Positive Mathematical Programming and Maximum Entropy :

Economic tools for applied production analysis

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INRA Seminar on Production Economics

28/29 November 2000

Paris

General Principles of Mathematical Programming Models

Main purpose: Modelling the economic behaviour of rational agents

Linear Programming models:

$$\underset{x}{Max} \quad c.x$$

$$s.t. \quad A.x \leq B$$

corner response ; stepwise response

Positive Mathematical Programming models:

$$\underset{x}{Max} \quad F(x)$$

$$s.t. \quad A.x \leq B$$

Replication of base year data (Positive)
Non jumpy behaviour
Few data (compared to econometric models)

Illustration with a simple example: one crop farmer, two activities, one limiting factor

Representation of the behaviour of this farmer using a **LP** model:

$$\underset{x_1,x_2}{Max} \quad \pi_1 x_1 + \pi_2 x_2$$

$$s.t. \quad \frac{x_1}{r_1} + \frac{x_2}{r_2} \leq \bar{l}$$

$$\underset{l_1,l_2}{Max} \quad m_1 l_1 + m_2 l_2$$

$$s.t. \quad l_1 + l_2 \leq \bar{l}$$

Representation of the behaviour of this farmer using a **PMP** model:

$$\underset{x_1,x_2}{Max} \quad \pi_1(x) x_1 + \pi_2(x) x_2$$

$$s.t. \quad \frac{x_1}{r_1} + \frac{x_2}{r_2} \leq \bar{l}$$

$$\underset{l_1,l_2}{Max} \quad m_1(l) l_1 + m_2(l) l_2$$

$$s.t. \quad l_1 + l_2 \leq \bar{l}$$

One possible specification: $m_i(l) = p_i.(a_i - b_i.l_i) - c_i$

Review of European applications of the PMP method

Rhöm O., Sinabell F., Dabbert S., Hofreither M. (1997). The method of "positive mathematical programming" to evaluate farm and market effects of countryside stewardship policies. Working paper, Universität Hohenheim

Linear yield functions

Carles R., Decouvelaere F., Millet G., Revel A., Sourie J.C. (1998). Nouveaux outils pour analyser les effets de la prochaine réforme de la PAC sur les exploitations agricoles. *Economie rurale*.

Quadratic margin functions

Barkaoui A., Butault J.P. (1998). Modélisation de l'agriculture meusienne et Paquet Santer. *Economie rurale*.

Barkaoui A., Butault J.P. (2000). Programmation mathématique positive et offre de céréales et d'oléagineux dans l'Union européenne sous l'Agenda 2000. *Economie et prévision*.

Quadratic cost functions and econometrically estimated yield functions

Jùdez L., Chaya C., Martinez S., Gonzàlez A. (1999). Effects of the measures envisaged in Agenda 2000 on arable crop producers, beef and veal producers and dairy farms. An application of Positive Mathematical Programming to representative farms of a Spanish region. Presented to Ixth European Congress of Agricultural Economists, Warsaw.

Quadratic cost functions

Chantreuil F., Le Roux Y., Levert F. (2000). Analyse of the French Crisis in the Market of Wine. Working paper, INRA-ESR-Rennes.

Quadratic cost functions

Bouamra Mechemache Z., Réquillart V. (2000). Oral presentation. FAIR project, Rome. (Presentation of their European model for the dairy sector with particular attention paid to the modelling of the milk processing level)

Quadratic cost functions

Review of methodological papers:

Arfini F., Paris Q. (1995). A positive mathematical programming model for regional analysis of agricultural policies. 40th EAAE seminar, Ancona, Italy.

Presentation of the methodology and discussion related to functional form

Howitt R. (1995). A calibrated method for agricultural economic production models. *Journal of Agricultural Economics.*

Introduction of a CES functional form

Heckelei T. (1997). Positive Mathematical Programming: Review of the Standard Approach. CAPRI Working paper 97-03.
Jùdez L., Martinez S., Fuentes-Pila J. (1999). Positive Mathematical Programming Revisited. Working paper, Ciudad Universitaria, Madrid.
Gohin A., Chantreuil F. (1999). La programmation mathématique positive dans les modèles d'exploitation agricole. Principes et importance du calibrage. *Cahiers d'Economie et de Sociologie Rurales.*

Discussion about calibration process.

Remaining issues of the PMP method

Aggregation over individuals (Chambers, 1988)

Risk behaviour, Price expectations, Dynamic Modelling (Arfini, 2000 ; Paris, 2000)

Validation of PMP models

Review of papers combining PMP and Maximum Entropy

Paris Q., Howitt R. (1998). An analysis of Ill-Posed Production Problems Using Maximum Entropy. *American Journal of Agricultural Economics*.

Heckelei T., Britz W. (1998). Maximum Entropy specification of PMP in CAPRI. CAPRI Working paper 99-08.

Gohin A., Chantreuil F., Guyomard H., Levert F. (2000). MECOP: A Regionalised European Arable Crops' Sector Model. Illustration for the French Case. Presented at the 65th EAAE Seminar, Bonn.

Graindorge C., Henry de Frahan B., Howitt R. (2000). Analysing the Effects of Agenda 2000 Using a CES Calibrated Model of Belgian Agriculture. Presented at the 65th EAAE Seminar, Bonn.

Gohin A., Guyomard H. (2000). The Agenda 2000 CAP Reform in the WTO Context: Distortion Effects of Area Compensatory Payments and Set-Aside Requirements. Presented at the XXIV IAAE Conference, Berlin.

Maximum Entropy:
A Powerful Technique of Econometric Estimation

Main characteristics:

Well suited for Ill-Posed and Ill-Conditioned estimation problems.

Robust Estimator

Implementation in a simple ill-posed problem:

Find one β such that $Y^0 = X^0.\beta$

Re-parameterisation by defining support spaces: $\beta = Z.p$

Rule: Choose the probability vector p that maximises the entropy function $H(p) = -p.\ln p$ subject to consistency constraints.

Suggested readings:

Methodology:

Golan A., Judge G., Miller D. (1996). *Maximum entropy econometrics: Robust Estimation with limited data*. New-York: John Wiley & Sons.

Golan A., Judge G., Perloff J.M. (1997). Estimation and inference with censored and ordered multinomial response data. *Journal of Econometrics*.

Golan A., Perloff J.M., Shen E.Z. (2000). Estimating a Demand system with Nonnegativity Constraints: Mexican Meat Demand. Working paper, Berkeley.

Agricultural applications:

Lence S.H., Miller D.J. (1998). Estimation of multi-output production functions with incomplete data: a generalized maximum entropy approach. *European Review of Agricultural Economics*.

Oude Lansink A. (1999). Generalised maximum entropy estimation and heterogeneous technologies. *European Review of Agricultural Economics*.