-MWh	Capacity $m-MW	Life Years
Node 1	P1	1	1 270	14.8	1.2	30
	P2	1	861	14.3	1.4	30
	P3	1	2 540	14.0	1.3	30
	P4	1	500	18.5	1.2	30
Node 2	P5	1	unlimited	32	0.5	30
	P6	1	unlimited	23.5	0.85	25
	P4	1	500	18.5	1.2	30
	P12	1	500	18.5	0.8	30
Node 4	P8	1	1 268	12.1	1.45	30
	P9	1	960	13.0	1.3	30
	P10	1	1 268	12.2	1.4	30
	P11	1	890	14.8	1.25	30

although the same type of line can connect nodes N2–N3 and nodes N1–N2, the overall characteristics of the line vary significantly. In particular, the total resistance along a line N2–N3 is 6.4 times as great as between N1–N2, because the distance between N1–N3 is 6.4 times as large as N1.

7. PRODUCTION MODEL

The data for the power stations located at each node are shown in Table 2. There are several types of plants available, ranging from baseload (coal) through to peaking (gas).

8. RESULTS AND DISCUSSION

In this section, the physical and quantity variables are presented first, followed by price, revenue, and cost results.

8A. Results in Summary

Figure 3 summarizes the optimal solution of the numerical model. It shows the loading of power stations in each time block, the number of power lines on each link, and the direction of power flows in each block.

Generation occurs at nodes 1, 2, and 4. Node 4 provides baseload power. It is able to have four coal-fired power stations operating,

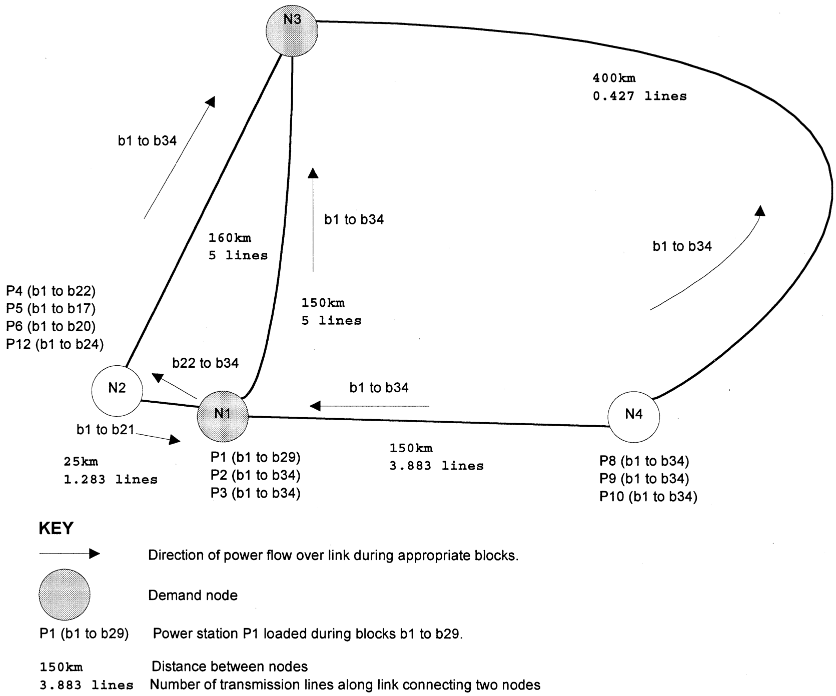

Figure 3. Optimal transmission structure, power flows, and loading.

but at the optimum, only three are built, all of which run at maximum capacity over all time blocks. Three power stations are built at node 1. Two of these stations (P3 and P2) provide baseload capacity, and the third (P1) provides intermediate and peaking capacity. Node 2 has four power stations that provide intermediate and peaking capacity.

8B. Demand

The levels of demand for each node over the load blocks are shown in Figure 4. The load duration curve has the expected shape. However, there is one point worth noting. Demand reaches a peak reasonably early in the load duration, which is different to that in the original load curve (Figure 2). The reason is that this model simulates a long-run equilibrium using peak load principles. A peaking plant is only installed if there is sufficient profit (revenue exceeds total short-run marginal costs) to cover the long-run marginal cost of capacity. For the very last units of peaking

