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ANALYSIS

Dynamic efficiency of soil erosion and phosphor reduction policies combining economic and biophysical models

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Abstract

In this paper, we propose the use of the metamodeling approach to determine the optimal intertemporal management of soil and phosphorus losses from agricultural land. This approach enables to find a common equilibrium of the economic and biophysical systems. In contrast to the existing literature, the model takes into account nonlinear biophysical relationships and land-use choices. As a solution to the mathematical problems arising from this complex setup, we propose and employ a modified Cobb Douglas function in the empirical part of the paper. Most importantly, we allow for the comparison of different soil erosion and phosphorus reduction policies. The results show that an indirect policy in the form of soil protection scores (SPS) is highly inefficient, while another indirect policy in the form of land-use taxes is nearly as efficient as a direct policy.

Keywords: Soil erosion; Phosphorus runoffs; Land-use tax; Soil protection scores; Dynamic optimization

1. Introduction

The cultivation of arable crops may cause soil loss, particularly at locations that are highly vulnerable. Soil loss leads to a decrease in productivity at the field level that can only be compensated in part by an increase in the amount of input (Lal et al., 1983). Moreover, it causes runoff of particulate phosphorous into surface water, which is responsible, together with soluble phosphorous, for the eutrophication of surface water (Wehrli and Wüest, 1996).

Putman et al. (1988) estimate that soil loss over 100 years causes productivity losses of 2.3% within

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the entire US, while productivity for particular areas may decline up to 25%. A number of studies indicate that the costs associated with the productivity loss, the so-called on-farm costs, are relatively low for agricultural farms within the temperate zones. According to Colacicco et al. (1989), the on-farm costs of soil erosion are negligible for 10 analyzed regions within the United States. They are only about 0.2 to 1 US$ per ton/acre of lost soil. Smith and Shaykewich (1990) obtain values from 0 to 0.73 US$ per ton/acre for different soils in Manitoba, Canada.[2] Both studies analyzed a planning horizon of 100 years. Schmid et al. (1998) situate on-farm costs for Swiss conditions between 0.59 and 0.76 US$ ton/acre with a 1 year planning horizon.[2] These cited studies may provide evidence that on-farm costs for farmers in temperate zones on average are not significant and there is no incentive for farmers to employ erosion control measures.

Off-farm costs consider the effects of soil erosion that occur beyond the limit of the farm, mostly as a result of the degradation of the water quality and/or sedimentation processes. For example, these additional costs could arise due to additional purification and treatment costs for water utilities or additional maintenance cost for rivers, canals, dams and water reservoirs. Moore and McCarl (1987) calculated that the average off-farm costs under agricultural use are 0.87 US$ per ton/acre of lost soil for the Willamette Valley, Oregon, USA. Ribaudo (1986) and Colacicco et al. (1989), however, present results showing that off-farm damages are at least twice as large as on-farm damages. In contrary to most findings in the literature, a study by Pimentel et al. (1995) reports that on-farm costs in the United States are 3.43 US$ and off-farm costs are 1.17 US$ per ton/acre of lost soil per year. Although these figures may presently be considered somewhat too high, a great discrepancy between the different findings in the literature remains (Glanz, 1999).

In view of these conflicting findings, the empirical part of this paper, concerned with a particular area in Switzerland, helps to shed more light on these divergent results. Most importantly, however, the results of this paper show the comparison of direct and indirect environmental policies with respect to their dynamic efficiency to induce a reduction in soil erosion and phosphorous loss at the farm level. Accordingly, we are able to answer the important question: to what extent the theoretical inefficiency of indirect policies, compared to the higher efficiency of direct policies, matters for actual policy design? Additionally, the results are obtained from the metamodeling approach that allows for an economic and biophysical equilibrium.

Moreover, in contrast to the previous literature, nonlinear relationships between yield and soil depth on one hand and between soil loss, phosphorous loss and soil depth on the other hand are explicitly considered, and not linearly approximated (Baffoe et al., 1986, Smith and Shaykewich, 1990, Wossink, 1993). There are examples in the literature in which nonlinear biophysical relationships were taken into account; however, these studies were limited to the analysis of one (Yadav, 1997), or two crops (Goetz, 1997) on a field level, or conceptual level, respectively. Thus, they ignore the full effect of an optimally chosen crop rotation together with crop-specific cultivation techniques on the reduction in soil and phosphorus losses. In this respect, we present in this paper an extension of the current literature by taking into account the nonlinear biophysical relationships, and by simultaneously considering the determination of the optimal choice of crop (decision at the extensive margin). Consequently, the chosen methodology is not only relevant for the specific problem at hand, but in general where policies at the extensive margin, for example land-use regulations, are analyzed together with nonlinear biophysical relationships. Given this setup, the resulting economic model is nonlinear in the state variable. In contrast to the method of linear programming, the method of nonlinear programming is often associated with problems of existence and/or uniqueness of the solution if the mathematical problem is non-convex. This problem is present particularly in this paper since the endogenous determination of the optimal crop rotation leads, for most functions employed in economic analysis, to a non-convex decision problem. In order to guarantee that a unique solution of the economic model can be obtained, we propose and employ a special production function (modified Cobb Douglas function).

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[2] The result was obtained based on a rate of change of 1 US$=0.74 CA$=1.35 CHF.

The paper is organized as follows: Section 2 analyzes the effects of different approaches, which link economic and biophysical models, with respect to the possibility of finding a common equilibrium. Section 3 compares direct and indirect environmental policies and Section 4 closes out the paper with a summary and conclusions.

2. The metamodeling approach and the economic and biophysical equilibrium

Previous approaches in the literature to link an economic system with a biophysical system can be classified into three categories: (A) economic models with biophysical parameters, (B) economic models in combination with biophysical models, and (C) economic models with partial integration of biophysical models.

2.1. Economic models with biophysical parameters

Examples in this category include the works of Johnsen (1993) and Moxey and White (1994). Johnsen (1993) evaluates the cost-effectiveness of nine different measures to reduce phosphorous runoffs from agricultural land in Norway. The economic data was obtained from a survey and from agricultural statistics. The biophysical data is based on the statistical analysis of a large number of field trials. Moxey and White (1994) modeled the economic relationships with an economic decision model in the form of a linear programming model. In this model, Moxey and White avoided the pure statistical approach, which was utilized by Johnsen, and instead explicitly modeled economic decision processes, with respect to the intensive and extensive margin of the production. However, what both works have in common is that they use exogenously predetermined biophysical parameters. Consequently, the economic equilibrium obtained does not correspond to an equilibrium of the biophysical system.[3] Changes in the biophysical system, as a result of the optimally chosen economic activities, would require adjusting the values of the biophysical parameters utilized previously in the economic model. As a result, the economic equilibrium has to be determined once again. Unfortunately, the continuation of this reciprocal process does not guarantee finding equilibrium of economic and biophysical systems.

2.2. Economic models in combination with biophysical models

As a result, a new approach was proposed which links biophysical processes with economic decision models more closely. The works of Louhichi et al. (1999) and Dabbert et al. (1999) are representative of this particular approach. Typically a series of data, generated with a biophysical model, is employed in the economic decision model. The results of the economic model in turn change the biophysical bases of production. Thus, it is necessary to once again generate biophysical data that can be incorporated as input for the economic decision model. This process of mutual dependent interaction comes to a rest only if the economic equilibrium coincides with the biophysical equilibrium. In any case, the iterative search of an economic and biophysical equilibrium is usually extremely time- and resource-consuming.

