The development of a mathematical model to investigate Irish beef production systems

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Abstract

Recent reform of European agricultural policy has resulted in substantial changes to the criteria by which premia payments are made. Beef farmers, who have been particularly dependent on premia payments to maintain margins, must re-evaluate their systems to identify optimal systems in these new circumstances. A mathematical model, the Grange Beef Model, is presented and used to identify optimal beef production systems in Ireland. The objective function maximises farm gross margin and the model is primarily constrained by animal nutritional requirements. Model applications are illustrated through the analysis of a series of scenarios concerning variation in beef and concentrate prices; technical development through the integration of an alternative forage and the impacts of participation in an agri-environmental scheme.

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1. Introduction

In many developed countries, agricultural production is supported through managed commodity prices and/or production linked direct subsidies. These support policies have sought to maintain the viability of family farms on the basis of their role in
maintaining the countryside and their contribution to the social fabric of rural communities. However, the future direction of agricultural support policies in developed countries is being questioned. A key driver of this change has been the imminent World Trade Organisation (WTO) Doha round of negotiations.

In anticipation of these negotiations, the EU members agreed to a reform of the Common Agricultural Policy (CAP) in Luxembourg in 2003. The Luxembourg Agreement (LA; European Commission, 2003) resulted in a shift in farm supports from product specific payments to single farm payments (SFP) based, in Ireland’s case, on historical farm support payments and area farmed. The reform returns the focus of producer decisions to market-based considerations, thereby reducing the distortions that have been caused by headage-based livestock subsidies. In this new situation, farmers not making adequate market returns may reduce or cease production whilst others may react by adjusting their production strategies and/or improving the cost competitiveness of their enterprises.

Beef produced in Ireland is sourced from the national cow-herd of 2.4 m of which approximately half are beef cows (CSO, 2005). This results in an annual calf-crop of approximately 1.8 m calves to be raised for beef production. Irish beef is produced predominantly using a diet of grazed and ensiled grass as these have been the cheapest forms of feed available (O’Riordan and O’Kiely, 1996). Research by Binfield et al. (2003) indicates that implementation of the LA in the EU will result in 20% increase in beef prices between 2005 and 2010 due to a reduction in beef supply as beef cow numbers decrease. However, in the intervening period, prices will fluctuate in line with the supply of beef. There is, thus, the need to investigate possible beef system adaptations to these changing circumstances and the potential to improve the cost efficiency of production in response to future challenges.

Mathematical models provide the opportunity to investigate beef production systems within a set of farm constraints and management alternatives (Tess and Kolsstad, 2000a; Nielsen et al., 2004; Rotz et al., 2005; Veyssset et al., 2005). Ireland’s unique situation within Europe with regard to soil type and climate promotes a long growing season and hence, grass based systems of production (Department of Agriculture and Food, 2004). Thus an appropriate model with an emphasis on grassland-based beef production is required.

This paper has two principal objectives. Firstly, to describe the structure of a mathematical model of Irish beef production systems, the Grange Beef Model. Secondly, to demonstrate the application of the model by investigating how farmers might optimally react to a series of scenarios comprising: (a) potential variations in beef and concentrate prices, (b) technical development through the use of an alternative forage (maize silage) and (c) participation in an agri-environmental scheme that limits nitrogen usage.

2. Model details

Simulation modelling and mathematical programming are the two methodologies typically used to model agricultural systems. Simulation consists of modelling the
strategies and biological processes of agricultural systems and simulating the interactions between these processes (Cros et al., 2004). Among the many examples of simulation models in the literature, changes in animal properties (Sanders and Cartwright, 1979; Tess and Kolstad, 2000a,b; Rotz et al., 2005) and forage properties (Doyle et al., 1989; Topp and Doyle, 1996a,b; Cros et al., 2004) have been investigated with respect to their impacts on farm production. However, in the context of the imminent difficulties facing Irish farmers, the challenge is to make optimum use of farm resources and to find the best combination of alternative management strategies to combat these difficulties. Linear programming offers the potential to identify optimal systems and was, thus, identified as the most appropriate modelling technique.

Linear programming has been extensively used to investigate livestock production systems. Many models are concerned with dairy systems (Conway and Killen, 1987; Berentsen and Giesen, 1995; Herrero et al., 1999; Ramsden et al., 1999; Berentsen et al., 2000; Van Calker et al., 2004) with fewer models which study beef systems (Nielsen et al., 2004; Costa and Rehman, 2005; Veysset et al., 2005). The latter models put little emphasis on the production and utilisation of grazed grass instead modelling feeding strategies biannually (Nielsen et al., 2004) or elaborating on the impacts of stocking intensity (Costa and Rehman, 2005) or the impacts of policy changes on production balance and type of produce sold (Veysset et al., 2005).

The aim of the Grange Beef Model is to specify a detailed and integrated set of management alternatives based on grazing temperate grassland within the feeding and livestock specifications of Irish beef production systems and to identify optimal systems based on these alternatives and farm resources available. To accurately capture the possible adaptations to the range of scenarios investigated, it was necessary to include a comprehensive set of the forage and animal production alternatives available to Irish beef farmers. In addition, due to the seasonal nature of these systems, monthly specifications were considered most appropriate.

The model employs a single year steady-state design that comprises 1013 activities and 432 constraints. The equations are specified in a Microsoft Excel spreadsheet and solved using the ‘add-in’ optimisation software “What’s Best!” (Lindo Systems Inc, 2002). A schematic outline of the model framework is presented in Fig. 1. Budgets are formulated for each activity using the most recently available Irish data (Teagasc, 2004). These budgets assign a cost or revenue to each activity and based on these the model identifies the optimal beef production system. The objective function of the model maximises farm gross margin. The mathematical formulation of the model is given in Appendix A.

A number of key assumptions underpin model construction:

– Connolly et al. (2004) estimate that 22% of cattle rearing farms are between 30 and 50 ha and thus, farm size is assumed to be 40 ha.
– The model is constructed around a typical suckler beef herd based on spring-calving Limousin x (Limousin x Friesian) cows (Drennan, 1999) with animal groups based on the average animal within that group.
Animal feed requirements and forage characteristics are based on well established biological functions (Jarrige, 1989; O’Mara et al., 1997; Crowley, 2001; INRA, 2003; O’Kiely, 2004).

Grass production response to nitrogen is estimated using data from experiments conducted at Teagasc, Grange Research Centre for the period 2001–2004 (McGee, 2002; D. Hennessy, pers. comm).

