Design for renewable energy systems with application to rural areas in Japan

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Abstract

This study uses optimization modeling to study efficient ways to integrate renewable energy systems to provide electricity and heat in rural Japan. The model provides minimum cost system configuration and operation taking into account hour-by-hour energy availability and demand. Grid electricity is available to rural areas of Japan, but it is relatively expensive. Local renewable energy generation can be economic while using grid electricity to compensate for the intermittency of the renewable generation. In the model, renewable electricity can be provided by a combination of wind, photovoltaic, and biomass. Heat can be provided by petroleum, LPG, and geothermal heat pumps (GHPs). We find that due to the relatively high cost of grid electricity, there is significant penetration of wind generation. In turn, the penetration of wind creates economic conditions that encourage GHP penetration. The integrated renewable system reduces the annual cost of the entire system by 31\%, and reduces the carbon emissions by 50\%.

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1. Introduction

Since the 1990s, global environment concerns have been increasing. In particular, world attention has been focused on global warming caused by greenhouse gases such as CO\textsubscript{2}. In 1997, the third session of the Conference of the Parties (COP3) was held in Kyoto. The Kyoto protocol, requiring reduction of greenhouse gases, was adopted. Because CO\textsubscript{2} emissions from renewable energy sources are lower than the emissions from fossil energy systems, renewable energy is expected to play a major role in the 21st century.

Additionally, Japan’s energy supply depends heavily on imported fossil fuels. In 2001, 99\% of primary energy fossil fuels were imported (The Energy Data and Modeling Center, 2003). With its attendant risks to energy security, it is necessary to reduce reliance on fossil fuels to achieve a stable energy supply (Uchiyama, 2002). As an alternative to fossil fuels, renewable energy has high sustainability and has the potential to provide a large share of energy supplies in the future (Agency for Natural Resources and Energy, 2001a, b). In order to promote installation of renewable energy systems in Japan, a green certificate market system has been enforced, and a renewable portfolio standard is being implemented (Espey, 2001; Fuchs and Arentsen, 2002; Jensen and Skytte, 2002). No renewable energy is unconstrained, since wind and hydro potentials are limited, biomass and geothermal fields do not exist in every place, and even solar energy is available only during specific hours of sunny days. However, on the scale required by rural energy systems, renewable energy, such as wind power, is attractive as an alternative power source.

When considering the penetration of renewable energy systems, we recognize that most renewable energy sources are not dense, requiring large land areas to achieve substantial levels of power (Agency for Natural Resources and Energy, 2001a, b). This is an obstacle to serving urban areas using renewable energy, particularly wind, since large installations would be required at some distance from the urban area and large new transmissions systems would be required. Photovoltaic (PV), if it is competitive, could be installed on...
rooftops in urban areas. Given the land requirements for renewable technologies, it may be useful to use renewable energy in rural towns and villages since sufficient land area may be nearby for installing generation capacity without requiring new, long distance transmission. This paper evaluates the penetration of renewable energy sources in a rural area of Japan as a complement to the conventional energy system.

Designing and evaluating a renewable system requires that we address the fact that renewable power from wind and PV are intermittent, and the annual energy available from other renewable sources such as biomass may be limited. Within Japan, rural areas are connected to the main electric grid. Although the grid is a valuable resource, it is relatively expensive on a cost per kWh basis. An efficient strategy for serving rural areas can use local renewable resources, complemented by the use of the grid electricity to mitigate the intermittency of renewable resources.

Generally, effective system designs will combine several technologies. Here we have developed a model which can make use of wind, PV, biomass, and the grid for electricity. The system also uses heat from petroleum, LPG, and geothermal heat pumps (GHPs). Through optimization, we identify the set of technologies that work effectively together, and find the most efficient set of capacities. We find here that there is a useful synergy between these technologies in that the penetration of wind generation improves the economic performance of GHP and encourages its penetration. This paper examines the structure and economic performance of an optimal renewable energy system and explores the economic interactions between renewable electric resources and the heating supply system.