For this reason, current work is limited to a small number of iterations, i.e. we only observe an approximation to the economic and biophysical equilibrium but not the equilibrium of both systems. Yet, the simultaneous determination of an economic and biophysical equilibrium is indispensable since disequilibrium of one system inevitably leads to disequilibrium of the other system. For example, the determination of the least-cost strategy, exclusively based on an economic model to comply with an exogenously specified environmental standard, will only yield a strategy, which is optimal for a short period of time as the strategy itself alters the underlying biophysical system. Thus, in a long-term perspective the exogenously specified environmental standard will most likely not be met, and if it is met, it is only by chance.

Within this context it seems important to emphasize that the necessity to search for a common economic and biophysical equilibrium is not related

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[3] An economic equilibrium is attained if the economic agents have no incentive to alter their behavior, i.e. the agents have no possibility of improving their situation. Likewise, a biophysical equilibrium is reached if the properties of the biophysical do not change at the macro-level, while the reactions at the micro-level continue, however, in such a way that they offset each other.

to the question of whether the model is static or dynamic. This necessity depends only on the interdependency of the biophysical and economic model. However, a dynamic model not only requires the search for a common economic and biophysical equilibrium for one particular point in time, as a static model does, but simultaneously over all points in time of the decision maker's planning horizon. On this account, the computational requirements for the determination of the common equilibrium increase in such a way that the approach of this category is hardly possible to realize computationally.

2.3. Economic models with partial integration of biophysical models

As a first step in advancing the interlocking of economic and biophysical models, Vatn et al. (1996) calibrated the economic and biophysical models before the actual optimization in order to come close to an equilibrium point. Lakshminarayan et al. (1996) made a more systematic approach with lower computational requirements. For this purpose, the economic model and the part of the biophysical model that is of interest were joined into one model. In fact, a part of the physical model becomes an integral part of the economic model. This approach is referred to as metamodeling. The key element of this approach is the incorporation of interdependent feedback of the economic and biophysical systems.

The feedback of the biophysical system is specified in the form of functions based on the results of a carefully constructed series of simulations that are generated with process-orientated biophysical models. For example, production as a function of soil depth can be presented as a feedback function if the relationship between output and soil depth is estimated with data generated by a process-orientated biophysical model. Basically, a biophysical feedback function presents the econometric evaluation of data series generated by simulations, and data series of the biophysical factors that were used in the simulations. The obtained biophysical feedback function is integrated into the economic model. The feedback of the economic model is given by the outcome of the economic decision process that in turn alters the values of biophysical factors. Hence, the parameters of the biophysical feedback function take on new values. The simultaneous consideration of economic and biophysical feedback leads to the metamodeling approach. It allows the greatest possible flexibility to evaluate the effects of different polices, as it is not necessary to coordinate exogenously the economic and biophysical models; therefore, it enables saving resources and time. On the other hand, the high degree of flexibility has its price. The amount of data obtained from the evaluation of the series of simulations increases rapidly with the number of different policies analyzed, such that the data management and the statistical evaluation of the data present a challenge. Nevertheless, the metamodeling approach compared to the previous approach is less time- and resource-consuming, and is therefore utilized for the work presented in this paper.

3. The model

Corresponding to the farm level approach of this work, our model reflects the decision problem of a farmer. The metamodel was specified utilizing biophysical data, which was previously generated with a process-orientated biophysical model (Erosion Productivity Impact Calculator, EPIC, Sharpley and Williams, 1990a,b). The generated data helps to determine the functional relationships between yield, soil and phosphorus losses subjected to cultivated crops, biophysical characteristics (soil type, etc.), weather and cultivation techniques (Sharpley and Williams, 1990a, 1990b). These relationships were econometrically estimated and the obtained feedback functions were integrated into the economic part of the model. The determination of feedback functions on the basis of empirical data is difficult to imagine since the data is often not available, or the existing time series do not allow isolation without ambiguity between endogenous variables and exogenous variables of interest (Goetz et al., 1998). Moreover, empirical data would allow only feedback functions, which are based on policies employed in the past. Thus, the use of empirical data would exclude the economic evaluation of new policies.

The intensification of agriculture, the expansion of arable land, the development of new agricultural land and the cultivation of erosive crops at vulnerable locations have aggravated the situation of soil loss in

Switzerland.[4] Soil erosion and phosphorous runoffs are both of prime importance within the watershed areas of the lakes located in central Switzerland, particularly the areas of Lake Baldegg, Sempach, and Hallwil. For the empirical analysis, the Lake Baldegg watershed has been chosen since soil and phosphorous losses are of great importance in this region. Also, we can revert to previous experience with the utilization of EPIC for this region (Maurer, 1995).[5,6]

For the economic part of the model, we assume that the farmer maximizes his/her farm gross margin over calendar time t, with t=0,. . ., T, and he/she is risk neutral (see Eq. (1) below). The decision variables for the decision-maker are the type of fertilizer, the type of tillage and the choice of crops. The farmer can choose between potatoes, corn, winter wheat, winter barley, summer oat, maize, annual or biennial grassland and summer oat with a cover crop over winter. The specified farm model presents a typical farm in the watershed of the Lake Baldegg with 20 ha arable land (Eidgenössische Forschungsanstalt für Landwirtschaftlichen Pflanzenbau, 1983 and Schudel et al., 1992). The planning horizon for the farmer is assumed to comprise two generations, i.e. T=66 years. Given the regional focus of the analysis, the prices are not influenced by production decisions and thus, they are exogenous. The dynamic economic decision can therefore be formulated as

$$\max_{y_{tijm}} \sum_{t=0}^{T}\sum_{i=1}^{9}\sum_{j=1}^{2}\sum_{m=1}^{2}\left(\frac{1}{1+\delta}\right)^{t} \times \left[\left(p_i f_{ijm}(n_t) - c_i f_{ijm}(n_t) - k_{ijm}\right)y_{tijm}\right] \tag{1}$$

subject to

$$n_{t+1} - n_t = \sum_{i=1}^{9}\sum_{j=1}^{2}\sum_{m=1}^{2}\left[\phi_{ijm}(n_t) - \gamma_{ijm}(n_t)\right]y_{tijm}/\bar{y}$$

$$n_0 \text{ given} \tag{2}$$

$$\sum_{i=1}^{9}\sum_{j=1}^{2}\sum_{m=1}^{1} y_{tijm} = \bar{y} \leq y_{\max} \tag{3}$$

$$I\bar{y} \leq \sum_{i=1}^{9}\sum_{j=1}^{2}\sum_{m=1}^{2} \alpha_{ijm} y_{tijm} \tag{4}$$

$$\varphi \sum_{i=1}^{9}\sum_{j=1}^{2}\sum_{m=1}^{2}\left[\phi(n_t)y_{tijm}\right] \leq \beta\bar{y} \tag{5}$$

$$y_{tijm} \in Y,\, Y \subset R^{i}, \quad i = 1,\cdots,9, \quad j = 1,2, \quad m = 1,2 \tag{6}$$

with the indices:

i	crop
j	type of fertilizer (mineral or organic fertilizer)
m	type of tillage (minimal or standard)

parameters:

p_i	price of crop i (CHF/ton)
c_i	costs of crop i that are related to the yield (harvest cost, drying cost, etc.) (CHF per ton)
k_{ijm}	cost of the cultivation technique of crop i, (capital, labor and costs that depend on the type of fertilizer j and on the type of tillage m) (CHF/ha)
Y	set of crop rotation restrictions with respect to y_{tijm}
δ	discount rate
α_{ijm}	soil protection scores of crop i, cultivated with fertilizer j and with tillage m
I	minimal required average soil protection scores per ha
β	maximal average admissible phosphorus loss in kg per ha
φ	content and transfer coefficient with respect to bioavailable phosphor, P, per ton of eroded soil
n_0	initial value of the soil depth in cm
$\bar{y}$	cultivated land in ha
$y_{\max}$	available land in ha

and variables:

y_{tijm}	cultivated land of crop i with fertilizer j applied and tillage m as a function of time t per ha
n_t	soil depth in cm at time t, and

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[4] Precipitation leading to soil erosion affects about 10–20% of the area of arable land (Mosimann et al., 1990).