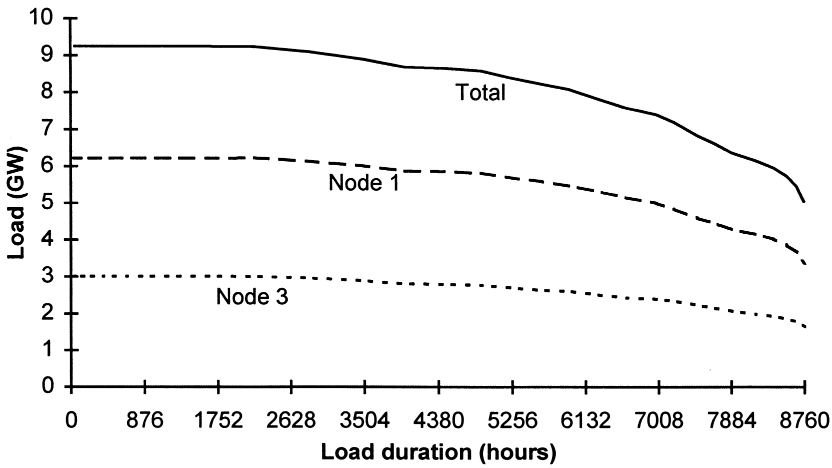

Figure 4. Load at demand nodes (GW).

capacity installed, this profit margin is generated by rationing demand so that price exceeds short-run marginal cost. This is analogous (in proposed electricity markets) to setting price equal to the value of foregone load when generating capacity is limiting.

In the original load data (Figure 2), prices to consumers did not vary as much by load block, particularly in the extreme peak periods; this allowed demand to rise above the levels resulting here.

8C. Generation by Location

Figures 5, 6, and 7 show the installed capacity and merit order dispatch of power stations located at three nodes in the network. It follows the expected pattern. The point raised in the previous section is evident here. Installed capacity reached a maximum well before the extreme peak period.

8D. Network Flows

The average power flow for each link over time is shown in Figure 8. A negative number indicates that the direction of flow is the reverse of that implied in the name. For example, for link N1–N4, the flow is from N4 to N1.

The flow is steady over time for some links because they represent major connections between demand and base load generation.

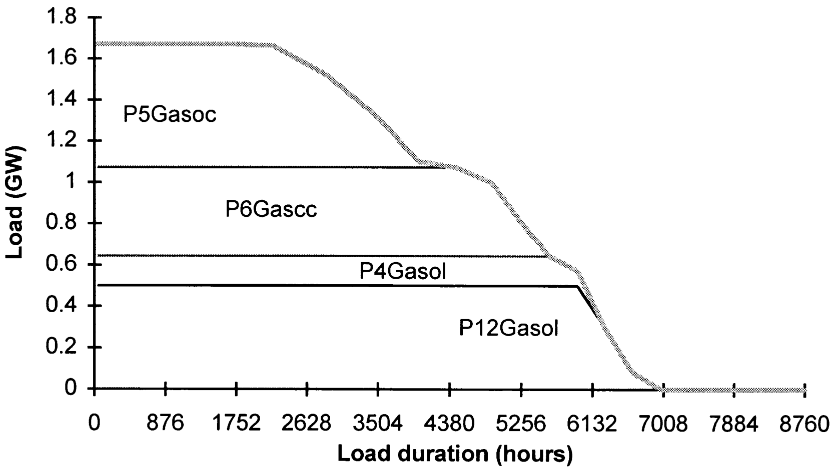

Figure 5. Merit order generation at node 2.

Other flows vary over time, and one (N1–N2) reverses direction. These links are associated with peaking and intermediate power stations. The dynamics of power flows vary as these plants are dispatched through the merit order process.

The average losses are shown in Figure 9. For the lines with large steady loads, the losses are stable. For lines where power flows vary and change direction, the losses vary over time.

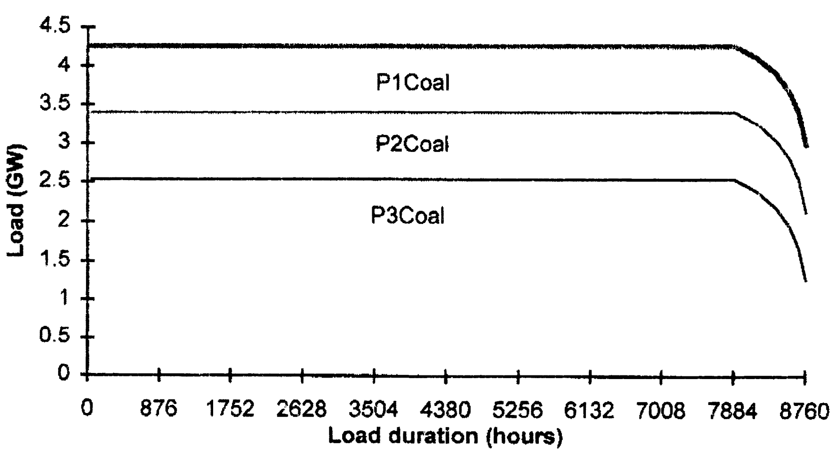

Figure 6. Merit order generation at node 1.