2. Renewable energy systems in rural areas

Many studies have been conducted on the economics and design of renewable energy sources (Hutter, 1996; Lund and Freeston, 2001). For example, Santisirisomboon et al. (2001) evaluated the possibility of installation of biomass energy, and Kara and Yuksel (2001) evaluated the availability of GHPs. However, because renewable energy sources are intermittent, it would be difficult to provide a stable energy supply using only one renewable energy source. Studies of an energy system in which two or more sources of renewable energy are combined have been conducted recently (Berry and Lamont, 2001; Berry et al., 2001; Elhadidy and Shaahid, 1999; Elhadidy, 2002; Hamada et al., 2001; Lamont, 2001a; Riesch, 1997). For example, Bassam (2001) has evaluated the energy system for Northern Germany; Rozakis et al. (1997) and Kaldellis and Kavadias (2001) have evaluated energy systems in Greece. Renewable energy sources may also provide a stable energy supply in combination with appropriate energy storage systems (Lamont, 2001b; Berry and Lamont, 2001).

In Japan, no study has been conducted on a system that combines several sources of renewable energy. The system that we consider here is structured around the renewable and conventional resources and the grid electric prices prevailing in a specific village in rural Japan. The sections below describe the village and the resources available to it.

3. Rural energy system model

3.1. Target area

We have analyzed the village of Kuzumaki in Iwate prefecture in northern Japan. The city administration requested that Tohoku University assess the potential for introducing renewable energy in the village, partly in response to the high cost of grid electricity.

Kuzumaki’s current population is 8870. The major industry is agriculture, primarily dairy. The annual electricity demand is 38,079 MWh, with a peak demand of 6.69 MW. The annual mean temperature is 8.8°C. The total annual demand for heating is estimated to be 40,579 MWh of heat energy. Over the year, the daily mean global solar radiation of this village is 3.32 kWh/m². For comparison that of Tokyo is 3.92 kWh/m².

This village lies mostly at over 400 m elevation, and is surrounded by mountains as much as 1000 m high. There are many good locations for wind power in and around the village where the mean wind speed is greater than 5 m/s. For example, wind power has already been developed at a nearby plateau where the mean wind speed is 7.9 m/s at a height of 36 m.

Manure from livestock appears to be a significant potential resource. Approximately 515 ton/day are available. Using fermentation, this can be turned into methane as another energy resource. The amount of methane reaches 15,500 Nm³/day.

3.2. Components and structure of the model

Four kinds of renewable sources were considered in the model, including: PV, wind power, biomass generation and GHP. This study optimizes the design of an energy system for target area by selecting technologies and their capacities, and determines the most economic operation. The network of the system is shown in Fig. 1.

3.3. METANet modeling approach

We have used the METANet economic modeling system developed at the Lawrence Livermore National
Laboratory. METANet allows a user to build and solve complex economic models as a network of processes such as end-uses (price-sensitive demands), markets (which allocate market shares based on relative prices), conversion processes (which compute inputs required to meet output requirements based on efficiencies, and compute prices based on capital and operating costs), and resources (which can be exhaustible, or can follow a set price track). Previous versions of METANet were developed to evaluate the long-term development of a system over a long time frames of several decades (Lamont, 1994; Nakata, 2000; Nakata and Lamont, 2001). It has recently been extended to optimize the design and operation of energy systems which include intermittent resources. In this formulation, it models a system over a single year on an hour-by-hour basis. The inputs to the model include the hourly demands for energy, and availability of intermittent resources, as well as the capital and operating costs of technologies. The model finds the least cost design and operation of the system. Appendix A describes the specific optimization problem solved by the system and its algorithm.

As is discussed in Appendix A, accounting for the system marginal costs for heat and electricity each hour, and the marginal costs of production for each of the technologies each hour is key to optimizing both the system configuration and operation. In this model, we find that the operating cost of the GHPs is dependent on the marginal cost of electricity each hour, which is in turn dependent on the installed capacity of wind generation. The underlying optimization in METANet allows us to take this into account in developing the optimal design for the overall electricity and heat supply systems.

