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Optimal Feed Mill Blending

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Commercial feed blending is a complex process consisting of many potential raw ingredients and final products. The sheer number of daily orders and final products at a typical feed mill means that raw ingredients cannot be mixed to directly produce final products in an economical fashion. As a result, the intermediate production of pellets with prespecified nutritional content is a necessity that makes the feed blending problem highly nonlinear. We discuss a nonlinear approach to feed blending and compare results from an empirical application to those from a sequential linear programming approach common to most feed mills.

Modern U.S. commercial feed mills that blend feed for sale to agricultural operations are engaged in a complex production process consisting of many potential raw ingredients and final products. The typical feed mill likely produces and delivers feed for poultry (turkeys and broilers), swine, beef cattle, and dairy operations.[1] Each of these operations requires several types of feed rations since they are typically designed around the nutritional requirements of animals in various stages of growth or production. For example, a lactating dairy cow requires a different feed ration than a bred heifer and a just weaned pig requires a different feed ration than a hog approaching market weight.

Although feed ration balancing is a prototype-blending problem found in many introductory linear programming (LP) textbooks (for example, Paris or Taylor), there are many reasons why strict application of the model is not used at feed mills. First, the sheer number of final products produced in conjunction with multiple daily orders of varying sizes, means that raw ingredients cannot be mixed to directly produce final products in an economical or practical manner. Also, storage and other production-capacity constraints limit the usefulness of such a basic approach. As a result, feed mills combine raw ingredients to produce a set of intermediate products that can be used in a variety of ways to produce final products possessing the appropriate nutritional composition. These intermediate products, called *pellets,* typically possess nutritional characteristics (or at least

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tolerances) specified by management.[2] Once the content of the pellets is known, they are blended to meet the nutritional characteristics of final products.

Management's objective is to meet the demand for final products with specific nutritional characteristics at least cost, given the availability and cost of raw ingredients. The intermediate step of producing pellets is a necessity, which makes the feed blending problem nonlinear (NL). To see this, consider a single nutritional requirement for a pellet, say crude protein. The amount in a pellet is essentially a decision variable since it is the choice over a set of raw ingredients with varying amounts of crude protein that determines the composition of the pellet. However, this information becomes technical coefficient data when pellets are mixed to produce final products and results in a NL feed blending problem. This nonlinearity is demonstrated mathematically in the first three sets of constraints in the NL feed blending model presented in the Appendix.

One way around the nonlinearity is to model the blending problem as a two-stage LP framework. In this setting, raw ingredients are turned into pellets at minimum cost with the recipe for a given pellet determined by an LP model. Subsequently, another LP model determines the minimum cost recipe for a given final product using the pellets with nutrient content determined by the first LP model. Obviously, the recipes for the final products are only truly optimal if the pellet recipes (including which pellets to make) were also optimal.

Another problem encountered by feed mills is that some raw ingredients do not need to be or cannot be turned into pellets. Molasses, for example, is added to many feeds to improve palatability. However, molasses is not incorporated directly into pellets, but rather, added along with pellets to make a final product. Thus, some raw ingredients enter the feed blending process during pellet production while others are blended with pellets to create final products. Despite these drawbacks, feed mills continue to use the LP approach most often because of the lack of commercially available software for handling the more complex nonlinear problem.

Constructing a nonlinear pooling model is one alternative to the two-stage LP that deals with the issue of nonlinearity as well as when certain raw products enter production. This is in contrast to a blending model that simultaneously determines the composition of the pellets and final products from a set of raw ingredients. Fieldhouse described an example of this sort of problem. Feed blending becomes a pooling problem rather than a blending problem since the nutrient content of the pellets is unknown prior to solution of the model, and is determined endogenously by the constraints of both the pellets and the final products. The pooling problem has been examined in many different industries, such as oil and gas, chemical, and food ingredient industries (see, for example, Amos et al.; Karmarkar and Rajaram). In the oil and gas pooling problem, ingredients available to the pooler typically are different types of crude oil. These oils are fractionated and pooled to achieve quality standards specified by the refinery (Amos et al.).