Diets for animal groups are based on a combination of grazing, grass silage, concentrate and maize silage if available. All feeding activities are specified on a monthly basis to incorporate the seasonal variation in animal diets during the year.

Price and cost estimates are those of Teagasc (2004).

2.1. Animal activities

The activities included are those that occur in spring-calving suckler beef production systems in Ireland. Included are suckler beef cow, replacement heifer, calf, store (yearling) and finishing activities. Cows are described as either young (first lactation) or mature (more than one lactation). Because of the predominance of spring-calving in Irish suckler beef herds, all breeding females are assumed to calve in early March. Expected liveweight changes of cows throughout the year are specified. A 20% replacement rate is assumed with replacement heifers purchased as 1-month-old calves. The heifers are assumed to calve at 24 months of age. The milk yield of mature and young beef suckler beef cows follow typical lactation curves found in Irish suckler beef systems, yielding 12 and 10 kg fresh weight milk per day respectively at peak lactation (McGee, 1997). All breeding cows and heifers
are mated to a Charolais sire with the progeny taken to beef within an integrated suckler calf to finish system. Trading options are specified facilitating sale of weanlings and stores. Male progeny can be finished as bulls at 16 months or as steers at 20, 22, 24, 26, 28 or 30 months. Female progeny can be finished at 20, 22 or 24 months.

Nutritional specifications are described in terms of animal energy requirements subject to a maximum intake capacity. The energy requirements of growing and lactating animals are specified in UFL’s (Feed Unit for lactation; Jarrige, 1989) and the energy requirements of finishing animals are specified in UFV’s (Feed Unit for maintenance and meat production; Jarrige, 1989). The intake capacity of all animals are specified in CFU’s (Fill Unit for cattle; Jarrige, 1989). The INRAtion software feeding program (INRAtion 3.0; INRA, 2003), which has been adapted for Irish conditions (O’Mara et al., 1997; Crowley, 2001; Crowley et al., 2002), was used to estimate the energy requirements of growing and finishing beef cattle. The functions used to calculate intake capacities and the maintenance and lactation energy requirements of cows are taken from Jarrige (1989) and are presented in Table 1. Pregnancy requirements are also taken from Jarrige (1989) and are as follows: sixth month, 0.56 UFL d⁻¹; seventh month, 1.08 UFL d⁻¹; eighth month, 1.86 UFL d⁻¹ and ninth month, 2.93 UFL d⁻¹. For growing animals, discrete growth rates (Drennan, 1999) are specified in the model. These can be changed within allowable limits at the input stage.

Intake capacities of growing and finishing animals are calculated using equations by Crowley (2001) which are presented in Table 1. The breed composition of progeny in the model is 50% Charolais, 37.5% Limousin and 12.5% Friesian. Equations to determine intake capacity for this mixed breed of animal are not available. Thus the relevant equations for Charolais cattle (which represent 50% of the genes of the modelled animals and are quite representative of another 37.5% of their genes

Table 1
Daily animal intake and energy requirement equations used in the Grange Beef Model

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nelc ≥ 0.041 × W⁰.⁷⁵</td>
<td>Maintenance net energy requirements of lactating cows (UFL kg d⁻¹); nel = lactation net energy requirements (UFL kg d⁻¹); nedc = net energy requirements of dry, pregnant cows (UFL kg d⁻¹); iclc = intake capacity of lactating cows (CFU kg d⁻¹); icdc = intake capacity of dry, pregnant cows (CFU kg d⁻¹); icgs = intake capacity of growing steers/heifers (CFU kg d⁻¹); icfs = intake capacity of finishing steers/heifers (CFU kg d⁻¹); icfb = intake capacity of finishing bulls (CFU kg d⁻¹); W = animal liveweight (kg); MP = milk production (kg fresh weight d⁻¹).</td>
</tr>
<tr>
<td>nel ≥ 0.45 × MP</td>
<td></td>
</tr>
<tr>
<td>nedc ≥ 0.037 × W⁰.⁷⁵</td>
<td></td>
</tr>
<tr>
<td>iclc ≤ 0.083 × W⁰.⁷⁵ + 0.244 × MP + 2.52</td>
<td></td>
</tr>
<tr>
<td>icdc ≤ 0.09 × W⁰.⁷⁵ + 1.46</td>
<td></td>
</tr>
<tr>
<td>icgs ≤ 0.0368 × W⁰.⁹</td>
<td></td>
</tr>
<tr>
<td>icfs ≤ 0.2087 × W⁰.⁶</td>
<td></td>
</tr>
<tr>
<td>icfb ≤ 0.1970 × W⁰.⁶</td>
<td></td>
</tr>
</tbody>
</table>

Where nelc = maintenance net energy requirements of lactating cows (UFL kg d⁻¹); nel = lactation net energy requirements (UFL kg d⁻¹); nedc = net energy requirements of dry, pregnant cows (UFL kg d⁻¹); iclc = intake capacity of lactating cows (CFU kg d⁻¹); icdc = intake capacity of dry, pregnant cows (CFU kg d⁻¹); icgs = intake capacity of growing steers/heifers (CFU kg d⁻¹); icfs = intake capacity of finishing steers/heifers (CFU kg d⁻¹); icfb = intake capacity of finishing bulls (CFU kg d⁻¹); W = animal liveweight (kg); MP = milk production (kg fresh weight d⁻¹).

a Taken from Jarrige (1989).
b Taken from Crowley (2001).
in terms of intake capacity) are used. Transfer rows in the model facilitate the movement of animals through subsequent stages in the life cycle. Mortality rates are also factored into these transfer rows.

In the current version of this model, protein contents of the diets have not been considered. For the livestock categories specified in the model, the fulfillment of energy requirements by the forage simultaneously satisfies protein requirements. A cross-check is made in the model output to ensure that the protein requirements of animals, as specified in INRAtion 3.0, are satisfied. If the protein requirements have not been satisfied, the user must specify to feed appropriate concentrates until requirements are met.