3.4. Economic parameters of the model

Table 1 shows the economic parameters for this analysis. The capital costs are typical turnkey values in Japan. The interest rate is 5%. For this analysis, we have used estimates of Japanese typical costs based on US DOE data. In general, capital costs in Japan are around twice the cost in other countries. For example, the capital cost of a gas turbine combined cycle is around $500/kW in most countries, while in Japan the cost is around $1000/kW.

The electricity demand data were developed based on Tohoku-Electric Power Company’s gross demand records in 2000. We created the following six demand patterns: weekday pattern and holiday pattern in spring and fall season (April, May, September, October, November), summer days (June, July, August), and winter days (December, January, February). We estimated the total electricity demand based on the population of the village, and applied the demand patterns to determine the hourly demands.

The heat demand data are based on the research on residential environment (Shoda et al., 1973). We have created the hourly demand pattern of heating and hot-water supply similar to the electric demand.

Solar radiation and wind condition time-series data were obtained from the local government and electric utility (Kuzumaki Municipal Government, 2001; Eco-Power, 2001).

Fig. 1. Network for the renewable energy system in rural area.
4. Result of the analysis

4.1. System configuration and operation

Without renewable resources the village relies on grid electricity and petroleum heating. With the grid electricity, GHP is too expensive to penetrate the market to any significant degree. Similarly, LPG heating is considerably more expensive than petroleum heat and is rarely used.

When renewable resources are made available, the optimal system configuration changes significantly, as is shown in Table 2. Substantial wind and GHP capacity are introduced. In the renewable system 71% of the electricity comes from wind. Fig. 2 shows the breakdown of energy resources used in the optimal renewable system.

For the grid as a whole, large capacities of intermittent resources such as wind can create problems of stability and require larger amounts of spinning reserve. In Denmark, the share of wind generation is about 15% (Lund and Münster, 2003). Other studies have suggested that spinning reserve should not be a significant problem for wind penetrations below about 20% (Watson et al., 1994). Here the wind penetration is substantially higher which raises questions about possible stability problems. It should be noted that although the wind penetration is a large fraction of the power for the village, it is not a large fraction of the total grid. The more significant problem might be local stability conditions. This should be explored in further investigations.

Table 1
Input data

<table>
<thead>
<tr>
<th>Type</th>
<th>Life (years)</th>
<th>Specific capital cost ($/kW)</th>
<th>Fixed operating cost ($/kW/yr)</th>
<th>Variable operating cost (cents/kWh)</th>
<th>Efficiency (%)</th>
<th>Fuel cost (cents/kWh)</th>
<th>Emission factor of CO₂ (kg C/kWh)</th>
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</thead>
<tbody>
<tr>
<td>Electricity</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>20</td>
<td>5000a</td>
<td>12.86a</td>
<td>0a</td>
<td>100</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>20</td>
<td>2000a</td>
<td>50.02a</td>
<td>0a</td>
<td>100</td>
<td>0</td>
<td>0</td>
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<td>Biomass</td>
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<td>47,486e</td>
<td>589.43e</td>
<td>1.63e</td>
<td>25d</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Grid</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>18</td>
<td>0.101182e</td>
</tr>
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<td>Heat</td>
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<td>0b</td>
<td>0b</td>
<td>85b</td>
<td>4</td>
<td>0.066135f</td>
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<tr>
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<td>92b</td>
<td>0b</td>
<td>0b</td>
<td>80b</td>
<td>12</td>
<td>0.059017f</td>
</tr>
</tbody>
</table>

SCC, specific capital cost; AOC, ancillary operating cost (all non-fuel variable operating costs). Discount rate, 5%.

bPower generation due to methane fermentation.
cNakagawa (2003).
dKuzumaki Town (2001).
fTakasugi et al. (2000).
gThis is the electricity requirement.

Table 2
System capacity configurations

<table>
<thead>
<tr>
<th>Component</th>
<th>Non-renewable (MW)</th>
<th>Renewable (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric system</td>
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<td></td>
</tr>
<tr>
<td>Peak grid electricitya</td>
<td>6.69</td>
<td>6.67</td>
</tr>
<tr>
<td>Wind</td>
<td>0.00</td>
<td>9.64</td>
</tr>
<tr>
<td>PV</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Biomass</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>Heat system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petroleum</td>
<td>17.87</td>
<td>16.69</td>
</tr>
<tr>
<td>GHP</td>
<td>0.00</td>
<td>1.69</td>
</tr>
<tr>
<td>LPG</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

aPeak power used.