While it is common knowledge that the feed mill's ration formulation problem is most appropriately modeled as an NL program, empirical applications have not appeared in the agricultural literature like they have in the energy literature. In addition, the computational burden associated with any NL modeling approach, even for today's computers, is still quite high. As a result, comparing the solution times, costs, and overall performance of NL and sequential LP models is a useful academic exercise that can lead to better NL model and algorithm development. In this paper, we present the results of a simple NL blending model using hypothetical data and compare the results to those suggested by a sequential LP blending model.

Empirical Application

The essence of the feed-mill's blending problem can be captured through a relatively simple example. In this example, the feed mill wishes to determine a least-cost means of producing four final products using at most, six different pellets and four raw ingredients that enter production in the second stage (i.e., when the pellets are combined to make the final products). The objective is to determine how much of each raw ingredient to procure and how to blend the ingredients to make a set of pellets. These pellets are then combined with other raw ingredients to make the final products, within the limits specified, at minimum cost. A standard set of raw ingredients[3] and nutritional constraints[4] are assumed.

The first four of the six different pellets are differentiated by crude protein or fiber content. For example, pellet 1 has a low crude protein percentage, but high fiber content, and pellet 4 has high crude protein and low fiber. Pellets 5 and 6 differ from the first four due to the addition of supplemental minerals and vitamins. These two pellets differ by crude protein and phosphorus levels. The final products vary by the levels of crude protein and raw ingredients added after the intermediate production (pellet) stage.

Tables 1 and 2 present the results of a standard sequential LP approach to the feed blending problem at current prices.[5] In this setting, the feed-mill manager would procure the ingredients listed and combine them in the manner suggested to form each of the six pellets from which final products may be produced. At this point, the exact nutritional content of each pellet can be determined since the nutritional content of each raw ingredient is known and the LP provides the recipe for the pellets. These outputs can then be used in a second-stage LP model along with other raw ingredients to form the final products.

An LP approach such as this circumvents the nonlinearity by breaking the problem into two sequential problems where the output of the first LP (the pellet solution) is imposed on the second LP (the final product problem). Table 2 presents an example of the model results and shows how each of the final products should be produced using the available pellets and other raw ingredients. Notice that pellets 2 and 5 are not used in the set of final products. A priori, the feed mill cannot afford to ignore these two pellets when deciding on a production run as relative prices may mean that one week they should be made, and the next otherwise. However, it is clear that another drawback of the sequential LP approach is that recipes need to be determined for all potential pellets irrespective of whether they will be used or not.

The Appendix presents an NL model depicting the feed mill's problem. In the model, the appropriate pellets to produce are optimally determined at the same time that the recipes for pellets and final products. As table 3 shows, the optimal recipes suggested by the NL model are somewhat different than the results from the sequential LP. Most notably, it is not optimal to produce pellet 3 in the NL approach.

Table 1. Optimal linear programming solutions for pellet ration formulation

	Intermediate Products					
Ingredient	**Pellet 1**	**Pellet 2**	**Pellet 3**	**Pellet 4**	**Pellet 5**	**Pellet 6**
Alfalfa meal	-	-	-	-	-	-
Broiler vitamin premix	-	-	-	-	-	-
Calcium sulfate	-	-	-	-	-	-
Copper sulfate	-	-	-	-	-	-
Cottonseed meal	-	-	-	-	-	10.8%
Dicalcium phosphate	-	0.4%	-	1.1%	6.6%	4.6%
Distiller's grain	-	-	-	-	6.2%	7.7%
Dynamate	-	0.1%	-	-	-	-
Fat	-	-	-	-	-	-
Gluten feed	-	-	-	-	-	-
Ground corn	-	8.3%	-	-	5.8%	-
Limestone	5.4%	2.3%	3.7%	7.4%	-	-
Magnesium oxide	-	-	-	0.3%	0.6%	0.6%
Monosodium phosphate	-	-	-	-	0.8%	1.1%
Salt	-	1.5%	1.5%	3.2%	2.5%	2.5%
Selenium premix I	-	-	-	-	-	0
Selenium premix II	-	0.1%	0.1%	0.6%	1.1%	1.0%
Soybean hulls	-	-	-	-	-	-
Soybean meal	-	1.2%	15.2%	81.3%	2.6%	7.6%
Trace mineral premix	-	0.1%	-	-	-	1.2%
Vitamin A, D, E	-	-	-	-	-	-
Vitamin E	-	-	-	-	0.2%	0.2%
Vitamin premix	-	0.1%	0.1%	0.2%	0.7%	0.7%
Wheat middlings	94.6%	85.9%	79.5%	5.9%	72.7%	61.7%
ZinPro	-	-	-	-	-	-
Cost ($/ton)	**$78.75**	**$88.45**	**$96.36**	**$169.29**	**$137.59**	**$157.13**