2.2. Feeding activities

The feeding activities available in the model are pasture, grass silage, maize silage and concentrates. Due to the predominance of pasture-based systems in Ireland, the model specifies a detailed set of grazing options that are typical of those available to Irish cattle farmers. Grass growth is modelled on historical data from Teagasc, Grange Research Centre and is responsive to inorganic nitrogen (N) application rates with annual yields ranging from 6.2 t ha\(^{-1}\) for 0 kgN ha\(^{-1}\) to 13.3 t ha\(^{-1}\) for 300 kgN ha\(^{-1}\) for the grazing area (McGee, 2002; D. Hennessy, pers. comm). The distribution of this herbage throughout the year is given by the following: March, 3.5%; April, 13.0%; May, 21.3%; June, 16.7%; July, 15.3%; August, 13.2%; September, 8.4%; October, 4.3% and November, 3.4%. The balance of the annual herbage yield is available as opening cover at turnout to pasture. The expected yield of herbage is thus specified for each month. In addition, pasture is available for grazing after either one-cut or two-cut grass silage harvest regimes with maximum N application rates of 120 and 60 kgN ha\(^{-1}\), respectively. The model allows transfer of pasture into subsequent months with consequential losses in quality and quantity. The nutritional specifications of the herbage are based on functions taken from Jarrige (1989). The functions describing the energy content of pasture are presented in Table 2. Fill values of pasture are modelled as discrete values of 0.92 and 0.95 CFU kgDM\(^{-1}\) for early season and late season grazing, respectively.

Thresholds for minimum pasture herbage cover regulate turnout and housing. Insufficient growth in the winter period compels the model to provide conserved forage and/or concentrates as the feed source. A number of options are included to facilitate winter feeding and feeding in periods of temporary grass shortage during the grazing season. Within grassland-based Irish beef production systems, two grass harvests for silage are often taken. The first harvest is typically in late May or early June with the second harvest approximately 8 weeks later. In more extensive systems, i.e. lower stocking rates, a single harvest is taken in June. These conservation strategies are provided for in the model. The growth duration prior to harvest may be modified to reflect system options. These growth durations correspond to early, intermediate and late first harvest in one-cut and two-cut systems and short, medium and long regrowths for second-cut harvest in two-cut systems. Thus, in total there are three (first-cut harvest options) \(\times 3\) (first harvest of two-cut options) \(\times 3\) (second...
harvest of two-cut options) grass silage harvesting options available. Energy and fill values for grass silage are taken from O’Mara et al. (Table 2; 1997). Grass silage yields and digestibilities are based on Teagasc, Grange Research Centre data (O’Kiely, 2004). Planned nutrient input recommendations for grass grown for pasture and silage production and for maize silage are those of Teagasc (2001).

Maize silage has also been included as a feed option in the model. Yields and digestibilities used are typical of those achieved under Irish conditions (Crowley, 2000) with nutritional specifications as per INRAtion 3.0. Thus energy contents are 0.75 UFV kgDM$^{-1}$ and 0.81 UFL kgDM$^{-1}$. Concentrate feeding, being a crucial element of beef production, is a key activity available to all animal categories. The default purchased concentrate ration is a barley-based mixture containing soyabean meal and maize gluten feed. The respective proportions of the feed ingredients can be adjusted as required. Default values represent concentrates with energy values of 1.01 UFV kgDM$^{-1}$ and 1.08 UFL kgDM$^{-1}$. Substitution rate, the reduction in forage intake caused by the addition of concentrate feedstuffs to the diet, was addressed using the ‘apparent fill’ method outlined by Jarrige (1989).

2.3. Labour

Labour data used in the model are based on studies carried out by Leahy (2003); and Leahy et al. (2003, 2004) on labour use on Irish beef farms. A constraint on available farmer and family labour is included and where labour is required above this level, there is the option to hire temporary labour at a cost specified by the model user. Silage harvesting and slurry spreading operations are assumed to be carried out by agricultural contractors.

2.4. Environmental considerations

To avail of various government grants and EU premia and to be compliant with legislation, farmers must operate within codes of good practice and must avoid over-
application of organic and inorganic fertiliser. The two primary programmes currently operated are:

1. Rural environment protection scheme (REPS). REPS is a program co-funded by the EU and the Irish government whereby farmers are rewarded financially for operating to a set of guidelines consistent with an agri-environmental plan drawn up by an approved planning agency. Farmers are paid an annual fee based on the area of land farmed and receive €200 ha\(^{-1}\) for the first 20 ha farmed, €175 ha\(^{-1}\) for the next 20 ha, €70 ha\(^{-1}\) for the next 15 and €10 ha\(^{-1}\) for each ha over 55 ha. A significant requirement is the N application limit of 170 kg organic N ha\(^{-1}\) and 260 kg total N ha\(^{-1}\) imposed on the area farmed. Over 38,000 farmers participated in REPS in 2004 with Connolly et al. (2004) estimating that 76% were beef farmers.

2. Nitrates directive. The nitrates directive of the EU (Directive 91/676/EEC) requires measures be taken in respect to farm practices so as to ensure the EU standard for nitrates in potable water of 50 mgNO\(_3\) l\(^{-1}\) is not breached. The implications of this directive are that farmers cannot exceed organic nitrogen (N) application rates of 170 kg N ha\(^{-1}\). In addition closed periods for application of organic manure together with minimum requirements for slurry storage facilities are specified.

The ultimate impact of these programmes is a limit on inorganic and organic N use and thus on the maximum stocking rates on farms.

Within the model, environmental issues are considered by means of maximum organic and inorganic N application constraints. Each animal is assumed to produce a specified quantity of organic N and this combined with inorganic N applied to grassland represents the total farm N usage. Maximum organic and total N usage limits are imposed either in adherence to environmental regulations, e.g. if the farm is participating in REPS or as user defined limits and these constrain production intensity.

2.5. Model evaluation

During the model building process, systems researchers at Teagasc, Grange Research Centre were routinely consulted to ensure appropriate biological relationships were specified and to verify coefficients used in the model. This satisfies the “validation by construct” provisions of McCarl and Apland (1986). Due to the absence of a robust dataset of representative suckler beef farms, expert opinion involving subjective assessment by “knowledgeable individuals” was also used to evaluate the model (Rykiel, 1996). Presentations were made at Grange Research Centre to systems researchers whereby a number of scenarios involving changing resource constraints (land area, animal facilities, N application limits, etc.), input and beef prices and performance indicators (liveweight gains, carcass weights, forage yields, etc.) were outlined. Following a number of such group meetings in addition to a number of individual meetings, researchers were satisfied that the model accurately replicated system processes in terms of financial (revenues, costs, net and gross mar-
gin) and technical (stocking rates, N use, land use, concentrates fed) performances within the range of expectations.