[5] As wind erosion has little importance for the analyzed region, the empirical part of the work considers exclusively the case of water erosion.

[6] Soil losses for arable crops vary for instance between 1 ton/ha for grassland and 25 ton/ha for corn within the watershed of Lake Baldegg. The average is about 11 ton/ha (Maurer, 1995).

functions:

f_{ijm}	crop yield in tons per ha as a function of soil depth
ϕ_{ijm}	erosion in tons per ha as a function of soil depth
γ_{ijm}	soil genesis in tons per ha as a function of soil depth.

The difference Eq. (2) describes the change in the soil depth in general. For our empirical study, however, it was assumed that the soil genesis function γ_{ijm} is constant and equal to zero. This assumption is based on the work by Bork (Chapter 1.1, 1988) that shows virtually no soil genesis can be observed for cultivated land in Western Europe. Restriction (3) limits the area of cultivated crops that belongs to the farm. The fourth restriction puts a lower limit on the number of soil protection scores, to be introduced later, which have to be achieved per hectare. Restriction (5) puts an upper limit on the average phosphorous loss per hectare. Moreover, it demonstrates the relationship between soil loss and P-runoffs employed in Bork's work. The restrictions concerning crop rotations are summarized in Eq. (6).[7] The restrictions (4) and (5) are specific for particular policy scenarios considered in Section 3.1. Although these restrictions do not form part of the farmer's decision problem without any government intervention, they are stated here for completeness.

The functions f_{ijm} and ϕ_{ijm} were estimated based on data generated with EPIC. In order to include weather induced yield variations and soil loss, particular weather conditions were selected. For the selection of the weather conditions, we did not evaluate the weather itself but the density function of the events of soil loss which was known from an earlier study (Maurer et al., 1995).

As an approximation of this density function, the median value of soil loss of the lower 34%, the mid 46% and the upper 20% of the density function was determined. Next, the weather conditions that have caused these erosion events were selected and utilized for the operation of the weather generator of EPIC. The simulated yield and soil loss were weighted with the probabilities (0.34, 0.46 and 0.20) of the erosion events. Hence, f_{ijm} and ϕ_{ijm} present weighted functions. The weight itself was selected based on the probability of the erosion event, since the erosion itself, as a trigger for phosphorus loss, is the center of interest. Weighing according to weather conditions, for instance dry, normal and wet, might have resulted in wrongly specified functions f_{ijm} and ϕ_{ijm}, since a dry year may result in the same amount of soil loss as a wet year. This would be the case if the precipitation of a wet year is distributed equally over the years while those of a dry year are concentrated on a few days, for example at the beginning of the vegetation period.

To evaluate the long-term effects of soil erosion on soil productivity, phosphorus and soil losses, it would have been necessary to generate a series of simulations over several centuries. Alternatively, it is possible to specify and use different depths of the soil horizon for the EPIC runs. In utilizing the second option, we set the soil horizon at 105, 90, 70, 60 and 35 cm, respectively. Together with the cut of the depth of the soil horizon, the specification of the C/N ratio was adjusted to actual eroded soil. According to the mathematical model, and given the set of alternatively specified EPIC parameters, one obtains 540 series of simulations, which are available for the estimation of the functions f_{ijm} and ϕ_{ijm} (9 crops×2 types of fertilizer×2 types of tillage×3 weather conditions×5 soil horizons=540). Each series of simulation generates a plenitude of data (1 Mbyte) that requires careful data management. The number of simulations could be reduced slightly since not all crops can be combined with the two different types of fertilizer and types of tillage. Thus, instead of 36 combinations of crops, fertilizer and tillage we have 30.

A unique solution to the nonlinear programming problem, specified in Eqs. (1)–(6), can theoretically only be guaranteed if the decision problem is convex, i.e. the objective function has to be at least pseudo-concave in the case of maximization and the left-hand side of the functions of inequalities, put into normal form, are at least quasi-convex (Bazaraa et

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[7] We used the recommendations of the Swiss extensions service with respect to crop rotation constraints, possible fertilizer and tillage combinations. In this way, we evaded modeling explicitly the growth of weeds, pests, and the carryover of nutrients. However, it implies that the problem, Eqs. (1)–(6), is linear in the choice variables.

al., 1993). The specification of functions f_{ijm} and ϕ_{ijm} by seven commonly employed functional forms in economics (Cobb Douglas, quadratic, transcendental, constant elasticity of substitution, translog, generalized Leontief or Miniflex Laurent) would violate the convexity requirement of the nonlinear programming problem (Keusch, 2000).[8] The requirements from an analytical or numerical perspective, however, are not identical (Fletcher, 1987). In other words, a convex problem might be hard to solve numerically, even though convexity suggests that it should be easily solved. Similarly, it may occur that a problem can be easily solved numerically, although the problem is not convex. Ideally, one uses different solvers based on distinct algorithms to verify that the obtained solution corresponds to a global solution. Due to resource constraints, we did not follow this path but decided to analyze the problem employing a modified Cobb Douglas function that guarantees the optimization problem itself is convex.[9] In this way, it seems more likely to find the global optimum. Furthermore, the economic model was qualitatively analyzed to verify that its implications are in line with economic theory. For this purpose, we derived and analyzed all necessary conditions for the optimum, with respect to its economic content. Moreover, we conducted a comparative static analysis for certain parameters to verify that the signs of a change in these parameters are theoretically correct. This validation is by no means mathematical gimmick but an imperative for the utilization of the model in empirical research. Only in this way is it possible to interpret the results justly and to attribute them to the underlying data. The mathematical validation of the model makes it possible to rule out unintentional interferences between the specified model and the utilized data.

The parameters of functions f_{ijm} and ϕ_{ijm} were both estimated based on algorithms for nonlinear least square regression techniques offered by the Software EVIEWS (Quantitative Micro Software, 1998). Before estimating, both functions were written in logarithmic form so that the relative, and not the absolute, deviations of the estimated from the observed value matter. Although the specification of functions f_{ijm} and ϕ_{ijm} in the form of a modified Cobb Douglas is required for the optimization problem to be convex, we also specified both functions in quadratic form. The latter specification was chosen as it is widely applied in empirical work and it allows us to compare the fit of the estimated modified Cobb Douglas functions with the estimated quadratic functions. The results of these estimations of yield and erosion functions show that the coefficient of determination, R^2, for both functional forms is between 0.97–0.99 for nearly all combinations of crops, fertilizer and tillage. Only three erosion functions, specified in both ways, yield an R^2 between 0.82 and 0.95 for all combinations of crops, fertilizer and tillage. Thus, we can conclude that the modified Cobb Douglas function fits the data well and can be considered a good "summary" of the biophysical model. As an example of the estimated 60 functions, we present the modified Cobb Douglas yield and erosion function of winter wheat with organic fertilizer and standard tillage given by

$$\log(\text{yield}/y) = \log(-48.549/y + 98.594(n/y)^{[1-0.805]} - 1.354(n/y)^{[1+3.949]})$$

$$\log(\text{erosion}/y) = \log(37.945/y - 30.568(n/y)^{[1-0.920]} + 0.445(n/y)^{[1+1.917]}).$$

Further information about the remaining 58 estimated functions can be found at Keusch (2000).