4.2. Electricity supply

Although the maximum electric load is 6.69 MW, the total peak wind capacity installed is 9.64 MW. The higher level of wind capacity means that the wind generation can meet more of the load even during periods with lower wind speeds.

Fig. 3 illustrates the dispatch of electricity generation for the renewable system during early August. In this period, the peak electric load of the village is around 6 MW. During windy periods wind provides most, or all, of the total electricity supplied to the village. When wind electricity cannot provide enough power to meet
demand, grid electricity makes up the deficiency. We note that the wind generation exceeds the electricity demand since some additional generation is needed to power the GHPs.

The hourly variation in the price of electricity is shown in Fig. 4. The electric market price is the marginal cost of production in each hour. During windy periods, wind is the marginal electric resource and the marginal price of electricity drops to the marginal cost of wind generation. When there is insufficient wind to meet demand, the grid electricity becomes the marginal supplier and the price rises to $180/MWh.

Some of the key results in this study are driven by the high marginal cost of grid electricity. In general, Japan does have high electricity prices, particularly in rural areas. Special rate structures, including time of day pricing, can be negotiated for large users, however, the typical customer in this village would not qualify.

4.3. Heat supply and economic interaction of wind and GHP

The dispatch of the heating technologies is illustrated in Fig. 5 for early August (the same period as in Figs. 3 and 4). Essentially, all of the heat is provided by GHP, and petroleum. LPG heat is considerably more expensive and is not used. Fig. 6 shows the marginal costs of heat during the same period. Fig. 6 also shows the price of GHP. Because the GHP runs on electricity, its price fluctuates with the price of electricity.

In Fig. 5, we can see a significant penetration of GHP. This occurs in spite of the fact that the capital cost of GHP is more than 10 times the capital cost of petroleum heat. The penetration of GHP is determined by the economic value that GHP capacity can provide to the system by reducing overall system costs. The economic value of additional capacity is determined by the difference between the system marginal heat price and the marginal operating cost of GHP during those hours when GHP is dispatched to its full capacity. The figures showing system dispatch and marginal costs illustrate the economic interaction between the electricity market and the penetration of GHP. For example, in Fig. 4 the marginal cost of electricity between the hours 119 and 152 is essentially zero because wind is the marginal resource in those hours (see Fig. 3). During those same hours, the price of GHP is generally low, except for those few hours when it is dispatched to capacity (at around hour 132, and hours 139–145, see Fig. 6).

In those hours when the heat demand exceeds the GHP capacity, and the electric cost is low, GHP is

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Fig. 2. Annual production of electric energy under the optimal renewable system configuration.

Fig. 3. Electricity generation in early August.
dispatched to its capacity and petroleum becomes the marginal producer at a cost of $47/MWh. When GHP is dispatched to capacity, its price in the market increases to $47/MWh to match the system marginal market price. The high price is sufficient to limit the demand to GHP to be equal to its capacity, the remaining heat demand is sent to the petroleum heat.\footnote{The algorithm in the GHP node seeks a price such that the demand from the market node is equal to (or no greater than) its capacity. A price that is very close to, or slightly greater than, the system marginal price will cause the demand to the GHP node to drop to the level equal to GHP’s capacity.} The GHP is paid the system marginal price. Thus GHP “earns” the difference between its actual marginal cost (essentially 0) and the system marginal price. This can be regarded as net revenue to the GHP, which can be applied to covering its capital costs. At the same time, it is the marginal value that an additional increment of GHP capacity would provide to the system if it were available in those hours. That is, with another increment of GHP capacity the petroleum heat could be backed down by an increment, saving $47/MWh. The output of the GHP would be increased at a cost of essentially 0, saving the entire system $47/MWh.