3. Application

Recent changes in European agricultural policy coupled with international currency fluctuations and the imminent WTO negotiations have created an unsettled situation within which a number of components influencing farm production are subject to change. Strategies by farmers to mitigate these effects may include adjustment of their animal and forage production systems, the use of different production technologies and/or the participation in agri-environmental programs. Farmers require information on optimal adjustment strategies given these alternatives and thus a range of scenarios based on these alternatives were designed and solved for using the Grange Beef Model.

Immediately following implementation of the Luxembourg Agreement in 2005, a large reduction in suckler beef cow numbers was predicted to substantially increase the supply of beef to the market resulting in a decrease in the reference beef price of over 5.5% in 2005 (Binfield et al., 2003). However, unforeseen increases in the demand for beef in Britain and continental Europe subsequently led Dunne (2004) to predict that Irish beef prices could increase in 2005. It can be seen that beef price predictions are uncertain at best and subject to unanticipated increases and decreases. A report on the challenges facing Irish agriculture has noted that, “Farmers . . . can expect output prices that will become somewhat more volatile than heretofore” (Agri Vision 2015, 2004).

Input prices for beef farms are also rather unpredictable with cereal prices particularly subject to change. In projecting the outlook for tillage farmers in Ireland, Thorne (2004) observed that much uncertainty exists due to the unpredictable nature of weather, production and price. As a main component of compound concentrate rations, the price of cereals has a direct impact on the cost of these rations to livestock farmers. This in turn could significantly alter the types of systems beef farmers would chose to adopt.

Whilst farmers can respond to these price fluctuations by adjusting their systems of production, they will also have to reduce costs of production and improve their productivity through adoption of different production technologies to survive the economic challenges of the new environment (Agri Vision 2015, 2004). The suitability of the Irish climate to grow grass at low cost is well documented (O’Riordan and O’Kiely, 1996; O’Riordan et al., 1998; McGilloway and O’Riordan, 1999; Humphreys et al., 2001; Department of Agriculture and Food, 2004). The efficient use of this herbage is essential for cost-effective production. Humphreys et al., 2001) observed utilisation rates of between 45% and 77% for treatments based on pre-grazing pasture mass and N fertilisation rate. It is clear that a wide range of rates of grass utilisation by grazing occur on farms.

Grass silage is the predominant form of winter forage conserved on Irish farms (Connolly et al., 2004) and accounts for over 50% of direct costs on beef farms where
progeny are finished (Teagasc, 2004). Depending on the cost of other feeds and on beef price, farmers may chose to increase the percentage of animal feed intake as grazed grass and decrease the quantity of grass silage fed. Thus, the role of grass silage may have to be reconsidered. Furthermore, the use of alternative forages offers the opportunity to reduce feed costs on many beef farms. Maize silage area in Ireland has increased from 8000 ha in 1999 to 15,600 ha in 2003 (CSO, 2005). This increase was facilitated by improved earlier varieties and the widescale use of plastic mulch (Crowley, 2000).

In the context of these challenges and opportunities a number of scenarios were investigated. These scenarios are presented in Table 3. The first scenario is a base scenario and represents the conditions typically found on Irish beef farms in 2005. The second and third scenarios represent an increase and decrease of 10% in beef price. The fourth and fifth scenarios represent an increase and decrease of 10% in concentrate price. The following two scenarios investigate varying herbage utilisation of grazing animals. Changes of 15% from the base scenario of 65% are investigated. The integration of maize silage into beef production systems and its potential to increase gross margin is then investigated. The value of REPS both in contributing to farm gross margin and in limiting N usage is studied by including a scenario where non-participation is assumed.

For all scenarios, land area owned is 40 ha with additional land available for rent at €210 ha⁻¹ and the SFP payable is €400 ha⁻¹. It is assumed that rented land does not have SFP entitlements attached.

4. Results

Table 4 presents the key production results for all the scenarios investigated. The base solution is characterised by a high proportion of land area used exclusively for
Table 4
Selected production results of the nine scenarios investigated using the Grange Beef Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Base</th>
<th>Beef price</th>
<th>Concentrate price</th>
<th>Grass utilisation</th>
<th>Harvest maize</th>
<th>No REPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area farmed (ha)</td>
<td>40.0</td>
<td>60.6</td>
<td>40.0</td>
<td>40.0</td>
<td>54.2</td>
</tr>
<tr>
<td></td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
<td>Good</td>
<td>Poor</td>
</tr>
<tr>
<td></td>
<td>Pasture area (ha)</td>
<td>22.1</td>
<td>33.5</td>
<td>23.0</td>
<td>22.1</td>
<td>30.2</td>
</tr>
<tr>
<td></td>
<td>Early silage harvest (ha)</td>
<td>5.5</td>
<td>8.4</td>
<td>5.2</td>
<td>5.5</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>Late silage harvest (ha)</td>
<td>17.9</td>
<td>27.1</td>
<td>17.0</td>
<td>17.9</td>
<td>24.0</td>
</tr>
<tr>
<td></td>
<td>Maize silage harvest (ha)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Concentrates fed per cow unit (tDM)</td>
<td>0.8</td>
<td>0.8</td>
<td>8.8</td>
<td>0.8</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Inorganic N applied (kgN ha$^{-1}$)</td>
<td>114.9</td>
<td>114.7</td>
<td>99.9</td>
<td>114.4</td>
<td>105.9</td>
</tr>
<tr>
<td></td>
<td>Organic N applied (kgN ha$^{-1}$)</td>
<td>145.1</td>
<td>145.3</td>
<td>138.5</td>
<td>145.6</td>
<td>154.1</td>
</tr>
<tr>
<td></td>
<td>Total N use (kgN ha$^{-1}$)</td>
<td>260.0</td>
<td>260.0</td>
<td>238.4</td>
<td>260.0</td>
<td>260.0</td>
</tr>
<tr>
<td></td>
<td>Suckler beef cow numbers</td>
<td>38.6</td>
<td>58.6</td>
<td>36.9</td>
<td>38.8</td>
<td>60.2</td>
</tr>
<tr>
<td>Males finished$^d$</td>
<td>24St; 17.4</td>
<td>24St; 26.4</td>
<td>24St; 16.6</td>
<td>24St; 17.4</td>
<td>24St; 8.8, 16Bu; 18.3</td>
<td>24St; 26.5</td>
</tr>
<tr>
<td>Females finished$^e$</td>
<td>22Hf; 17.4</td>
<td>22Hf; 26.4</td>
<td>22Hf; 16.6</td>
<td>22Hf; 17.4</td>
<td>22Hf; 27.1</td>
<td>22Hf; 26.5</td>
</tr>
</tbody>
</table>

$^a$ Land used for grazing only.