Finally, the mathematical problem was programmed in Algebraic Modeling Programming Language (AMPL; Fourer et al., 1993) and solved with the solver MINOS (Murtagh and Saunders, 1983; revised 1995).

The described processes of the different components of the model and the relationships among each other are presented schematically in Fig. 1. It illustrates at the same time the temporal sequence and the interaction of the different components of the model.

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[8] See the appendix—pseudo-convexity—for details.

[9] See the appendix—modified Cobb Douglas function—for details.

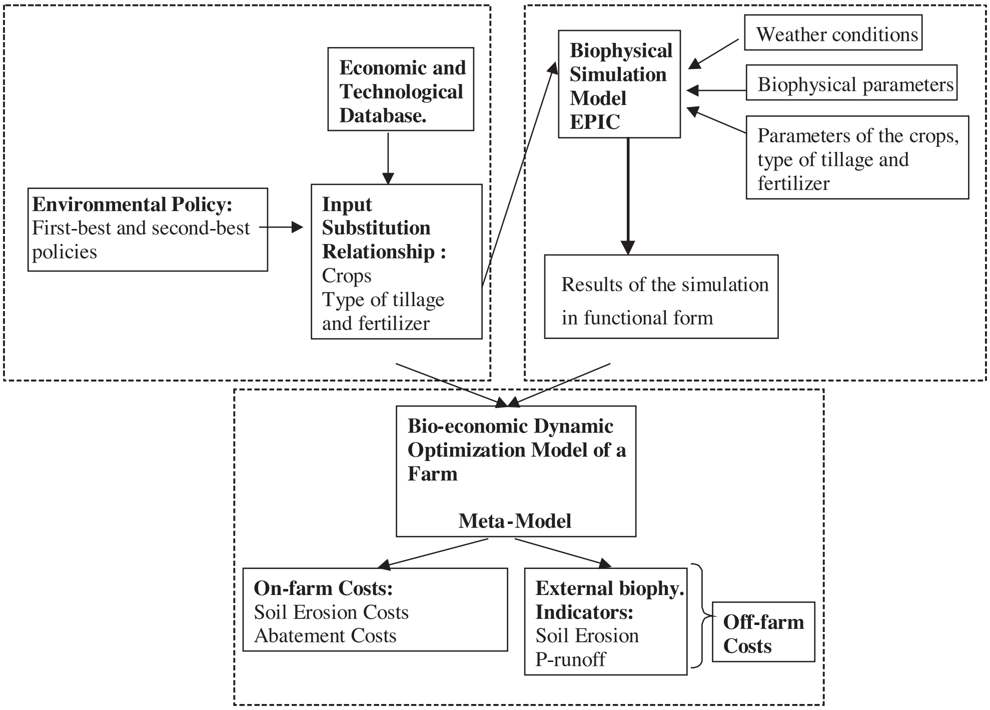

Fig. 1. Components of the model and scheme of the economic analysis.

3.1. Environmental policies

In the following sections, several polices aimed at reducing phosphorous and soil losses are compared with respect to their dynamic efficiency, i.e. with respect to the value of abatement and damage costs over time. Other costs related to the different policies such as transaction, administration, control and enforcement costs are also important; however, they do not form part of this analysis. The valuation of damage cost often poses a serious, if not unsolved, problem in practice. For this reason, policymakers often apply standards in order to avoid monetary valuation of the damage. We employ this approach so that the value of abatement costs is the only criteria for evaluation of the efficiency of different policies. Even though a threshold value, which has been defined explicitly by the legislator, does not exist, the Swiss law of water protection implicitly defines one. The law requires an oxygen content of 4 g O_2/m^3 in all parts of the watercourses. This threshold level in turn implies for the analyzed region that the phosphorous loss should not exceed 0.3 kg/ha (Wehrli and Wüest, 1996).

3.2. Private optimum

The private optimization problem is given by Eq. (1) and subject to Eqs. (2) (3) (6). To examine whether there are incentives to limit soil erosion, we calculate the on-farm costs of soil erosion. For this purpose, we compare the monetary value of lost soil for a short-term planning horizon with the monetary value of the lost soil for a long-term planning horizon. The shadow price of differential equation (2) was used to evaluate the value of the lost soil for the respective optimization problems. Thus, the monetary value of the lost soil presents the on-farm cost for the farmer. We assume that the discount rate is 0.06 and off-farm costs are not taken into account (private optimum).

If the planning horizon is classified as short-term, the economic model is optimized recursively, i.e. given the value of the stock variable for the previous period; the economic model is optimized for every year individually in a sequence of 66 years. For a long-term planning horizon, however, we do not use recursive optimization but dynamic optimization. This approach allows optimization for all 66-time periods simultaneously. The distinction between short-term and long-term planning horizon enables the determination of relevance in the length of the planning horizon.

The results of the calculations are summarized in Table 1.

A variation in the discount rate (3% and 0%) leads, for a short-term planning horizon and an initial soil depth of 105 cm, to average on-farm costs between 0.19 (3%) and 0.46 (0%) CHF per mm of lost soil and hectare, and for an initial soil depth of 35 cm to 0.72 (3%) and 1.68 (0%) CHF. Likewise, a variation in the discount rate for a long-term planning horizon leads to higher on-farm costs, given by 4.75 (3%) and 16.31 (0%) CHF per mm of lost soil and hectare for an initial soil depth of 105 cm, and by 17.27 (3%) and 57.85 (0%) CHF for an initial soil depth of 35 cm.

Albeit these apparent differences in the on-farm costs, for instance with respect to the average on-farm costs per ha, the optimal crop rotation, given a discount rate of 0%, 3% or 6%, is identical for soils with an initial depth of 105 or 70 cm. Only for an initial soil depth of 35 cm and a discount rate of 6% (3% or 0%) the crop rotation for a long-term planning horizon up to year 1 (4 or 10) is less erosive than the crop rotation of a short-term planning horizon. Thereafter, the crop rotation plans are identical. Moreover, irrespective of the initial soil depth and the length of the planning horizon, the on-farm costs are too low to give incentives for applying soil conserving cultivation techniques or soil structure improving organic fertilizer. Thus, conventional tillage and mineral fertilizer is applied independently of the length of the planning horizon and the initial soil depth. Consequently, soil loss associated with a long-term planning horizon differs only from that of a short-term planning horizon for an initial soil depth of 35 cm. Yet these differences are negligible as they sum up to only 1.21 ton/ha over the entire planning horizon. In other words, the results show that the optimal choice is almost completely insensitive to the length of the planning horizon and the initial soil depth. Changes in the optimal choice rarely occur since on-farm costs (forgone gross margin) are lower than the losses in gross margin resulting from soil conserving behavior.

3.3. Social optimum

Even though there is no reason to reduce soil loss from a private perspective, it might be advantageous from a social perspective (McConnel, 1983). This is most likely the case if soil erosion leads to damages, in the form of phosphorous runoffs and the subsequent eutrophication of surface waters (off-farm costs), which are not included in private considerations. Therefore, from a social perspective the question of how farmers can be encouraged to apply soil conserving and P-runoffs reducing cultivation techniques is raised.