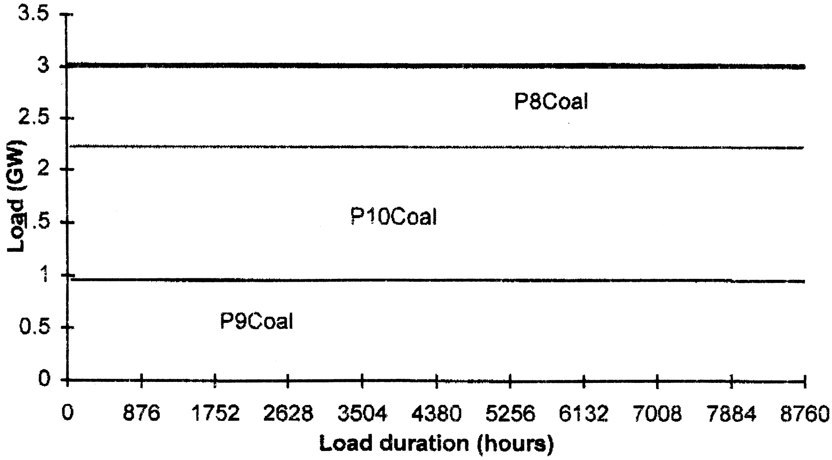

Figure 7. Merit order generation at node 4.

8E. Prices

As expected, the nodal prices are high in peak-load periods and low in base-load periods (Figure 10).

An interesting feature occurs at the very peak demand periods. Here, the welfare optimizing solution involves rationing demand rather than increasing the capacity of plants to satisfy very high

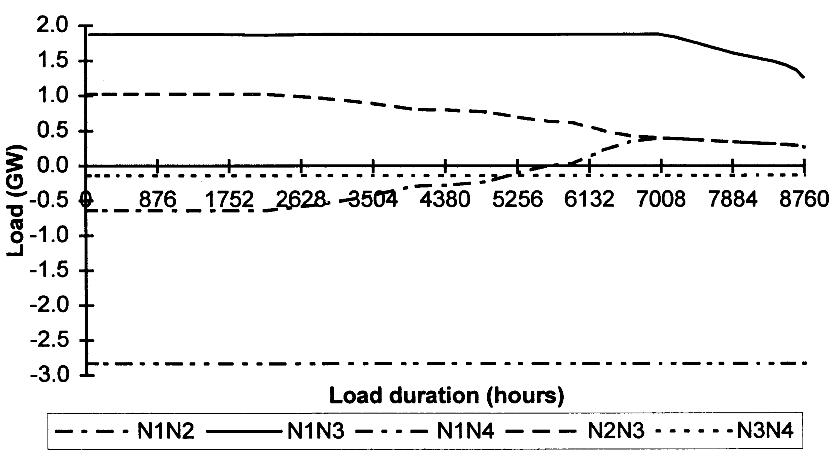

Figure 8. Average power flow by link (GW).

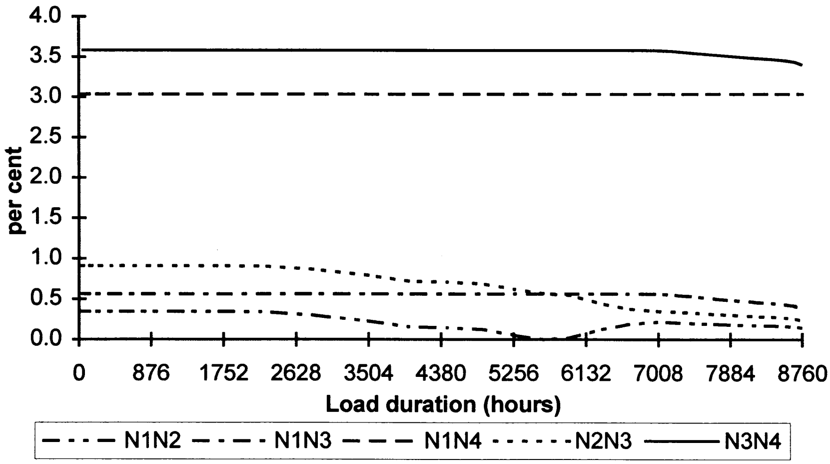

Figure 9. Losses as a percentage of power flows.

demands for relatively short periods of duration. Consequently, at the very peak demands prices increase even further.

There are some periods of time when relatively large differences emerge between nodal prices.

The variation between nodal prices arising from capacity constraints on transmission and the peak load-type cost recovery of

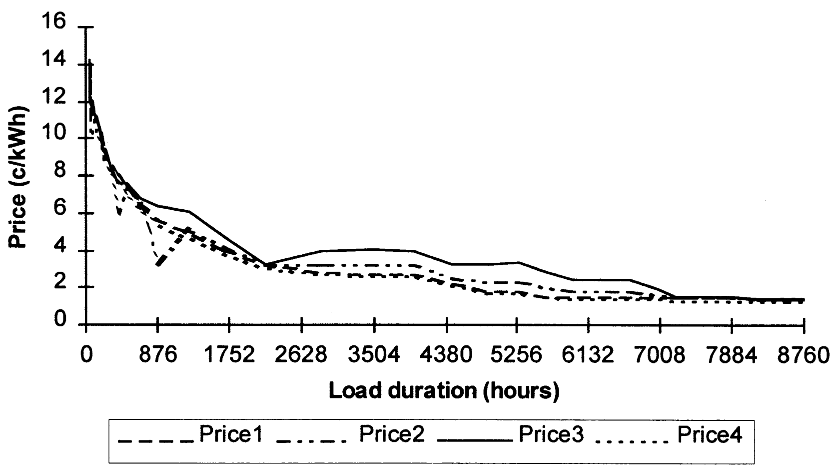

Figure 10. Nodal prices.