$^b$ Land can also be used for late silage harvest and is available for grazing.

$^c$ Land is also available for grazing.

$^d$ 24St, steers finished at 24 months; 16Bu, bulls finished at 16 months.

$^e$ 22Hf, heifers finished at 22 months.
grazing and the main part of the grass silage conserved as late harvested silage. Late
harvest is defined in this case as those harvests taken after 20 June. N usage is limited
by REPS specifications and thus total application is therefore 260 kg N ha\(^{-1}\). The
preferred finishing option for male and female progeny identified at 24 months
and 22 months, respectively, on grass silage/concentrate diets.

An increase in beef price leads to an increase in area farmed facilitated by renting
over 20 ha of land. The relative proportion of grassland and grass silage area is sim-
ilar to the base solution. Suckler beef cow numbers increase by 52% with male prog-
eny finished as steers at 24 months of age and heifers finished at 22 months. If beef
price were to decrease, results indicate that the optimal system involves a slight
reduction in animal numbers; in this scenario suckler beef cow numbers are 4% less
than the base solution. Therefore, there is no requirement to rent land in this sce-
nario and N usage is somewhat reduced.

An increase in concentrate price leads to little change in the production system
when compared to the base solution. Land usage, N application rates and animal
production system are similar. In contrast, decreasing concentrate price has a consid-
erable impact on the optimal system of production. A sizable increase in the area of
land farmed of over 14 ha is allied to an increase in animal numbers. In this case,
suckler beef cow numbers are 56% greater than in the base solution. Finishing of
male progeny also shifts somewhat towards finishing as bulls at 16 months with a
consequential increase in concentrates fed.

Improvement in grassland utilisation also leads to an increase in the area of land
farmed when compared to the base solution; in this case over 16 ha is rented. Suckler
beef cow numbers increase by 53% compared to the base with male and female prog-
eny finished at 24 and 22 months of age, respectively. Where herbage utilisation is
poor, there is no land rented. N usage and animal numbers are lower than the base
solution. Finishing of male progeny, whilst primarily based on 24 months, also
includes some finishing of bulls at 16 months of age. Heifer finishing is at 22 months.

With maize harvest included in the production system, land area farmed increases,
concentrate feeding decreases and suckler beef cow numbers increase compared to
the base solution. In this scenario, the area rented is 12 ha and finishing is at 24
months and 22 months for steers and heifers respectively. There is no grass silage
harvested and all forage conserved is as maize silage with 16 ha grown.

The final scenario investigates the impact of not participating in REPS. The result
is an intensification of production with an increase in animal numbers and area
farmed when compared to the base solution. Additional feed requirements are met
largely by an increase in N usage; in this instance N application rates are not limited
by the REPS limit of 260 kg total N ha\(^{-1}\) and thus the total N application rate is
365 kg ha\(^{-1}\). Finishing is similar to the base solution with steers finished at 24
months and heifers finished at 22 months.

Table 5 presents the financial results of the scenarios investigated and follows
from the production systems specified in Table 4. The major revenue item in all sce-
narios is animal sales. Despite this, non-production-based revenue, REPS payments
and SFP receipts, contribute substantially to revenue ranging from 20% for the non-
REPS scenario to 38% for the beef price decrease and the grass utilisation scenarios.
The highest revenue earned is where an increase in beef price is investigated which is 43% greater than the base solution. The scenarios investigating decreases in concentrate price, good grass utilisation and maize harvesting have revenues more than 30% greater than the base solution. Where beef prices decrease by 10% and in the poor grass utilisation scenario, the lowest revenue are observed being 9% lower than the base solution.

Feed costs are the primary costs in all scenarios although, in particular in scenarios where land is rented and animal numbers are greater, animal expenses and land rental costs are also sizeable. The highest costs are for the beef price increase scenario and the non-REPS scenario with total direct costs of over 60% greater than the base solution. The lowest costs are for the poor grass utilisation scenario with direct costs 8% less than the base solution.

The base solution has a gross margin of €32,500. Relative to this the highest gross margins are earned by the scenarios investigating increases in beef price and maize harvesting where gross margins are approximately 20% greater than the base solu-

Table 5
Selected financial results of the nine scenarios investigated using the Grange Beef Model (all results in €000's)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Base</th>
<th>Beef price</th>
<th>Concentrate price</th>
<th>Grass utilisation</th>
<th>Harvest maize</th>
<th>No REPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Good</td>
<td>Poor</td>
</tr>
<tr>
<td>Animal sales</td>
<td>42.5</td>
<td>71.0</td>
<td>36.4</td>
<td>42.6</td>
<td>63.3</td>
<td>64.8</td>
</tr>
<tr>
<td>Interest¹</td>
<td>1.3</td>
<td>1.9</td>
<td>1.1</td>
<td>1.3</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>REPS payments</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>SFP² receipts</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
</tr>
<tr>
<td>Total</td>
<td>67.1</td>
<td>96.2</td>
<td>60.8</td>
<td>67.2</td>
<td>87.8</td>
<td>89.8</td>
</tr>
</tbody>
</table>

Direct costs

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pasture</td>
<td>7.2</td>
<td>10.9</td>
<td>6.7</td>
<td>7.3</td>
<td>9.7</td>
<td>8.6</td>
<td>7.0</td>
<td>9.9</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>Grass silage</td>
<td>11.8</td>
<td>17.9</td>
<td>11.2</td>
<td>11.8</td>
<td>14.1</td>
<td>17.9</td>
<td>10.6</td>
<td>0.0</td>
<td>18.3</td>
<td></td>
</tr>
<tr>
<td>Maize silage</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>20.1</td>
<td></td>
</tr>
<tr>
<td>Concentrate purchases</td>
<td>6.7</td>
<td>10.2</td>
<td>6.4</td>
<td>7.4</td>
<td>13.7</td>
<td>10.2</td>
<td>6.5</td>
<td>4.1</td>
<td>10.1</td>
<td></td>
</tr>
<tr>
<td>Animal expenses²</td>
<td>8.9</td>
<td>13.5</td>
<td>8.5</td>
<td>8.9</td>
<td>13.7</td>
<td>13.6</td>
<td>7.8</td>
<td>13.3</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>Land rental and interest³</td>
<td>0.0</td>
<td>4.3</td>
<td>0.0</td>
<td>0.0</td>
<td>3.0</td>
<td>3.4</td>
<td>0.0</td>
<td>2.5</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>34.6</td>
<td>56.8</td>
<td>32.8</td>
<td>35.4</td>
<td>54.2</td>
<td>53.7</td>
<td>32.0</td>
<td>49.8</td>
<td>56.5</td>
<td></td>
</tr>
<tr>
<td>Gross margin</td>
<td>32.5</td>
<td>39.4</td>
<td>28.0</td>
<td>31.8</td>
<td>33.6</td>
<td>36.1</td>
<td>29.2</td>
<td>38.9</td>
<td>25.1</td>
<td></td>
</tr>
<tr>
<td>Gross margin relative to base</td>
<td>1.21</td>
<td>0.86</td>
<td>0.98</td>
<td>1.03</td>
<td>1.11</td>
<td>0.90</td>
<td>1.20</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