In the following sections, different policies aimed at the reduction of P-runoffs are presented and compared with respect to their efficiencies. The comparison of these policies is based on the different abatement costs associated with each policy. Initially the abatement costs are calculated for the policy of the introduction of a P-emission standard and of a P-emission tax. Although these direct measures cannot be applied in the context of nonpoint source pollution, the calculated abatement costs are of great importance. They allow

Table 1
On-farm costs for a short- and long-term planning horizon for a discount rate of 6% and an initial soil depth of 105 cm

	Short-term planning horizon		Long-term planning horizon	
	First year on-farm costs in CHF		First year on-farm costs in CHF	
	Soil depth 105 cm	Soil depth 35 cm	Soil depth 105 cm	Soil depth 35 cm
Per ha	0.31	1.62	5.91	27.71
Per mm soil loss/ha[a]	0.36	1.53	6.88	27.44
Per ton of soil loss/ha	12.06	14.08	12.06	14.87
	average on-farm costs in CHF over the entire planning horizon		average on-farm costs in CHF over the entire planning horizon	
Per ha	0.10	0.41	1.80	6.75

[a] The soil density is 1.4 ton/m^3.

for the quantification of the inefficiency of indirect measures from a least-cost point of reference. We analyze the concept of soil protection scores and land-use taxes as indirect measures.

3.4. P-emission standard and P-emission tax

As mentioned above, the admissible P-emission for the analyzed region of the Lake Baldegg watershed is 0.3 kg P/ha. This standard can be obtained in a single farm model at the same abatement costs, either with a P-emission standard, ψ, or with a P-emission tax, τ_{ψ}.[10]

For a comparison of different environmental policies, however, not only the abatement costs associated with a particular threshold value are of interest, but also the abatement costs as a function of different threshold values, as it allows for a more general evaluation of the different policies. The graphical presentation of the abatement costs, to be presented later on, is based on the function of the marginal abatement costs. Besides the marginal abatement costs, it supplies the abatement costs given by the area below the graph of the marginal abatement costs function. The marginal abatement costs were determined for different threshold values, which result in different P-emission standards or P-emission taxes.

A P-emission standard is incorporated into the model (Eqs. (1–3) by the introduction of a P-runoffs restriction, Eq. (5). The starting point of the P-runoffs restriction is the unrestricted optimum[11] of 1.2 kg P/ha. Thereafter, the P-runoffs restriction is reduced stepwise by 0.1 kg P-emission/ha in order to produce data that permits the calculation of the abatement cost function. The abatement costs associated with a P-emission standard are given by the difference between the farm gross margin with and without a P-runoffs restriction.

As an alternative to a P-emission standard, ψ, one can also apply a P-emission tax, $\tau_{t\psi}$, which leads to P-emissions of ψ. The value of the P-emission tax corresponds to the shadow value of the P-runoffs restriction for each moment in time. For the implementation of this environmental policy, the objective function needs to be modified such that the social cost, generated by complying with the P-emission limit, is incorporated into the private objective function. Adding the product—$\tau_{t\psi}\times$kg of P-emission—to the private objective function, the modified model allows the determination of the socially desired outcome for a short-term as well as for a long-term planning horizon.

As required by economic theory, the shadow price of the P-emission standard of 0.3 kg P/ha produces the correct time-dependent P-emission tax. Thus, the optimal choices and the P-emissions of both environmental policies coincide for each year of the planning horizon (Keusch, 2000). To generate the data necessary to estimate the time dependent marginal abatement costs function, the time path of the P-emission taxes is calculated for each of the successively more restrictive P-emission limits. This procedure yields the necessary data to estimate the marginal abatement costs function for the first, second, ..., until the 66th year of the planning horizon.

The abatement costs, F, as a function of the abated P-emission, P_a, are estimated on the basis of an exponential function and take the following form:

$$F(P_r) = -534.27 + e^{0.194*P_a+6.26} \qquad R^2 = 0.994. \tag{7}$$

The marginal abatement costs function per kg P, $f(P_a)$, yields

$$f(P_a) = 0.194e^{0.194P_a+6.26}. \tag{8}$$

A variation of the initial soil depth has a very limited effect on the form and magnitude of the marginal abatement functions, and thus it is not presented here.

Fig. 2 shows the graph of the estimated first year marginal abatement cost function of the farm per kg P given a short-term planning horizon. We consider the case where the initial soil depth is 105 cm, and the discount rate is 6%. Marginal abatement costs for P-emissions for the entire farm increases from 102 to 5409 CHF, where a total 20.43 kg P are abated. The graph presents the estimated least-cost marginal abatement cost and serves as a reference for the evaluation of other policies. Please note that the mentioned values of the marginal abatement costs are taken directly from the outcome of the optimization

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[10] For models with more than one farm that are distinct from each other, the abatement costs would not be identical for a P-emission standard and a P-emission tax.

[11] Without any P-runoff restriction the farm emits 22.48 kg P, i.e. 1.12 kg P/ha. Thus, 1.2 kg P/ha is the lowest restriction with just one decimal that yields the unrestricted optimum.

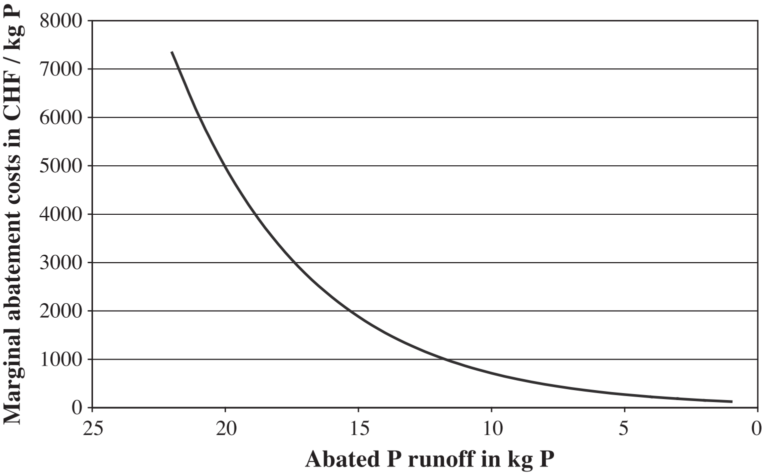

Fig. 2. Estimated marginal abatement cost of phosphorus at the farm level based on a P-emission tax or a P standard.

process of the economic model. For this particular reason, the mentioned marginal abatement costs may not coincide completely with the graph of the estimated marginal abatement cost function as presented in Fig. 2. For the same account, Figs. 3–6) may display some differences between the estimated and mentioned values in the text.

Our calculations show that the abatement costs are independent of the length of the planning horizon, i.e. independent of the consideration of on-farm costs. Short-term and long-term planning yield the same abatement costs per kg P in every year of the planning horizon. Therefore, Fig. 2 is also representative of the first year marginal abatement costs given a long-term planning horizon.

The abatement costs are decreasing slightly over time. In the case of a discount rate of 6% and a soil depth of 1.05 m, the abatement costs per ha decline from 584 CHF in the first year to 567 CHF in the 66th year. The sign and magnitude of the change in the abatement costs over time is maintained for the different values of the initial soil depth (0.7 and

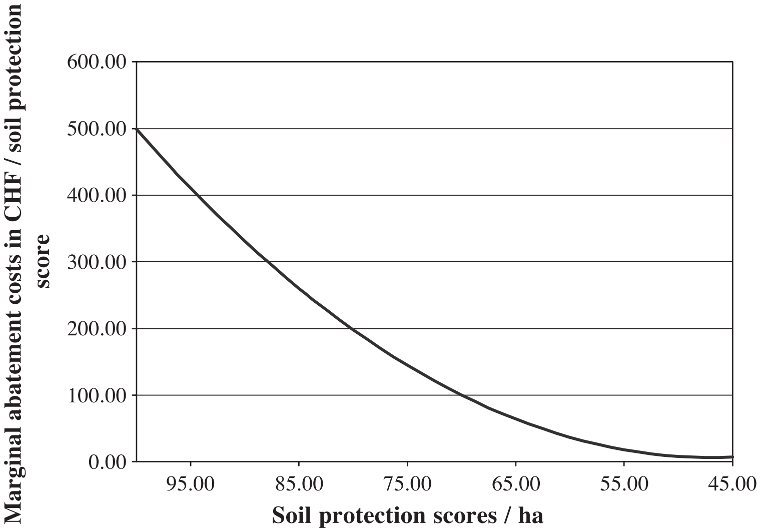

Fig. 3. Estimated marginal abatement cost of phosphorus at the farm level based on soil protection scores.