Table 4 presents recipes for the final products suggested by the NL model. All the tables show the cost of pellets and final products produced in dollars per ton. Somewhat counterintuitively, the cost per ton of pellet produced suggested by the sequential LP solution is always less than the suggested NL solution. It is important to point out, however, that the two models suggest different recipes for a given pellet so the nutritional content can vary depending on the method (i.e., linear vs. nonlinear) chosen to formulate it.

Even so, the cost per ton of final product is always in favor of the NL program solution. Comparing the last two rows of tables 2 and 4 reveals that the cost per ton differences are $8.37, $8.18, $19.08, and $29.80 for Products A, B, C, and D. These differences amount to savings per ton of 6.6%, 5.0%, 14.3%, and 23.7%, respectively, for the four final products.

This result is not unanticipated given the potential superiority of the NL approach over sequential LP. For example, using the sequential LP solution, Product A should be mixed using pellets 3, 4, and 6. The NL program suggests using pellets 1, 4, and 5 is optimal. It is tempting to conclude that since the LP solution

Table 2. Optimal linear programming solutions for final product ration formulation

	Final Products			
Ingredient	**Product A**	**Product B**	**Product C**	**Product D**
Pellet 1	-	0.7%	0.7%	-
Pellet 2	-	-	-	-
Pellet 3	35.6%	3.3%	38.8%	27.7%
Pellet 4	23.1%	80.9%	23.3%	34.8%
Pellet 5	-	-	-	-
Pellet 6	18.6%	14.3%	34.1%	3.5%
Molasses	2.5%	0.8%	3.2%	14.0%
Soybean oil	-	-	-	-
Steam crimped barley	20.2%	-	-	20.0%
Steam flaked corn	-	-	-	-
Cost ($/ton)	**$127.58**	**$163.89**	**$133.69**	**$125.91**

uses less expensive ingredients, the NL solution is less desirable. However, the recipes for pellets 1, 4, and 5 in the NL program differ from the LP recipes for the same pellets. This is because the NL program has more information from which to determine the optimal composition of the pellets. In short, the recipes for pellets 3, 4, and 6 (from the LP solution) are not optimal from the perspective of the final products.

It is also important to point out that another potential advantage of the NL model, as formulated in the Appendix, is that final product demands drive the pooling process. While the results have been presented in percentage terms, the model can show how much to procure (in tons) of each raw ingredient. This allows for better management of a feed mill because final products with higher demands are allowed to have a greater influence over the amount and type of pellets produced.

Practical Implementation Issues

While it appears that the NL approach to the feed blending problem is far better, a few caveats are in order. First, NL programs such as the one presented do not satisfy the convexity requirements that insure a globally optimal solution. Therefore, it is conceivable that in some settings, the NL model may produce multiple local optima, some of which may actually be inferior to the LP solution. Also, for even relatively small applications such as the one presented in this paper, model runtimes can be considerably longer than LP. For a problem such as the one presented, runtime is inconsequential for modern LP solvers while the NL model takes on average of about a minute to solve.