¹ Interest earned on cash surpluses.
² Single farm payments.
³ Expenses include veterinary, transport, breeding and miscellaneous animal costs.
⁴ Interest payable on overdrafts.
The lowest gross margin is earned in the non-REPS scenario which has a gross margin 23% lower than the base solution. All the other scenarios are intermediate between these margins.

5. Discussion

In this paper, a new mathematical model is described. Model specifications and evaluation have been described. The model operation involves a complex interaction of feed costs, animal maintenance costs, beef price, animal intake requirements, farm capacity and policy environment. Application of the model is demonstrated by using it to investigate optimal strategies in the face of changing circumstances brought about by policy and economic developments.

Key concerns for beef farmers are beef prices and concentrate prices. Solutions indicate that changes in beef price can result in substantial adjustments to optimal beef production systems and leads to considerable variation in farm gross margins. A 10% increase in beef price results in an intensification of production with regard to area farmed and animal numbers and a 21% increase in gross margin whereas a similar decrease in beef price leads to more extensive production and a 14% decrease in gross margin. Increase in concentrate price of the magnitude modelled here, indicates little change in production and a marginal 2% decrease in gross margin relative to the base solution. However, where concentrate price decreases by 10%, land area farmed and suckler beef cow numbers increase although gross margin only increases by 3% relative to the base scenario. The more concentrate intensive bull beef finishing at 16 months is preferred for male progeny with resulting increases in concentrate feeding levels.

In addition to optimal adjustment strategies in the face of price changes, farmers also require information on how different production technologies might improve their gross margins. Close attention to grassland management has the potential to increase grass utilisation rates. Results indicate that increases and decreases in grass utilisation rates of 15% relative to the base scenario, results in a 11% increase and 10% decrease in gross margin, respectively. The harvesting of maize silage rather than grass silage also has the potential to increase gross margin by 20% relative to the base scenario. It is therefore, apparent that farmers can be proactive in combating falling gross margin by making judicious use of different production technologies.

Agri-environmental programs, such as REPS, have proven attractive to farmers given that the income derived from such programs can represent a sizable proportion of total farm revenue. These programs can also provide real benefits to society by compelling farmers who participate to operate within a set of environmental guidelines and constraining application rates of inorganic and organic N fertiliser. Relative to the non-REPS scenario, it is apparent that participation in REPS increases gross margin by 23% in the base solution. In addition, inorganic and total N usage is much lower due to the limits imposed by REPS. The increased gross margin in the base scenario of €7,400 is comparable with
the REPS payment received of €7,300. This suggests that there is little change in enterprise output but that this more than offset by the REPS payment. In addition to limits on N application however, REPS also requires compliance with a range of biodiversity and farmyard maintenance measures. These measures must be assessed by individual farmers with regard to the increased overhead costs of compliance. However, REPS has remained an attractive program for beef farmers (Connolly et al., 2004) and will continue to be effective in constraining N application rates.

A conspicuous aspect of results is the importance of land availability. In each case where gross margin is greater than the base scenario, additional land area is rented. The area rented ranges from 13.5 ha in the maize harvesting scenario to 22.5 ha in the beef price increase scenario. Land rental prices in all scenarios are assumed to be €210 ha\(^{-1}\) and this price will be important in determining optimal systems.

It has been shown that the model can be used to analyse current or prospective scenarios of interest. Future changes in agricultural policy can be routinely investigated. The sensitivity of optimal systems to price changes can be analysed. Whilst much of the production data is based on performances obtained at Grange Research Centre, the parameters can be modified to reflect other situations.

Acknowledgements

The authors acknowledge the contributions of M.J. Drennan, R. Fallon, P. French, M.G. Keane, M. McGee and E.G. O’Riordan in this research and in evaluation of the model. The financial assistance for one of the authors (P. Crosson) of the Teagasc Walsh Fellowship and the National Development Plan 2000–2006 of the Irish Government is also acknowledged.

Appendix A. Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOTLAND</td>
<td>ha of land farmed</td>
</tr>
<tr>
<td>LANDO</td>
<td>ha of land owned</td>
</tr>
<tr>
<td>LANDIN, LANDOUT</td>
<td>ha of land rented in and rented out</td>
</tr>
<tr>
<td>PAS(_a), 1SIL(_b), 2SIL(_c)</td>
<td>ha of land used for pasture activity (a), one-cut silage activity (b) and two-cut silage activity (c)</td>
</tr>
<tr>
<td>1SILYD(_b), 2SILYD(_c)</td>
<td>Grass silage yield (kgDM) of one-cut silage activity (b) and two-cut silage activity (c)</td>
</tr>
<tr>
<td>GSIL</td>
<td>Total grass silage production (kgDM)</td>
</tr>
</tbody>
</table>