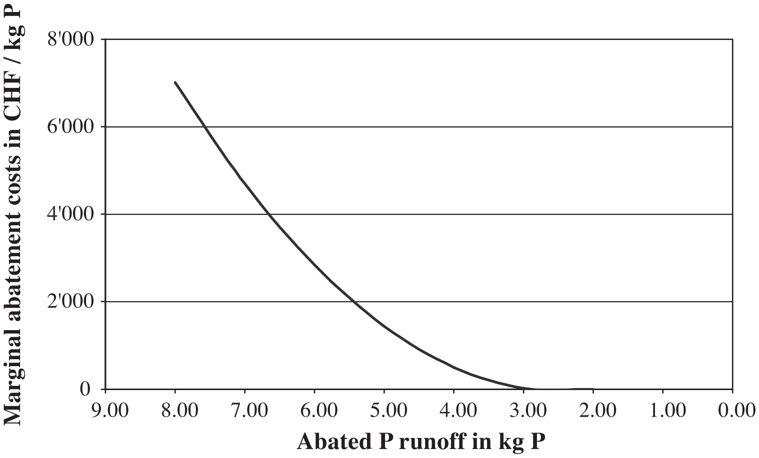

Fig. 4. Estimated marginal abatement cost of phosphorus at the farm level based on soil protection scores expressed as abated phosphorus.

0.35 m) and of the discount rate (3%, 0%). Due to this relatively insignificant decrease over time, the marginal abatement cost function in Fig. 2, as well as in the following figures, is only presented for the first year and not for the following years. The invariance of the abatement costs, with respect to the length of the planning horizon, may in part be explained by the linearity of the decision variables in the economic model.

The reduction of P is achieved by the introduction of minimum tillage, organic fertilizer, cover crops and changes in the crop rotation. For instance, in order to comply with a P-emission standard of 0.1 kg P per ha, 75% of the land is covered by pastures and the remaining 25% of the land is cultivated with summer oats and a cover crop during winter.

Even though the abatement costs barely change over time, the shadow price of the phosphor emission standard changes significantly. As this shadow price determines the time dependent P-emission tax, it drives a wedge between the optimal P-emission taxes that are derived from a short-term or long-term planning horizon. Given that a short-term perspective does not consider the long-term

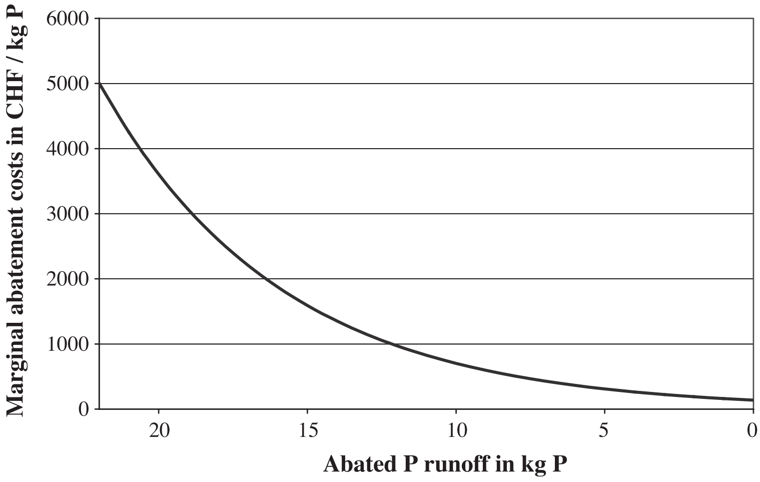

Fig. 5. Estimated marginal abatement cost of phosphorus at the farm level based on a land-use tax.

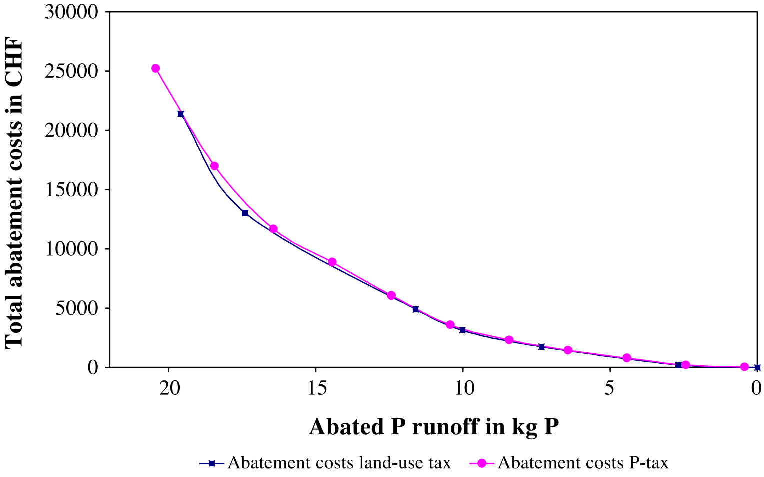

Fig. 6. Estimated marginal abatement cost of phosphorus at the farm level based on a land-use tax compared to P-emission tax.

on-farm costs, the P-emission taxes have to be higher in the myopic case than in the farsighted case. For instance, in the case of a discount factor of 6% and a soil depth of 1.05 m, the P-emission tax per kg is 1592 CHF for a short-sighted perspective and 1571 CHF for a long-sighted perspective. As time passes, P-emission taxes and the difference between short-sighted and far-sighted perspectives decrease and vanish completely in the final year of the planning horizon. The average P-emission tax for the case of a discount factor of 6% and a soil depth of 1.05 m is 414 CHF for a short-sighted perspective and 409 CHF for a long-sighted perspective over the entire planning horizon. Thus, P-emission taxes derived from a static or myopic economic analysis would impose taxes, in particular for the initial years of the planning horizon, that are too high in comparison to dynamic or far-sighted perspectives. The dynamic approach of this paper identifies this extra and unnecessary financial burden in comparison with a static approach and it facilitates the determination of the optimal adjustment of P-emission taxes over time. A variation in the initial soil depth or the discount rate confirms the already observed behavior of the optimal P-emission tax over time.

The average abatement costs per ha over the entire planning horizon are between 567.5 and 614.5 CHF, depending on the initial soil depth and the chosen discount rate. If the abatement costs are interpreted as the off-farm cost, a comparison of these values with the average on-farm costs per ha for the private optimum (Table 1) shows that the off-farm costs are substantially higher.

3.5. Soil protection scores

Indirect measures have been proposed as an alternative to the direct measures discussed in the previous section, since they can be applied more easily in practice. An example for such a measure is the concept of soil protection scores (SPS) according to the regulation of integrated production (Bundesamt für Landwirtschaft, 1999). The scores, α_{ijm}, relate to the unit of 1 ha and they are differentiated according to the cultivated crop, i, the type of fertilizer, j, and the utilized tillage technique, m. Based on this regulation, farmers are required to surpass a certain score, I, per ha at two particular dates of the year: November 15 and February 15. The multiplication of I with the entire cultivated land, $\bar{y}$, yields the minimum score to be achieved at the farm level. This regulation is reflected in the model (Eqs. (1–3) by adding Eq. (4).