In those hours when heat demand is less than GHP capacity, GHP is not dispatched to capacity and its price is just equal to its marginal cost. This sets the system

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\[\text{Electricity prices, $/MWh}\]

\[\text{Heat energy output, MWh}\]

---

\[\text{Grid price} \quad \text{Electric market price}\]

\[\text{Geothermal heat pump} \quad \text{Petroleum heat} \quad \text{LPG gas heat} \quad \text{Total heat demand}\]

---

Fig. 4. Electricity marginal costs in early August.

Fig. 5. Heat supply in early August.
marginal price. In those hours, the difference between the system marginal price and GHP’s marginal cost is 0. GHP earns nothing in those hours, consistent with the fact that additional GHP capacity would provide no value to the system.

At many hours of the year there is a potential savings from GHP capacity. The total of these potential savings over the entire year is the total value of adding an increment of capacity. If this total is greater than the incremental cost of adding capacity, then the total system cost will be reduced by adding an increment of capacity. The GHP capacity is optimal when the total potential value of an increment of GHP is just equal to the incremental cost of the capacity.

The penetration of GHP occurs in this case because there is enough wind capacity to make wind the marginal generator in many hours of the year. It is only in these hours that the value of GHP is high relative to its cost. If there were not enough wind capacity to put wind on the margin in some hours, there would be no penetration of GHP since grid electricity would always be on the margin. When grid electricity is on the margin, the cost of GHP heat is about the same as the cost of petroleum heat and it would not provide any potential system savings.

4.4. \( \text{CO}_2 \) emissions

\( \text{CO}_2 \) emission from renewable energy systems and the conventional energy system are shown in Fig. 7. The emission factors of fossil fuels are based on the report published by the Japanese government (Ministry of the environment of Japan, 2000). As for the grid electricity, we have made reference to the report of electric utilities (The Federation of Electric Power Companies of Japan, 2002). The emission factors of petroleum, gas and grid electricity are shown in Table 1. In the conventional energy system, the annual \( \text{CO}_2 \) emissions are 7023 ton C/yr, while the annual \( \text{CO}_2 \) emissions in renewable energy systems are 3494 ton C/yr. The reduction of emissions from generation of grid electricity is the largest component of the reduction since substantial grid electricity is replaced by wind. In addition, about a quarter of the emissions from petroleum heat are eliminated by the GHP. Evaluating the total change in the \( \text{CO}_2 \) emissions due to both construction and operation of facilities would require a life cycle analysis. But, since the emissions from renewable energy sources are considerably lower than the emissions from fossil fuel fired stations, the impact on this evaluation would be small.
4.5. *Comparison of system costs*

Fig. 8 compares the annual cost of a renewable energy system and that of a conventional energy system in the village. The costs include fuel, operating and capital expenses. The annual cost of a conventional energy system is US$8.9 million per year. While the annual cost of a renewable energy system in the village is US$6.1 million per year. Most of the reduction in costs comes from reducing purchases of grid electric. This is partly balanced by the capital cost of the wind capacity.

5. *Summary*

We have examined the least cost design of a local renewable energy system which considers wind electricity, PV, biomass generation, and GHP, while using grid electricity to compensate for the intermittency of the renewable generation. We find that the most economical system makes use of wind and GHP along with conventional petroleum heat and grid electricity. PV and biomass generations are not used due to their very high capital costs. Substantial wind penetration is economical due to the high cost of grid electricity ($180/MWh). This high penetration of wind energy results in a number of hours per year when wind is the marginal generator and, consequently, the marginal cost of electricity is very low in those hours. This low marginal cost of electricity interacts with the heat supply system. GHP becomes very inexpensive in many hours per year, particularly when compared with the alternative of petroleum heat. As a result, we see significant penetration of GHP.

The use of wind energy with GHP substantially reduces both the total energy cost and the carbon emissions from the village. The total annual cost is reduced by 31% and the carbon emissions are reduced by 50%.

**Acknowledgements**

We would like to thank Professor Hiroaki Niitsuma, Professor Hiroshi Yoshino, and Professor Yuichi Nibori of Tohoku University for giving us valuable advice.