The results in table 5 are designed to examine the possibility of multiple local optima, solution stability, and the impact of nonlinearities on solver runtime. The dollar cost per ton of each final product from the LP solution is presented for reference. Five additional solutions from the NL model generated from various starting points (initial solutions) and methods are also shown. For

Table 3. Optimal nonlinear programming solutions for pellet ration formulation

	Intermediate Products					
Ingredient	**Pellet 1**	**Pellet 2**	**Pellet 3**	**Pellet 4**	**Pellet 5**	**Pellet 6**
Alfalfa meal	-	-	-	-	-	-
Broiler vitamin premix	-	-	-	-	-	-
Calcium sulfate	-	-	-	-	-	-
Copper sulfate	-	-	-	-	-	-
Cottonseed meal	-	-	-	-	-	-
Dicalcium phosphate	-	2.7%	-	1.2%	6.7%	4.7%
Distiller's grain	-	-	-	-	-	-
Dynamate	-	21.8%	-	-	-	-
Fat	-	6.9%	-	-	-	-
Gluten feed	-	-	-	-	-	-
Ground corn	-	40.9%	-	-	5.0%	-
Limestone	2.0%	1.5%	-	7.1%	-	-
Magnesium oxide	-	-	-	0.5%	0.7%	0.7%
Monosodium phosphate	-	-	-	-	0.9%	1.5%
Salt	0.7%	1.5%	-	3.2%	2.5%	2.5%
Selenium premix I	0.2%	-	-	0.3%	-	-
Selenium premix II	-	0.1%	-	-	1.3%	1.0%
Soybean hulls	-	-	-	-	-	-
Soybean meal	0.5%	24.2%	-	87.2%	8.7%	26.8%
Trace mineral premix	0.2%	0.1%	-	0.5%	-	7.4%
Vitamin A, D, E	-	-	-	-	-	-
Vitamin E	-	-	-	-	0.5%	0.2%
Vitamin premix	-	0.3%	-	0.2%	2.3%	0.7%
Wheat middlings	96.4%	-	-	-	71.5%	54.4%
ZinPro	-	-	-	-	-	-
Cost ($/ton)	**$82.15**	**$173.08**	**$0.00**	**$176.33**	**$178.39**	**$193.81**

example, solution 1 uses the standard NL approach and zeros for starting values for all decision variables. The second solution uses the previous solution as starting values and shows only improvement in the number of iterations required to solve the model. In the third solution, the LP solution was used as the starting values and there is noticeable improvement in terms of cost with virtually no increase in runtime. The fourth solution uses the previous solution's starting values and shows no improvement. Notice that in comparing the two sets of solutions, the primary difference beyond cost is the inclusion of pellet 5 in the optimal solution when starting from the LP solution. The second set of solutions resulted in an overall cost reduction of about one-half percent when compared to the first set.

The fifth solution presented in table 5 is different in two respects. First, a heuristic was used to determine starting values (referred to as "crashing the initial solution"). Second, a steepest-edge algorithm was employed so that the nonlinear solver would spend more time selecting variables (to improve the objective function) by looking at the rate that the objective function would improve relative

Table 4. Optimal nonlinear programming solutions for final product ration formulation

Ingredient	Final Products			
	Product A	Product B	Product C	Product D
Pellet 1	44.8%	21.2%	64.9%	58.6%
Pellet 2	-	-	-	7.4%
Pellet 3	-	-	-	-
Pellet 4	30.3%	78.1%	32.0%	-
Pellet 5	2.4%	-	-	-
Pellet 6	-	-	2.1%	-
Molasses	2.5%	0.8%	1.1%	14.0%
Soybean oil	-	-	-	-
Steam crimped barley	20.0%	-	-	10.0%
Steam flaked corn	-	-	-	10.0%
Cost ($/ton)	**$119.21**	**$155.71**	**$114.61**	**$96.11**

Table 5. Comparison and stability of linear and nonlinear feed blending solutions