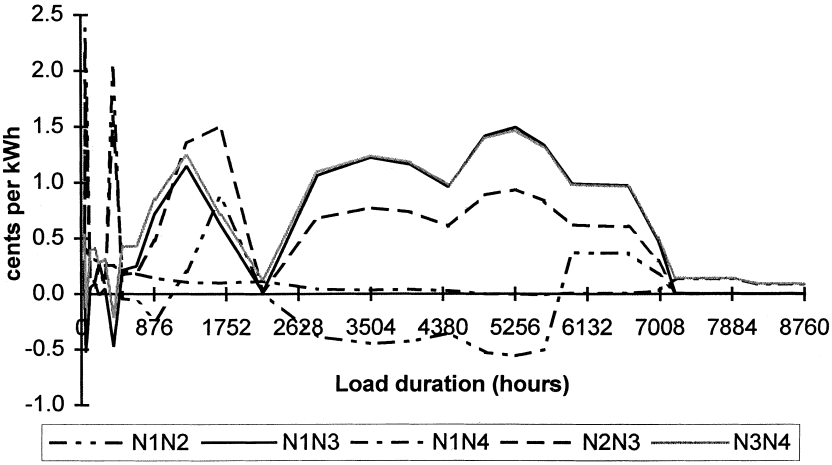

Figure 11. Transmission prices by link.

transmission costs are evident in Figure 11. Transmission prices are calculated as the revenue from buying and selling electricity over the link in each time period divided by the average quantity of power flow over the link during each period. This is the average merchandising surplus[6] per unit of power.

At some points in time, the transmission price is negative. This is the case referred to by Wu et al. (1996, p. 17), whereby economic dispatch can require power to flow from a node with a high price to one with a low price. In the presence of a transmission constraint and with Kirchoff's laws in operation, it is welfare maximizing and economically efficient for one link to make a loss at a point in time. This is because the change in power flow allows the generation and transmission of a lower cost source of electricity than could be obtained otherwise, resulting in net economic benefits.

8F. Sales Revenue

The revenue from sales to customers in each load block is shown in Table 3. This revenue is defined as the nodal price times the quantity sold. This is derived from the Kuhn-Tucker condition in Equation 11, in which total revenue is the nodal price times the quantity demanded, based on the linear demand functions.

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[6] Merchandizing surplus is a term used by Wu et al. (1996) and Chao and Peck (1996).

Table 3: Revenue from Sales to Customers, by Node and Load Block ($Million)

	Node		
Load block	**N1**	**N3**	**Total**
B1	32.26	15.89	48.15
B2	10.23	4.79	15.02
B3	17.80	8.45	26.26
B4	22.54	11.07	33.61
B5	38.32	18.90	57.22
B6	26.89	13.13	40.01
B7	40.10	19.71	59.80
B8	52.21	24.02	76.22
B9	49.37	24.84	74.21
B10	65.43	33.21	98.64
B11	78.63	43.32	121.95
B12	118.94	71.62	190.56
B13	104.56	58.89	163.45
B14	103.28	50.59	153.87
B15	114.35	76.63	190.98
B16	106.53	74.41	180.94
B17	80.09	54.64	134.73
B18	60.31	41.58	101.89
B19	46.75	39.84	86.59
B20	37.65	32.89	70.55
B21	29.28	25.96	55.23
B22	27.42	21.95	49.37
B23	26.32	20.74	47.06
B24	26.15	20.53	46.68
B25	26.69	17.05	43.74
B26	15.12	7.40	22.52
B27	19.21	9.45	28.65
B28	14.06	6.85	20.90
B29	13.52	6.59	20.11
B30	16.44	7.93	24.36
B31	12.50	6.05	18.55
B32	8.54	4.14	12.68
B33	6.17	3.01	9.18
B34	4.42	2.16	6.58
Total	1452.05	878.23	2330.28

As will be shown, the revenue from sales to customers is ultimately paid to the transmission network and generators.

The cost of generation (2144.35 in Table 5) plus the cost of transmission (185.93 in Table 4) is equal to the total value of sales to customers (2330.28 in Table 3).

8G. Transmission Revenue and Cost

Table 4 shows the revenue paid to each link of the transmission network.