Appendix A. Mathematical optimization in the METANet economic modeling system

**A.1. Introduction**

The METANet economic modeling system has been developed to simplify the analysis of commodity production and consumption systems. A system is modeled as a set of nodes representing resource production, conversion from one form to another (e.g. coal into electricity), demand processes, and markets. The links between nodes represent the physical movement of products. The links also pass information about prices. This form of model makes it easy to construct, modify, and analyze models.

This approach has been used in the past for modeling long-term development of the economic system (primarily for energy systems). The underlying mathematical formulation for that problem is documented in Hogan and Weyant (1982). In the long-term formulation capacities vary over the model horizon—capacity is added to production nodes in earlier periods and is retired away in later periods.

Recently, this approach has been modified at Lawrence Livermore National Laboratory to optimize a system with intermittent resources over a single year of operation. It finds the optimal configuration of capacities and system operation (dispatch of generation) over a 1-year period. The model input includes hour-by-hour availability of intermittent resources and demands for final output. This is useful not only for situations where
a system includes intermittent resources, but also for the case where the operating cost of one process depends on the operation of other processes. In this paper, the operating cost of the GHP depends on the marginal cost of electricity, which in turn depends on the availability and capacity of wind generation. In this case, the model is used to optimize the capacities and operation of both the wind generators and the heat pumps.

The modeling system optimizes the structure and operation of the system by optimizing the capacities of the components of the system and their dispatch from hour to hour. For each hour of the year the model inputs include demands to be met and the availability of the solar and wind energy. The model inputs also include the capital and operating costs of each technology. The remainder of this Appendix describes the formulation of the optimization problem solved by METANet.

Many readers will note that this is simply an extension of the screening curves approach to optimizing the set of capacities in a generation system needed to serve a given load duration curve.

### A.2. Nomenclature

There is a set of generation technologies each denoted by the subscript $g$. These are dispatched to meet hourly demands, $D_{mnd_h}$. Some of these technologies are intermittent so that in some hours their available capacity is only a fraction, $F_{g,h}$, of their nameplate (or peak) capacity. For a dispatchable generator, $F$ is 1.0 for each hour. Each generator will be dispatched to its available capacity for at least 1 h. For the formulation below, it is essential to keep track of these hours. We denote the set of hours that generator $g$ is dispatched to its available capacity as $H^*_g$. The full set of variables is as follows:

- $C_{cap_g}$: Annual capital cost of one unit of capacity for generator $g$ ($/yr$).
- $Cap_g$: The capacity of generator $g$, which is the peak output available from generator $g$ (kW).
- $Out_{g,h}$: Output of the $g$th generator in the $h$th period (kW).
- $Cop_{g,h}(Cap_g, Out_{g,h})$: Operating cost of generator $g$ in hour $h$. In general, this is a function of the output level and capacity ($/$).
- $Copm_g$: The marginal operating cost of generator $g$ for the case that the operating cost is essentially independent of the capacity and the output level. This the derivative of $Cop_{g,h}(Cap_g, Out_{g,h})$ w.r.t. $Out_{g,h}$.

### A.3. Analysis

We will minimize the total annual cost, which is the sum of the annualized capital costs for all the generators plus their operating costs over the year. However, the minimization is subject to the constraint that the total output each hour must equal the demand that hour, and each generator must be dispatched its available capacity, or less:

\[
C_{tot} = \sum_g Cap_g \times C_{cap_g} + \sum_h \sum_g Cop_{g,h}(Out_{g,h}, Cap_g) \tag{A.1}
\]

subject to

\[
\sum_g Out_{g,h} = D_{mnd_h} \text{ for all } h, \tag{A.2}
\]

\[
Out_{g,h} \leq F_{g,h} \times Cap_g \text{ for all } g \text{ and } h. \tag{A.3}
\]

The inequality can be removed from Eq. (A.3) by noting that a generator will only be dispatched to full capacity in a specific set of hours. Denote this set of hours as $H^*_g$ for generator $g$. Then Eq. (A.3) can be written as

\[
Out_{g,h} = F_{g,h} \times Cap_g \text{ for all } h \in H^*_g. \tag{A.3a}
\]