We consider the case of a short-term planning horizon with an initial soil depth of 1.05 m and a discount rate of 6%. The calculations show that P-runoffs recede only slightly as the lower limit of the number of soil protection scores is raised. An increase in the number of soil protection scores leads to a decrease in the average emissions over the entire planning horizon from 1.12 to 0.76 kg P/ha. Even though the minimum scores are raised from the lower limit of 45 to its upper limit of 100, it is not possible to reduce the emissions up to 0.3 kg P/ha, as required to meet the Swiss water quality regulations. Moreover, a variation in the initial soil depth, in the discount rate or a change in the planning horizon does not allow for cut backs of P-emissions beyond 0.76 kg P/ha.

Next, the abatement costs themselves were estimated based on a polynomial of third degree as a function of soil protection scores and also as a function of abated P. The abatement costs functions and their derivatives are given by

$$F(SPS) = -6092.67 + 385.95SPS - 8.12SPS^2 + 0.058SPS^3, (R^2 = 0.991) \quad (9)$$

$$f(SPS) = 385.95 - 16.23SPS + 0.174SPS^2, \quad (10)$$

where SPS=soil protection scores

$$F(P_\mathrm{a}) = -313.253 + 1334.30P_\mathrm{a} - 562.9P_\mathrm{a}^2 + 76.47P_\mathrm{a}^3, (R^2 = 0.991) \quad (11)$$

$$f(P_\mathrm{a}) = 1334.30 - 1125.78P_\mathrm{a} + 229.42P_\mathrm{a}^2, \quad (12)$$

where P_a=abated P in kg.

The graphs of these two marginal abatement costs functions are presented in Figs. 3 and 4. They show that the marginal abatement costs surge with an increase in soil protection scores, or in abated P.

A comparison of Figs. 2 and 4 produces evidence that the policy of soil protection scores is far more expensive than the benchmark solution. The reduction of P-runoffs by the introduction of soil protection scores, for example by 0.36 kg P/ha (from 1.12 to 0.76 kg P/ha), entails average farm abatement costs of 8736 CHF over the entire planning horizon. The introduction of a P-emission tax/standard, however, only produces average farm abatement costs of 1888 CHF. Thus, soil protection scores imply a loss of efficiency equivalent to 6848 CHF. The comparison of these measures demonstrates that the current definition of the soil protection scores according to the regulation of integrated production lessens the erosion and P-runoffs problem but in an inefficient way. Moreover, it is not capable of meeting the standard aimed at by Swiss water regulations.

As in the benchmark case, the abatement costs of soil protection scores do not vary with the length of the planning horizon, and only slightly over time. Moreover, the utilization of a dynamic approach shows that consideration of the on-farm costs reduces the necessary P-emission taxes to induce the social optimal outcome compared to a static approach. Thus, the use of the dynamic approach reduces the lower limit of soil protection scores in comparison with the use of a static approach, both in their futile effort to approximate the benchmark solution.

Although SPS are shown to be highly inefficient, this instrument still deserves attention since transaction costs are probably low in comparison with other instruments. Thus, if natural scientists were able to develop soil protection scores that relate closer to P-emissions, the efficiency of this instrument might improve substantially. A first step might involve the development of a P index that takes into account the site vulnerability of the land, the use of the land and the characteristics of the body of water to determine the potential P load of each site (Sharpley and Halvorson, 1994).

3.6. Land-use taxes

A further alternative policy constitutes the introduction of a land-use tax, denoted by σ_{tijm}, where i denotes the crop, j the type of fertilizer and m the tillage technique. The translation of this policy into the economic model, given by Eqs. (1–3, introduces the term, $-\sum_i \sum_j \sum_m \sigma_{tijm} y_{tijm}$ in the objective function.

The model based on the metamodeling approach determines the shadow prices of the P-emissions depending on a prespecified level of the P-emission restriction. The optimal land-use tax per hectare is given by the product of the crop, cultivation technique and fertilizer specific P-runoffs with the time-dependent shadow price for each prespecified P-runoffs restriction, σ_{tijm}=$(\varphi\phi_{ijm})\tau_{t\psi}$. The crop, cultivation technique and fertilizer specific P-runoffs cause the land-use tax to be differentiated with respect to the cultivated crop, the cultivation technique and the employed type of fertilizer. Since the time variant shadow price forms part of σ_{tijm}, the land-use tax varies over time as well.

The optimal land-use tax gives sufficient incentive for farmers to achieve the prespecified P-emission standard of 0.3 kg P/ha. The successive change of the prespecified P-emission standard and thus of the optimal land-use tax helps to determine the abatement costs for different tax rates. We consider the case of an initial soil depth of 1.05 cm and a discount rate of 6%. For a short-term planning horizon, the abatement cost function and its derivative for the first year are given by the following equations:

$$F(P_a) = -1064.12 + e^{0.164P_a+6.73}, \qquad (R^2 = 0.98) \tag{13}$$

$$f(P_a) = 0.164e^{0.164P_a+6.73}, \tag{14}$$

where P_a=abated P in kg.

Fig. 5 shows the graph of the marginal abatement costs of the land-use tax for the first year. The land-use tax differs largely between the different crops, but it is directly proportional to the erosion/P-runoffs of the crop. The immediate relationship between erosion/P-runoffs of a crop and the land-use taxes results in land-use taxes that are directly proportional to the changes in the optimal P-emission taxes. Moreover, it shows that land-use taxes decrease slightly over time. In particular, the higher land-use taxes, associated with a short-term planning horizon, approach the lower land-use taxes, associated with a long-term planning horizon, towards the end of the planning horizon.

Fig. 6 compares the total abatement costs of the land-use tax with the benchmark case—P-emission tax/standard for the first year. It illustrates that the graphs are nearly identical and as such the land-use tax can be classified as an efficient instrument. Moreover, land-use taxes can also be applied in practice since the use of the land is easily observed by the regulator. However, the associated information, control and administrative costs may be too high, such that the regulator may opt for a simplified version of the land-use tax. Moreover, they confront the legislator with the problem of the political acceptance of this measure since land-use tax imposes a high financial burden on farmers. Like the unfeasible P-emission tax, the land-use tax nearly consumes the entire profit of the farmer. Thus, the legislator may think about reimbursing the collected taxes to the agricultural sector, or tax reductions in order to mitigate the distributional effect of this policy.

With respect to the difference between static and dynamic approaches, we can employ the results we obtained in the previous two sections. As such, calculations of the optimal land-use tax within a static model would result in land-use taxes which are too high compared to the taxes that would result in the utilization of a dynamic model. The results of this section are not affected by a change in the initial soil depth (0.7, 0.35 m) or in the discount rate (3%, 0%).

4. Summary and conclusions

The utilization of an economic optimization model in combination with the results of a biophysical simulation model provides the basis for the metamodeling approach. It is necessary to apply this approach for analysis of environmental policy instruments, since it enables to find a common equilibrium of the economic and biophysical system. Moreover, the use of biophysical simulation models overcomes the lack of time series data of the relevant variables required for a dynamic analysis. The reflection of nonlinear biophysical relationships together with the incorporation of land-use decision (choice of crops) results in no-convex programming problem where a unique solution for most functions employed in economic analysis cannot be guaranteed. As a solution to this problem, in this paper we propose and employ a modified Cobb Douglas function within the economic analysis. However, the proposed methodology is not only important for the specific problem analyzed within this paper, but in general where policies directed towards the extensive margin, for example land-use regulations, are analyzed together with nonlinear biophysical relationships.