This revenue is referred to as merchandising surplus by Wu et al. (1996) and Chao and Peck (1996). It is the difference between the revenue from sales (exports from the network) and the cost of energy imported (injected) into the network. For both the entire network and each link, the revenue received is equal to the total cost of transmission (including any rents arising from the number of lines on the link being constrained). This is a zero profit condition on the network and each link, and is based on the Kuhn-Tucker conditions in Equation 18.

A point to note is that some nodes received negative revenue in some periods (load blocks). This situation arises when power flows along a line from a node with high price to one with a lower price. The reason this occurs is because of the externality arising from the power flow equations (based on Kirchoff's laws) when there are transmission capacity constraints. For instance, the flow of power from node 2 to node 1, node 1 to node 4, and node 1 to node 3 enables more power to flow from node 2 to node 4. These power transfers help alleviate the congestion between node 2 and node 3. This enables consumers to source lower cost electricity, with some of the savings used to offset the loss on the power flows that made it possible.

8H. Generation Revenue and Costs

The revenues received for total generation at each node in each load block are shown in Table 5. The grand total for all nodes is 2144.35. This result is based on the second Kuhn-Tucker condition in Equation 12, where the revenue received at the node for local generation is equal to the revenue paid for local generation.

The revenue received at each node is then paid to the individual generators that produce power at each node in each load block, as shown in Table 6.

This revenue is the product of the nodal price and the output of each plant in each load period. This result is embedded in the second Kuhn-Tucker condition in Equation 13.

Table 7 shows the total short-run operating costs of power stations in each load block. These results are also embedded in the Kuhn-Tucker condition in Equation 13.

Table 4: Revenue Received by the Transmission Network and Transmission Cost ($Million)

	Revenue received by each link					
	N1–N2	**N1–N3**	**N1–N4**	**N2–N3**	**N3–N4**	**Total revenue**
Load block						
B1	0.01	0.08	0.44	0.06	0.03	0.61
B2	0.20	−0.13	0.15	0.25	0.00	0.47
B3	0.25	−0.15	0.25	0.32	0.00	0.68
B4	0.01	0.04	0.31	0.04	0.02	0.41
B5	0.01	0.11	0.52	0.07	0.03	0.74
B6	0.08	−0.01	0.37	0.13	0.02	0.59
B7	0.01	0.07	0.55	0.06	0.03	0.73
B8	1.38	−0.92	0.77	1.71	−0.03	2.90
B9	−0.03	0.44	0.65	0.20	0.06	1.33
B10	−0.06	0.76	0.85	0.32	0.10	1.97
B11	−0.34	3.02	0.90	1.11	0.27	4.96
B12	0.50	8.33	1.15	5.37	0.67	16.02
B13	2.31	4.81	1.15	6.35	0.41	15.04
B14	0.04	0.17	1.69	0.15	0.10	2.15
B15	−1.40	13.16	0.83	4.35	1.01	17.95
B16	−1.21	14.72	0.66	4.44	1.12	19.73
B17	−0.60	10.67	0.55	2.92	0.81	14.35
B18	−0.44	8.37	0.40	2.26	0.64	11.22
B19	−0.52	11.70	0.01	3.03	0.86	15.09
B20	−0.21	10.32	−0.03	2.41	0.76	13.25
B21	−0.01	8.52	−0.05	1.82	0.63	10.91
B22	0.05	6.27	0.07	1.28	0.47	8.13
B23	0.26	6.13	0.07	1.04	0.46	7.95
B24	0.43	6.24	0.07	0.89	0.46	8.10
B25	0.26	3.26	0.27	0.43	0.25	4.46
B26	0.00	0.03	0.82	0.00	0.04	0.90
B27	0.00	0.03	1.10	0.00	0.06	1.19
B28	0.00	0.02	0.83	0.00	0.04	0.90
B29	0.00	0.02	0.83	0.00	0.04	0.89
B30	0.00	0.03	0.69	0.00	0.04	0.75
B31	0.00	0.02	0.54	0.00	0.03	0.59
B32	0.00	0.01	0.38	0.00	0.02	0.42
B33	0.00	0.01	0.29	0.00	0.01	0.32
B34	0.00	0.01	0.23	0.00	0.01	0.25
Total revenue	1.00	116.16	18.28	41.02	9.47	185.93
Transmission costs						
Rent for the lines	0.00	92.19	0.00	15.97	0.00	108.16
Line construction cost	1.00	23.97	18.28	25.05	9.47	77.77

Table 5: Revenue Received for Energy Produced at Each Node, by Load Block ($Million)