Based on these we form the Lagrangian as

\[
L = \sum_g Cap_g \times C_{cap_g} + \sum_h \sum_g Cop_{g,h}(Out_{g,h}, Cap_g) + \sum_h \tilde{\lambda}_h \left(D_{mnd_h} - \sum_g Out_{g,h}\right) + \sum_g \sum_{h \in H^*_g} \gamma_{g,h}(Out_{g,h} - F_{g,h} \times Cap_g). \tag{A.4}
\]

To determine the conditions to be met at the optimum, we differentiate with respect to the output of each generator for each hour, and with respect to the capacity
of each generator:

\[
\frac{\partial L}{\partial \text{Out}_{g,h}} = \begin{cases} 
\frac{\partial \text{Cap}_{g,h}(\text{Out}_{g,h}, \text{Cap}_{g})}{\partial \text{Out}_{g,h}} - \lambda_h & \text{if } h \notin H_g^*, \\
\frac{\partial \text{Cap}_{g,h}(\text{Out}_{g,h}, \text{Cap}_{g})}{\partial \text{Out}_{g,h}} - \lambda_h + \gamma_{g,h} & \text{otherwise,}
\end{cases}
\]  

(A.5)

\[
\frac{\partial L}{\partial \text{Cap}_{g}} = 0 = \text{Cap}_{g} + \sum_h \frac{\partial \text{Cap}_{g,h}(\text{Out}_{g,h}, \text{Cap}_{g})}{\partial \text{Cap}_{g}} - \sum_{g,h} (\gamma_{g,h} F_{g,h}).
\]  

(A.6)

The Lagrange multiplier \( \lambda_h \) is the shadow price on changes in demand in hour \( h \). This is the system marginal cost. The first case in Eqs. (A.5) and (A.5a) indicate that each technology should be dispatched to the point that its marginal cost is equal to the system marginal cost. If a generator’s marginal cost is greater than the system marginal cost, it is not dispatched.

The Lagrange multiplier \( \gamma_{g,h} \) is the multiplier on the constraint that a generator cannot be dispatched beyond its capacity. From the second case in Eqs. (A.5) or (A.5a), it is the difference between the system marginal price in hour \( h \) and the marginal operating cost of the generator \( g \). This is the net change in total system cost from increasing the output of generator \( g \) in hour \( h \) — that is, if generation from generator \( g \) is increased, the system cost is reduced by \( \lambda_h \) by backing down the marginal generator, but is increased by \( \text{Cop}_{g} \). The sum of \( \gamma_{g,h} F_{g,h} \) across all hours is the marginal value of an incremental increase in the capacity of generator \( g \), taking into account the fact that it cannot provide full rated capacity each hour due to intermittency. Conditions in Eqs. (A.6) and (A.6a) indicate that the marginal value of capacity for generator \( g \) should be equal to the marginal cost of capacity.

METANet solves these equations by exchanging price and quantity information between nodes each hour until the conditions in Eqs. (A.5) and (A.6) are satisfied. The algorithm is illustrated in Fig. 9. The demand nodes send down a quantity demanded. The market nodes allocate total demand among the generators based on prices provided by the generators (generators with lower prices receive higher allocations). When a generator’s allocation is less than its capacity, it sends a price equal to its operating cost. Such a low cost can elicit a demand that exceeds the capacity of the generator. In that case, over a series of iterations the generator increases its price until the price is approximately equal to the system marginal cost, \( \lambda_h \). Based on this higher price, the market node adjusts the allocation to the generator to the point where the allocation just equals its capacity. Based on this, the generator can make an accurate estimate of \( \lambda_h \). From this, it can estimate \( \gamma_{g,h} \) and, through a series of iterations, adjust its capacity until the condition in Eq. (A.6a) is met. This can be interpreted as a market in which each supplier to a market (i.e. each generator)

Fig. 9. METANet solution algorithm.
receives as payment the marginal cost in the market. It then can make the financial calculation as to whether or not additional increments of capacity would earn an acceptable rate of return.

References