Model Run[a]	Final Product Cost ($/ton)				Pellets	Iterations	Time[b]
	Product A	Product B	Product C	Product D			
LP solution	$127.58	$163.89	$133.69	$125.91	-	-	-
Nonlinear Solutions:							
1	$121.08	$156.66	$118.13	$92.34	1, 2, 4, 6	724	1:05
2	$121.08	$156.66	$118.13	$92.34	1, 2, 4, 6	289	1:06
3	$119.21	$155.71	$114.61	$96.11	1, 2, 4, 5, 6	326	1:06
4	$119.21	$155.71	$114.61	$96.11	1, 2, 4, 5, 6	333	1:07
5	$119.53	$155.42	$113.43	$92.07	1, 2, 3, 4, 6	238	1:10

[a]The five nonlinear solutions correspond to (1) zero initial values for all decision variables, (2) initial values for all decision variables equal to those from solution 1, (3) initial values for all decision variables equal to the LP solution, (4) initial values for all decision variables equal to those from solution 3, and (5) initial values for all decision variables determined via a heuristic and steepest edge strategy employed.
[b]All model solutions were generated on a Pentium 4 operating at 3.2 GHz with 1 GHz of RAM memory.

to movements in the other nonzero variables. While both these options have the potential to increase runtime dramatically, neither did in any of the cases studied. The optimal solution suggests that pellet 3 should be substituted for pellet 5 resulting in another 1% reduction in total cost.

From this analysis, it is clear that the nonlinearities inherent in the feed blending problem are potentially problematic in that local solutions are clearly being found by the solver. However, in all cases examined, the local solutions generated via NL programming outperform the LP solutions from a cost perspective. For example, the total cost of all four final products from the LP solution is $551.07, compared to the best NL solution of $480.45 (table 5). Costs savings such as this can translate into many thousands of dollars for even a moderate-sized feed mill over the course of a year. Even so, the NL approach is likely most useful as a complementary model to help management make better decisions about ration formulation. This is because runtimes likely rise dramatically as the number of raw ingredients and/or pellets and/or final products considered by the model increases.

Summary and Conclusions

In this paper, we describe an NL approach to feed blending and present an empirical application of the model. The results are compared to feed rations suggested by a more common two-stage LP approach. The final products formulated in the NL model were −5 to 24 percent per ton less costly than those determined by LP, even though the intermediate pellet costs were higher.

The NL model utilizes more complete information regarding the specification of the final and intermediate products than the two-stage model resulting in the potential for significant cost savings. Future research should be directed at expanding the capability of the NL model to find timely solutions when faced with a larger set of intermediate (i.e., pellets) and final products.

Appendix

Nonlinear Feed Blending Model

Notation:

Constants:

$C =$ minimum cost of producing all products (endogenous).

Subscripts:

$i =$ final products;
$j =$ ingredients used in making pellets;
$k =$ nutrients required for final products and/or pellets;
$n =$ pellets to consider making;
$m =$ ingredients used in making final products.

Parameters:

$w_j =$ unit cost of raw ingredient j (input data);
$v_m =$ unit cost of raw ingredient m (input data);
$a_{kj} =$ amount of nutrient k in raw ingredient j (input data);
$e_{km} =$ amount of nutrient k in raw ingredient m (input data);
$l^n_{kn} =$ minimum amount of nutrient k required in pellet n (input data);
$u^n_{kn} =$ maximum amount of nutrient k required in pellet n (input data);
$l^i_{ki} =$ minimum amount of nutrient k required in final product i (input data);
$u^i_{ki} =$ maximum amount of nutrient k required in final product i (input data);
$l^Z_{mi} =$ minimum amount of raw ingredient m required in final product i (input data);
$u^Z_{mi} =$ maximum amount of raw ingredient m required in final product i (input data);
$l^X_{jn} =$ minimum amount of raw ingredient j required in pellet n (input data);

$u_{jn}^{X} =$ maximum amount of raw ingredient j required in pellet n (input data);
$d_i =$ demand for (or scheduled production of) final product i (input data);
$s_j^X =$ supply of ingredient j available;
$s_m^Z =$ supply of ingredient m available.

Variables:
$X_{jn} =$ physical amount of ingredient j to use in pellet n (endogenous);
$P_{ni} =$ fraction of the nth pellet to use in final product i (endogenous);
$Z_{mi} =$ physical amount of ingredient m to use in final product i (endogenous);
$B_{kn} =$ physical amount of nutrient k in pellet n (endogenous).