Load block	N1	N2	N4	Total
B1	24.21	8.62	14.71	47.54
B2	7.68	2.22	4.65	14.55
B3	13.37	4.11	8.11	25.58
B4	16.92	6.01	10.27	33.20
B5	28.76	10.24	17.47	56.47
B6	20.18	6.99	12.25	39.43
B7	30.10	10.70	18.28	59.08
B8	39.19	10.40	23.74	73.32
B9	37.06	13.30	22.53	72.88
B10	49.12	17.68	29.87	96.67
B11	59.02	21.95	36.02	117.00
B12	89.28	30.55	54.71	174.54
B13	78.49	21.98	47.94	148.41
B14	77.50	27.46	46.77	151.73
B15	87.12	32.22	53.69	173.03
B16	82.89	27.09	51.22	161.21
B17	63.72	17.33	39.34	120.38
B18	48.14	12.80	29.73	90.67
B19	37.60	10.36	23.54	71.51
B20	30.95	6.94	19.42	57.30
B21	24.48	4.47	15.38	44.33
B22	23.04	3.59	14.61	41.24
B23	22.85	1.85	14.42	39.11
B24	23.27	0.52	14.79	38.58
B25	23.94		15.34	39.28
B26	13.36		8.26	21.62
B27	16.39		11.07	27.46
B28	11.66		8.34	20.01
B29	10.87		8.34	19.21
B30	12.80		10.81	23.61
B31	9.45		8.51	17.96
B32	6.22		6.04	12.26
B33	4.27		4.59	8.86
B34	2.76		3.57	6.33
Total	1126.65	309.37	708.32	2144.35

The difference between the revenue received and the operating cost in each load block is the gross margin for the plant. These are shown in Table 8, and represent a contribution towards the capital cost of each power station. The sum of these over all load blocks is the total gross margin. Again, these results arise from the Kuhn-Tucker condition in Equation 13.

Table 6: Revenue Received by Each Power Station, in Each Load Block ($Million)

Load block	P1	P2	P3	P4	P5	P6	P8	P9	P11	P12	Total
B1	6.58	4.46	13.17	0.74	3.09	2.21	3.85	4.68	6.18	2.58	47.54
B2	2.09	1.42	4.17	0.19	0.80	0.57	1.22	1.48	1.96	0.66	14.55
B3	3.63	2.46	7.27	0.35	1.47	1.05	2.12	2.58	3.41	1.23	25.58
B4	4.60	3.12	9.20	0.52	2.16	1.54	2.69	3.27	4.32	1.80	33.20
B5	7.82	5.30	15.64	0.88	3.67	2.63	4.57	5.56	7.34	3.06	56.47
B6	5.49	3.72	10.97	0.60	2.51	1.79	3.20	3.90	5.15	2.09	39.43
B7	8.18	5.55	16.37	0.92	3.84	2.74	4.78	5.82	7.68	3.20	59.08
B8	10.66	7.22	21.31	0.89	3.73	2.67	6.21	7.55	9.98	3.11	73.32
B9	10.08	6.83	20.15	1.14	4.77	3.41	5.89	7.17	9.47	3.98	72.88
B10	13.35	9.05	26.71	1.52	6.34	4.53	7.81	9.50	12.55	5.29	96.67
B11	16.05	10.88	32.10	1.89	7.88	5.63	9.42	11.46	15.14	6.56	117.00
B12	24.27	16.46	48.55	2.62	10.96	7.83	14.31	17.41	22.99	9.13	174.54
B13	21.34	14.47	42.68	1.89	7.88	5.63	12.54	15.26	20.15	6.57	148.41
B14	21.07	14.29	42.14	2.37	9.78	7.07	12.23	14.88	19.66	8.24	151.73
B15	23.69	16.06	47.37	3.04	9.55	9.06	14.04	17.08	22.57	10.57	173.03
B16	22.54	15.28	45.08	2.96	5.02	8.82	13.40	16.30	21.53	10.29	161.21
B17	17.32	11.75	34.65	2.26	0.47	6.74	10.29	12.52	16.53	7.86	120.38
B18	13.09	8.87	26.18	1.71		5.12	7.78	9.46	12.50	5.97	90.67
B19	10.22	6.93	20.45	1.49		3.69	6.16	7.49	9.89	5.18	71.51
B20	8.41	5.70	16.83	1.25		1.35	5.08	6.18	8.16	4.34	57.30
B21	6.66	4.51	13.31	1.00			4.02	4.89	6.46	3.47	44.33
B22	5.98	4.32	12.74	0.46			3.82	4.65	6.14	3.14	41.24
B23	6.00	4.26	12.58				3.77	4.59	6.06	1.85	39.11
B24	5.99	4.37	12.90				3.87	4.70	6.21	0.52	38.58

(Continued)