Apart from this methodological contribution, we analyze and compare three different abatement policies, in the first of which, emission standards (emission taxes) cannot be employed in the context of nonpoint source pollution since the regulator cannot observe the emission. However, these instruments serve as least-cost reference for alternative measures. Soil protection scores are shown to be highly inefficient. However, a more precise targeting of this instrument in the future may reduce abatement costs substantially. Land-use taxes, differentiated according to the cultivated crop, cultivation techniques and fertilizer type, are shown to be highly efficient. As the criteria for differentiation are easily observed, land-use taxes can be applied and differentiated in practice. As land-use taxes are fairly high and consume a large share of the farm profit, one has to consider the possibility of reimbursing farmers. One possibility might be to allow for special tax reduction in the tax declaration.

Appendix A. Pseudo-convexity

Let S be nonempty open set in R^n and let f: $S \rightarrow \mathrm{R}^1$ be differentiable in S. The function f is pseudo-convex, if for any x_1, $x_2 \in S$ with $\nabla f(x_1)^t(x_2 - x_1) \geq 0$, $f(x_2) \geq f(x_1)$ or equivalently if $f(x_2) < f(x_1)$, $\nabla f(x_1)^t (x_2 - x_1) < 0$ holds (Bazaraa et al., 1993, p. 113).

Convex and pseudo-convex functions share the characteristic that, if $\nabla f(\bar{x})=0$, the point $\bar{x}$ achieves a global optimum. Thus, the slope of the function has to be distinct from zero for all points other than $\bar{x}$. In contrast to convex functions, pseudo-convex functions may have an inflection point. However, the slope at this point has to be distinct from zero so that it cannot be a saddle point.

The decision problem, given by Eqs. (1)–(6), requires of the objective function that the sum of products of $f(\cdot)_{ijm} y(\cdot)_{tijm}$ and of $\phi(\cdot)_{ijm} y(\cdot)_{tijm}$ have to be pseudo-convex. Given the fact that a sum of quasi-convex functions is not necessarily quasi-convex (Sydsaeter and Hammond, 1995, Thm. 17.16) one can easily deduce that a sum of pseudo-convex functions is not necessarily pseudo-convex.[12] However, every nonnegative linear combination of convex functions is convex (Chiang, 1994). Thus, we need to require that the products of $f(\cdot)_{ijm} y(\cdot)_{tijm}$ and $\phi(\cdot)_{ijm} y(\cdot)_{tijm}$ have to be strictly convex. Unfortunately, it turns out that these products, with f or ϕ based on one of seven different functional forms commonly employed in economics, is not strictly concave (Keusch, 2000, chp. 10). According to a test described by Bazaraa et al. (1993, p. 90), concavity of these functions was analyzed by determining the sign of the principal minors of the Hessian matrix. Each principal minor was either minimized or maximized with Mathematica® to compare its sign with the sign required for concavity. Some functions, such as the Cobb Douglas can be formulated in such a way that the product of $f(\cdot)_{ijm} y(\cdot)_{tijm}$ is concave; however, the parameters need to be restricted severely such that they lose a high degree of flexibility and are only of limited interest for an economic analysis.

Appendix B. Modified Cobb Douglas function

The modified Cobb Douglas function in its general form is given by:

$$Q(\vec{X}^T, Y) = A + \omega\left(\prod_{i=1}^{l} X_i^{c_i}\right) Y^{1-\sum_{i=1}^{l} c_i} + \sum_{i=1}^{l} \times \zeta_i X_i^{e_i} Y^{1-e_i} - \sum_{i=1}^{l} \xi_i X_i^{g_i} Y^{1-g_i}, \qquad (15)$$

where i=1, ..., l denote the ith component of the input vector $\vec{X}$, Y the remaining aggregate input (in this paper the single input land), $Q\left(\vec{X}^T, Y\right)$ yield or erosion, and A the yield or erosion if no input is employed. The domain of the different parameters, specified below by the set (D), is limited by the following restrictions

$$\begin{array}{ll} A = \{A | A \in \mathbf{R}\} & \zeta_i = \{\zeta_i | \zeta_i \geq 0\} \\ \omega = \{\omega | \omega \geq 0\} & e_i = \{e_i | 0 \leq e_i < 1\} \\ c_i = \{c_i | 0 \leq c_i < 1\} \wedge \sum_{i=1}^{l} c_i \leq 1 & \xi_i = \{\xi_i | \xi_i \geq 0\} \\ & g_i = \{g_i | 1 \leq g_i\}. \end{array} \qquad (D)$$

The first term, A, of the sum of Eq. (15) presents the intersection with the y-axis, the second term a classical Cobb Douglas production function and the third and fourth term a concave and convex function, respectively, that reflects the input–output relation more precisely. If the set of restrictions (D) is satisfied, all terms of Eq. (15) with a positive sign in front are concave and all terms with a negative sign in front are convex. Thus, the modified Cobb Douglas function Q is jointly concave in its arguments.

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[12] Take for instance a function ξ, which is the difference between an S-shaped function v and a linear function ς. The functions v and ς are pseudo-convex but the sum $\xi = v - \varsigma$ is not pseudo-convex.

Expressing the input, X_1, per ha of land, Y, and employing the fact that l=1 (in this paper soil depth), Eq. (15) can be written as

$$Q(X_1) = A + \omega(X_1/Y)^c Y + \zeta(X_1/Y)^e Y - \xi(X_1/Y)^g Y,$$

$$\frac{Q(X_1)}{Y} = \frac{A}{Y} + \omega(X_1/Y)^c + \zeta(X_1/Y)^e - \xi(X_1/Y)^g. \qquad (16)$$

The parameters of Eq. (16) still have to comply with the set of restrictions (D). This can be achieved either by the estimation of the parameter subject to the set of restrictions (D) or by an adequate transformation of the parameters (squaring or addition or subtraction of 1) which ensures that the estimated function is jointly concave in its arguments. In contrast to the transformation of the model, the transformation of the parameters does not alter neither deterministic nor stochastic parts of the model. Moreover, the transformation of the parameters improves the convergence of the algorithms employed in the estimation process (Ruckstuhl, 1996).

For this reason, we transformed the parameters of the modified Cobb Douglas. The initial results of the estimations showed that the last term of the modified Cobb Douglas function was not necessary and therefore ξ was set equal to zero. The modified Cobb Douglas function is now given by

$$q = a + (\omega)^2 x^{[1-(c)^2]} - (\zeta)^2 x^{[1+(e)^2]},$$

with q=Q/Y, a=A/Y and x=X_1/Y. Thus, we obtain the specification of the yield and erosion function as presented in the main body of the paper.

The elasticity of scale, like the elasticity of substitution, is not constant (Keusch, 2000). Thus, the function provides a high degree of flexibility. In the model presented by Eqs. (1) (2) (3) (4) (5) (6), however, l=1, and thus, we obtain the desired quality of constant elasticity scale. Yet, the elasticity of substitution remains variable.

Takayama (1991, p. 115) noted that the requirement that the left-hand side of the functions of the inequalities, Eqs. (3)–(6), put into normal form, are at least quasi-convex implies that the constraint set is convex. To test the convexity of this set, the program MPROBE was employed.[13] This tool supports the mathematical analysis of a mathematical programming model by testing the effectiveness of the restrictions, the convexity of the constraint set, and it allows drawing iso-level curves of nonlinear functions. Additionally, the mathematical formulation of the model in AMPL (Algebraic Modeling Programming Language) (Fourer et al., 1993) establishes a direct link with MPROBE.

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[13] See Chinneck (2001, 2002).

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