Model:

The objective is to minimize cost

(A1) $$\min C = \sum_j w_j \left(\sum_n X_{jn}\right) + \sum_m v_m \left(\sum_i Z_{mi}\right) - 3$$

subject to:
Minimum and maximum nutritional requirements of the pellets

(A2) $$\sum_j a_{kj} X_{jn} \geq l_{kn}^{n} \left(\sum_j X_{jn}\right) \quad \text{and} \quad \sum_j a_{kj} X_{jn} \leq u_{kn}^{n} \left(\sum_j X_{jn}\right) \forall k, n.$$

Equations to transfer the nutritional content of the pellets for use in final products

(A3) $$\sum_j a_{kj} X_{jn} = B_{kn} \quad \forall k, n.$$

Minimum and maximum nutritional requirements on the final products (these are nonlinear)

(A4) $$\sum_n B_{kn} P_{ni} + \sum_m e_{km} Z_{mi} \geq l_{ki}^{i} \sum_n P_{ni} \left(\sum_j X_{jn}\right) \quad \text{and}$$

$$\sum_n B_{kn} P_{ni} + \sum_m e_{km} Z_{mi} \leq u_{ki}^{i} \sum_n P_{ni} \left(\sum_j X_{jn}\right) \forall k, i.$$

Pellet balance equations

(A5) $$\sum_j X_{jn} \geq \sum_i P_{ni} \left(\sum_j X_{jn}\right) \forall n.$$

Final product balance equations

(A6) $$\sum_n P_{ni} \left(\sum_j X_{jn}\right) + \sum_m Z_{mi} \geq d_i \ \forall i.$$

Maximum and minimum amounts of raw ingredients that can be used in making pellets

$$\text{(A7)} \qquad X_{jn} \leq u_{mi}^{X}\left(\sum_{j} X_{jn}\right) \forall j, n \quad \text{and} \quad X_{jn} \geq l_{mi}^{X}\left(\sum_{j} X_{jn}\right) \forall j, n.$$

Maximum and minimum amounts of ingredients that can be used in final products

$$\text{(A8)} \quad Z_{mi} \leq u_{mi}^{Z}\left[\sum_{n} P_{ni}\left(\sum_{j} X_{jn}\right)\right] \forall m, i \text{ and } Z_{mi} \geq l_{mi}^{Z}\left[\sum_{n} P_{ni}\left(\sum_{j} X_{jn}\right)\right] \forall m, i.$$

Ingredient supply (availability) constraints

$$\text{(A9)} \qquad \sum_{n} X_{jn} \leq s_{j}^{X} \quad \text{and} \quad \sum_{i} Z_{mi} \leq s_{m}^{Z}.$$

Variable Bounds (to aid in finding a solution faster)

$$\text{(A10)} \qquad 0 \leq P_{ni} \leq 1 \; \forall n, i.$$

Nonnegativity

$$\text{(A11)} \qquad X_{jn}, P_{ni}, Z_{mi}, B_{kn} \geq 0 \quad \forall j, n, i, m, \text{ and } k.$$

Endnotes

[1]In addition, the typical feed mill likely produces feed for noncommercial animals such as dogs, cats, chicks, ducks, etc.

[2]Management in this context typically includes a procurement specialist and/or a nutritionist.

[3]See, for example, those raw ingredients listed in tables 1 or 3. Raw ingredients listed in tables 2 or 4 are not pelletized.

[4]Nutritional constraints (maximums and minimums) modeled include those relating to crude protein, fat, fiber, undigestable protein (UDP), net energy lactation (NEL), non-fiber carbohydrate (NFC), phosphorus, calcium, potassium, selenium, magnesium, manganese, iodine, iron, zinc, copper, cobalt, salt, vitamins A, D, and E. Additionally, feed texture and product palatability constraints were also modeled. These nutritional requirements represent a comprehensive set of constraints used by typical feedmills.

[5]Prices are as of December 2003.

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