Table 6: *Continued*

Load block	P1	P2	P3	P4	P5	P6	P8	P9	P11	P12	Total
B25	5.79	4.60	13.56				4.01	4.88	6.44		39.28
B26	2.75	2.69	7.92				2.16	2.63	3.47		21.62
B27	2.17	3.60	10.62				2.90	3.52	4.66		27.46
B28	0.95	2.71	8.00				2.18	2.65	3.51		20.01
B29	0.15	2.71	8.00				2.18	2.65	3.51		19.21
B30		3.40	9.40				2.83	3.43	4.55		23.61
B31		2.67	6.77				2.23	2.70	3.58		17.96
B32		1.90	4.32				1.58	1.91	2.54		12.26
B33		1.44	2.83				1.20	1.45	1.93		8.86
B34		1.12	1.64				0.94	1.13	1.50		6.33
Total	286.93	214.14	625.58	30.67	83.94	84.09	185.27	225.33	297.72	110.68	2144.35

Table 7: Total Operating Cost for Each Power Station, by Load Block ($Million)

Load block	P1	P2	P3	P4	P5	P6	P8	P9	P10	P12
B1	0.69	0.45	1.34	0.10	0.71	0.37	0.35	0.46	0.57	0.34
B2	0.25	0.16	0.47	0.03	0.25	0.13	0.12	0.16	0.20	0.12
B3	0.45	0.29	0.87	0.06	0.46	0.24	0.23	0.30	0.37	0.22
B4	0.61	0.39	1.18	0.09	0.63	0.33	0.31	0.41	0.50	0.30
B5	1.10	0.71	2.13	0.16	1.13	0.59	0.56	0.73	0.91	0.54
B6	0.86	0.55	1.66	0.12	0.88	0.46	0.44	0.57	0.71	0.42
B7	1.39	0.89	2.68	0.20	1.42	0.74	0.71	0.92	1.14	0.68
B8	1.96	1.26	3.79	0.28	2.00	1.05	1.00	1.30	1.61	0.96
B9	2.04	1.31	3.95	0.29	2.09	1.09	1.04	1.36	1.68	1.00
B10	3.06	1.96	5.92	0.43	3.13	1.64	1.56	2.03	2.52	1.51
B11	4.21	2.70	8.13	0.59	4.30	2.26	2.14	2.79	3.46	2.07
B12	7.27	4.66	14.05	1.03	7.43	3.90	3.69	4.83	5.98	3.58
B13	7.72	4.95	14.91	1.09	7.88	4.14	3.92	5.12	6.35	3.80
B14	9.68	6.21	18.70	1.37	9.78	5.19	4.92	6.43	7.97	4.76
B15	12.41	7.96	23.99	1.76	9.55	6.66	6.31	8.24	10.22	6.11
B16	12.09	7.75	23.36	1.71	5.02	6.48	6.14	8.03	9.95	5.95
B17	9.23	5.92	17.83	1.30	0.47	4.95	4.69	6.13	7.60	4.54
B18	8.74	5.60	16.89	1.24		4.69	4.44	5.80	7.19	4.30
B19	8.29	5.32	16.02	1.17		3.69	4.21	5.50	6.82	4.08
B20	6.94	4.45	13.42	0.98		1.35	3.53	4.61	5.71	3.42
B21	6.41	4.11	12.39	0.91			3.26	4.26	5.28	3.16
B22	5.98	4.09	12.31	0.46			3.24	4.23	5.24	3.14
B23	6.00	4.03	12.15				3.19	4.18	5.18	1.85
B24	5.99	4.14	12.47				3.28	4.28	5.31	0.52

(Continued)

Table 7: *Continued*

Load block	P1	P2	P3	P4	P5	P6	P8	P9	P10	P12
B25	5.79	4.35	13.10				3.44	4.50	5.58	
B26	2.75	2.54	7.65				2.01	2.63	3.26	
B27	2.17	3.40	10.26				2.70	3.52	4.37	
B28	0.95	2.57	7.73				2.03	2.65	3.29	
B29	0.15	2.57	7.73				2.03	2.65	3.29	
B30		3.33	9.40				2.63	3.43	4.27	
B31		2.62	6.77				2.07	2.70	3.36	
B32		1.86	4.32				1.47	1.91	2.39	
B33		1.41	2.83				1.12	1.45	1.81	
B34		1.10	1.64				0.87	1.13	1.41	
Total	135.19	105.59	312.05	15.36	57.12	49.95	83.64	109.25	135.51	57.38

Table 8: Gross Margin for Each Power Station, by Load Block ($Million)