(continued on next page)
### Appendix A (continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PASECa, 1AGECbab, 2AGECc</strong></td>
<td>Energy content (UFL kgDM⁻¹ or UFV kgDM⁻¹) of grass produced under pasture activity <em>a</em> and of after-grass produced under one-cut silage activity <em>b</em> and under two-cut silage activity <em>c</em>.</td>
</tr>
<tr>
<td><strong>GSILEC</strong></td>
<td>Energy content (UFL kgDM⁻¹ or UFV kgDM⁻¹) of grass silage.</td>
</tr>
<tr>
<td><strong>CONC</strong></td>
<td>kgDM Concentrates fed annually.</td>
</tr>
<tr>
<td><strong>CONCEC</strong></td>
<td>Energy content (UFL kgDM⁻¹ or UFV kgDM⁻¹) of concentrates.</td>
</tr>
<tr>
<td><strong>STOCKd</strong></td>
<td>Number of animals in stock group <em>d</em>.</td>
</tr>
<tr>
<td><strong>STOCKERd</strong></td>
<td>Energy requirements (UFL day⁻¹ or UFV day⁻¹) of stock group <em>d</em>.</td>
</tr>
<tr>
<td><strong>PASFVa, 1AGFVbab, 2AGFVc</strong></td>
<td>Fill value (CFU kgDM⁻¹) of grass produced under pasture activity <em>a</em> and of after-grass produced under one-cut silage activity <em>b</em> and under two-cut silage activity <em>c</em>.</td>
</tr>
<tr>
<td><strong>GSILFV</strong></td>
<td>Fill value (CFU kgDM⁻¹) of grass silage.</td>
</tr>
<tr>
<td><strong>STOCKICd</strong></td>
<td>Intake capacity (CFU day⁻¹) of stock group <em>d</em>.</td>
</tr>
<tr>
<td><strong>PASLABa, 1SILLABb, 2SILLABc</strong></td>
<td>Labour requirements (hrs ha⁻¹) of pasture activity <em>a</em>, one-cut silage activity <em>b</em> and two-cut silage activity <em>c</em>.</td>
</tr>
<tr>
<td><strong>GSILLAB, CONCLAB</strong></td>
<td>Labour requirements (hrs kgDM⁻¹) of grass silage feeding and of concentrate feeding.</td>
</tr>
<tr>
<td><strong>STOCKLABd</strong></td>
<td>Labour requirements (hrs hd⁻¹) of stock group <em>d</em>.</td>
</tr>
<tr>
<td><strong>MISLAB</strong></td>
<td>Labour requirements (hrs) of miscellaneous tasks.</td>
</tr>
<tr>
<td><strong>FAMLAB, HIRELAbm</strong></td>
<td>Annual family labour available and labour hired in month <em>m</em> (hrs).</td>
</tr>
<tr>
<td><strong>PASNab</strong></td>
<td>Inorganic nitrogen application rate (kgN ha⁻¹) of pasture activity <em>a</em>.</td>
</tr>
<tr>
<td><strong>1SILN, 2SILN</strong></td>
<td>Inorganic nitrogen application rate (kgN ha⁻¹) of one-cut and of two-cut silage activity.</td>
</tr>
<tr>
<td><strong>1AGNab, 2AGNc</strong></td>
<td>Inorganic nitrogen application rate (kgN ha⁻¹) of after-grass produced.</td>
</tr>
</tbody>
</table>
Appendix A (continued)

under one-cut silage activity \( b \) and two-cut silage activity \( c \)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>STOCKNd</td>
<td>Organic nitrogen production (kgN hd(^{-1})) of animal group ( d )</td>
</tr>
<tr>
<td>TOTNLIM</td>
<td>Total nitrogen (inorganic and organic) limit (kgN ha(^{-1}))</td>
</tr>
<tr>
<td>ORGNLIM</td>
<td>Organic nitrogen limit (kgN ha(^{-1}))</td>
</tr>
<tr>
<td>COW</td>
<td>Number of cows</td>
</tr>
<tr>
<td>BCALF, HCALF, RCALF</td>
<td>Number of bull, heifer and replacement calves</td>
</tr>
<tr>
<td>YSTEER, YHEIFER</td>
<td>Number of yearling steers and heifers</td>
</tr>
<tr>
<td>FBULL, FSTEER, FHEIFER</td>
<td>Number of finishing bulls, steers and heifers</td>
</tr>
<tr>
<td>RHEIFER</td>
<td>Number of replacement heifers</td>
</tr>
<tr>
<td>YCOW, MCOW</td>
<td>Number of young and mature cows</td>
</tr>
<tr>
<td>SELLWBCALF(_m), SELLWHCALF(_m)</td>
<td>Number of weanling bull and heifer calves sold in month ( m )</td>
</tr>
<tr>
<td>SELLSSTEER(_m), SELLSHEIFER(_m)</td>
<td>Number of store steers and heifers sold in month ( m )</td>
</tr>
<tr>
<td>SELLFSTEER(_m), SELLFHEIFER(_m), SELLFBULL(_m), SELLCULL(_m)</td>
<td>Number of finishing steers, heifers and bulls sold in month ( m )</td>
</tr>
<tr>
<td>ADFAC, YFAC</td>
<td>Number of adult cattle and young cattle facilities available</td>
</tr>
<tr>
<td>PASBT(_a), 1SILBT(_b), 2SILBT(_c)</td>
<td>Budget of pasture activity ( a ), one-cut silage activity ( b ) and two-cut silage activity ( c ) (€ ha(^{-1}))</td>
</tr>
<tr>
<td>CONCCT</td>
<td>Cost of concentrates (€ tDM(^{-1}))</td>
</tr>
<tr>
<td>STOCKEX(_d)</td>
<td>Annual expenses of stock group ( d ) (€ hd(^{-1}))</td>
</tr>
<tr>
<td>WBCALFSV, WHCALFSV</td>
<td>Sale value of weanling bulls and heifers (€ hd(^{-1}))</td>
</tr>
<tr>
<td>SSTEERSV, SHEIFERSV</td>
<td>Sale value of store steers and heifers (€ hd(^{-1}))</td>
</tr>
<tr>
<td>FSTEERSV, FHEIFERSV, FBULLSV</td>
<td>Sale value of finishing steers, heifers and bulls (€ hd(^{-1}))</td>
</tr>
<tr>
<td>HIRELABCT</td>
<td>Labour cost (€ hr(^{-1}))</td>
</tr>
<tr>
<td>OH</td>
<td>Total overhead costs (€)</td>
</tr>
<tr>
<td>LE</td>
<td>Total living expenses (€)</td>
</tr>
<tr>
<td>OFE</td>
<td>Off-farm employment (€)</td>
</tr>
</tbody>
</table>
A.1. Land and feeding

\[ \text{TOTLAND} = \text{LANDO} + \text{LANDIN} - \text{LANDOUT} \]

\[ \sum_{a=1}^{7} \text{PAS}_{a} + \sum_{b=1}^{7} 1\text{SIL}_{b} + \sum_{c=1}^{7} 2\text{SIL}_{c} \leq \text{TOTLAND} \]