Load block	P1	P2	P3	P4	P5	P6	P8	P9	P10	P12	Total
B1	5.89	4.02	11.83	0.64	2.38	1.84	3.49	4.22	5.61	2.23	42.15
B2	1.84	1.26	3.70	0.16	0.55	0.44	1.09	1.32	1.75	0.54	12.65
B3	3.18	2.18	6.40	0.29	1.02	0.81	1.89	2.28	3.04	1.01	22.09
B4	3.99	2.73	8.02	0.43	1.53	1.21	2.38	2.86	3.81	1.50	28.45
B5	6.72	4.59	13.51	0.72	2.55	2.03	4.01	4.83	6.43	2.52	47.92
B6	4.63	3.17	9.32	0.48	1.63	1.33	2.77	3.33	4.44	1.67	32.77
B7	6.79	4.66	13.68	0.72	2.42	2.00	4.08	4.89	6.54	2.52	48.31
B8	8.69	5.97	17.52	0.62	1.73	1.61	5.21	6.25	8.36	2.14	58.11
B9	8.03	5.52	16.21	0.85	2.68	2.31	4.85	5.81	7.79	2.97	57.04
B10	10.29	7.09	20.79	1.09	3.22	2.89	6.26	7.47	10.03	3.78	72.90
B11	11.84	8.18	23.97	1.29	3.58	3.37	7.28	8.67	11.68	4.49	84.36
B12	17.01	11.80	34.50	1.60	3.53	3.94	10.62	12.58	17.01	5.56	118.13
B13	13.62	9.52	27.76	0.80		1.50	8.62	10.13	13.80	2.77	88.51
B14	11.39	8.08	23.44	1.00		1.88	7.32	8.46	11.69	3.48	76.73
B15	11.27	8.10	23.38	1.28		2.41	7.74	8.84	12.35	4.46	79.82
B16	10.45	7.53	21.72	1.25		2.34	7.26	8.27	11.58	4.34	74.74
B17	8.10	5.83	16.82	0.95		1.79	5.60	6.39	8.94	3.31	57.72
B18	4.35	3.27	9.29	0.48		0.43	3.34	3.66	5.30	1.67	31.78
B19	1.93	1.61	4.43	0.32			1.95	1.99	3.07	1.10	16.40
B20	1.47	1.25	3.41	0.27			1.55	1.57	2.45	0.92	12.89
B21	0.24	0.40	0.92	0.09			0.77	0.64	1.19	0.32	4.56
B22		0.23	0.43				0.58	0.42	0.90		2.56
B23		0.23	0.42				0.58	0.41	0.88		2.52
B24		0.24	0.44				0.59	0.42	0.90		2.59

(Continued)

Table 8: *Continued*

Load block	P1	P2	P3	P4	P5	P6	P8	P9	P10	P12	Total
B25		0.25	0.46				0.57	0.38	0.87		2.52
B26		0.15	0.27				0.15		0.21		0.78
B27		0.19	0.36				0.20		0.29		1.04
B28		0.15	0.27				0.15		0.22		0.78
B29		0.15	0.27				0.15		0.22		0.78
B30		0.07					0.20		0.28		0.55
B31		0.06					0.15		0.22		0.43
B32		0.04					0.11		0.16		0.31
B33		0.03					0.08		0.12		0.23
B34		0.02					0.06		0.09		0.18
Total	151.75	108.54	313.53	15.31	26.82	34.15	101.63	116.08	162.21	53.30	1083.31

Table 9: Summary of Power Station Revenue and Costs ($Million)

	Total revenue	Total variable cost[1]	Construction costs[2]	Plant capacity rents[3]
P1	286.93	135.19	135.37	16.37
P2	214.14	105.59	107.07	1.47
P3	625.58	312.05	293.31	20.22
P4	30.67	15.36	15.31	
P5	83.94	57.12	26.65	0.17
P6	84.09	49.95	34.15	
P8	185.27	83.64	101.63	
P9	225.33	109.25	110.86	5.22
P10	297.72	135.51	157.69	4.52
P12	110.68	57.38	35.53	17.77

[1] Total variable costs that include fuel costs and other variable costs of running power station.

[2] Construction costs that are equal to the annualized cost of building the power station of the capacity installed.

[3] Plant capacity rents are the rents accruing because the installed capacity of the power station is at the maximum allowed.

Table 9 shows that the revenue received by each power station is equal to total variable cost plus construction costs and plant capacity rents. It shows that there are zero profits. The plant capacity rents represent the opportunity cost of the limit on installed capacity of the particular power station. Together, Kuhn-Tucker conditions in Equation 13 and 14 determine these results.

9. CONCLUSION AND FURTHER RESEARCH

The model presented here provides insights into economic issues arising in electricity markets. It has integrated demand, transmission, and generation into a single model to simulate the economically efficient operation of an electricity market.

The pricing rules embodied in this long-run model allow nodal and transmission prices to be optimized over time and space. In this model, the costs of generation and transmission are fully recovered.

In the illustrative example, transmission prices fluctuate considerably, reflecting the externality effects of Kirchoff's laws on the flows of electricity embedded within a peak-load pricing approach to transmission cost recovery.

The approach used here contrasts with other models of transmission pricing where losses play a key role in determining nodal prices and Kirchoff's laws are omitted.

This model provides a framework against which to assess other approaches to transmission pricing, such as Backerman, Rassenti, and Smith (1996), and the approach being proposed by the Australian National Electricity Code (NEMMCO, 1996).

Future research could use the framework in this paper to compare the models of transmission pricing arrangements being proposed for access pricing to electricity network in deregulated electricity markets.

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