\[ \sum_{b=1}^{7} 1\text{SIL}_{b} \times 1\text{SILYD}_{b} + \sum_{c=1}^{7} 2\text{SIL}_{c} \times 2\text{SILYD}_{c} = \text{GSIL} \]

\[ \sum_{a=1}^{7} \text{PAS}_{a} \times \text{PASEC}_{a} + \sum_{b=1}^{7} 1\text{SIL}_{b} \times 1\text{AGEC}_{b} + \sum_{c=1}^{7} 2\text{SIL}_{c} \times 2\text{AGEC}_{c} \]

\[ + \text{GSIL} \times \text{GSILEC} + \text{CONC} \times \text{CONCEC} \geq \sum_{d=1}^{19} \text{STOCK}_{d} \times \text{STOCKER}_{d} \]

\[ \sum_{a=1}^{7} \text{PAS}_{a} \times \text{PASFV}_{a} + \sum_{b=1}^{7} 1\text{SIL}_{b} \times 1\text{AGFV}_{b} + \sum_{c=1}^{7} 2\text{SIL}_{c} \times 2\text{AGFV}_{c} \]

\[ + \text{GSIL} \times \text{GSILFV} \leq \sum_{d=1}^{19} \text{STOCK}_{d} \times \text{STOCKIC}_{d} \]

A.2. Labour

\[ \sum_{a=1}^{7} \text{PAS}_{a} \times \text{PASLAB}_{a} + \sum_{b=1}^{7} 1\text{SIL}_{b} \times 1\text{SILLAB}_{b} + \sum_{c=1}^{7} 2\text{SIL}_{c} \times 2\text{SILLAB}_{c} \]

\[ + \text{GSIL} \times \text{GSILLAB} + \text{CONC} \times \text{CONCLAB} + \sum_{d=1}^{19} \text{STOCK}_{d} \]

\[ \times \text{STOCKLAB}_{d} + \text{MISLAB} \]

\[ \leq \text{FAMLAB} + \sum_{m=1}^{12} \text{HIRELAB} \]

A.3. REPS N application constraints

\[ \sum_{a=1}^{7} \text{PAS}_{a} \times \text{PASN}_{a} + \sum_{b=1}^{7} 1\text{SIL}_{b} \times 1\text{SILN} + \sum_{b=1}^{7} 1\text{SIL}_{b} \times 1\text{AGN}_{b} \]

\[ + \sum_{c=1}^{7} 2\text{SIL}_{c} \times 2\text{SILN} + \sum_{c=1}^{7} 2\text{SIL}_{c} \times 2\text{AGN}_{c} + \sum_{d=1}^{19} \text{STOCK}_{d} \]

\[ \times \text{STOCKN}_{d} \leq \text{TOTLAND} \times \text{TOTNLIM} \]

\[ \sum_{d=1}^{19} \text{STOCK}_{d} \times \text{STOCKN}_{d} \leq \text{TOTLAND} \times \text{ORGNLIM} \]
A.4. Stock transfers

\[ \sum_{d=1}^{19} \text{STOCK}_d = \text{COW} + \text{BCALF} + \text{HCALF} + \text{RCALF} + \text{YSTEER} + \text{YHEIFER} + \text{FBULL} + \text{FSTEER} + \text{FHEIFER} + \text{RHEIFER} \]

\[ \text{COW} = \text{YCOW} + \text{MCOW} \]

\[ \text{COW} \times 0.9 = \text{BCALF} + \text{HCALF} \]

\[ \text{COW} \times 0.2 = \text{RCALF} \]

\[ \text{BCALF} = 0.99 \text{YSTEER} + \text{FBULL} + \sum_{m=1}^{12} \text{SELLWBCALF}_m \]

\[ \text{YSTEER} = \text{FSTEER} + \sum_{m=1}^{12} \text{SELLSSTEER}_m \]

\[ \text{FSTEER} = \sum_{m=1}^{12} \text{SELLFSTEER}_m \]

\[ \text{HCALF} = 0.99 \text{YHEIFER} + \sum_{m=1}^{12} \text{SELLWHCALF}_m \]

\[ \text{YHEIFER} = \text{FHEIFER} + \sum_{m=1}^{12} \text{SELLSHEIFER}_m \]

\[ \text{FHEIFER} = \sum_{m=1}^{12} \text{SELLFHEIFER}_m \]

\[ \text{FBULL} = \sum_{m=1}^{12} \text{SELLFBULL}_m \]

\[ \text{COW} \times 0.2 = \sum_{m=1}^{12} \text{SELLCULL}_m \]

A.5. Facilities

\[ \text{ADFAC} \geq \text{COW} + \text{FBULL} + \text{FSTEER} + \text{FHEIFER} + \text{RHEIFER} \]

\[ \text{YFAC} = \text{BCALF} + \text{HCALF} + \text{RCALF} \]

A.6. Objective function

\[ \text{MAX} : \sum_{a=1}^{7} \text{PAS}_a \times \text{PASBT}_a + \sum_{b=1}^{7} \text{1SIL}_b \times \text{1SILBT}_b + \sum_{c=1}^{7} \text{2SIL}_c \times \text{2SILBT}_c \]

\[ + \text{CONC} \times \text{CONCCT} + \sum_{d=1}^{19} \text{STOCK}_d \times \text{STOCKEX}_d \]
\[
+ \sum_{m=1}^{12} \text{SELLWBCALF}_m \times \text{WBCALFSV} + \sum_{m=1}^{12} \text{WHCALF}_m \times \text{WHCALFSV} + \sum_{m=1}^{12} \text{SELLSSTEER}_m \times \text{SSTEERSV} \\
+ \sum_{m=1}^{12} \text{SELLSHEIFER}_m \times \text{SHEIFERSV} + \sum_{m=1}^{12} \text{SELLFSTEER} \times \text{FSTEERSV} \\
+ \sum_{m=1}^{12} \text{SELLFBULL}_m \times \text{FBULLSV} + \sum_{m=1}^{12} \text{HIRELAB} \times \text{HIRELABCT} + \text{OH} + \text{LE} + \text{OFE}
\]

References


Department of Agriculture and Food., 2004. Proposal for a derogation from a limit of 170 kg of organic nitrogen per hectare per annum specified in the nitrates directive. Department of Agriculture and Food, Agriculture House, Kildare Street, Dublin 2, Ireland.


