Dynamic Positive Equilibrium Problem

Abstract

The Dynamic Positive Equilibrium Problem (DPEP) is a methodology for dealing with time series about economic agents' decisions, regardless of the amount of available information. The approach is articulated in three phases, as in the static counterpart Symmetric Positive Equilibrium Problem (SPEP), with the variant that it must be preceded by the estimation of the equation of motion which characterizes a dynamic model. Furthermore, the definition of marginal cost in the DPEP model is different from the same notion in the static SPEP. In this paper, the DPEP approach was applied to a panel data dealing with annual crops from California agriculture for a horizon of eight years. The dynamic character of the DPEP model is based upon then assumption of output price adaptive expectations that follows a Nerlove-type specification.

Introduction

The methodology of Symmetric Positive Equilibrium Problem (SPEP) presented by Paris and Howitt (2000) is extended in this paper to include a dynamic structure. Dynamic models of economic problems can take on different specifications in relation to different sources of dynamic information. When dealing with farms whose principal output is derived from fruit orchards, for example, the equation of motion is naturally represented by the difference between standing orchard acreage in two successive years plus new planting and minus culling. In more general terms, the investment model provides a natural representation of stocks and flows via the familiar investment equation

(1) $$K_t = K_{t-1} + I_t - \delta K_{t-1}$$

where K_t represents the capital stock at time t, I_t is the investment flow at time t, and δ is the depreciation rate. This dynamic framework, expressed by a relevant equation of motion, becomes operational only when explicit information about investment, initial stock, and depreciation is available. Unfortunately, information about new plantings and culling rarely exists.

Annual crops are also dynamically connected through decisions that involve price expectations and some inertia of the decision making process. We observe that farmers who produce field crops, for example, will produce these activities year after year with an appropriate adjustment of both acreage and yields. In this paper, therefore, we consider economic units (farms, regions, sectors) that produce annual crops. That is, production activities that, in principle, may have neither technological antecedents nor market consequences but that nevertheless are observed to be connected through time. We assume that the underlying dynamic connection is guided by a process of output price expectations.

In a static framework, the SPEP specification takes on the following structure:

$$\mathbf{A}\mathbf{x} + \boldsymbol{\beta} \leq \mathbf{b}, \qquad \mathbf{y} \geq \mathbf{0} \tag{2}$$

$$\mathbf{x} \leq \mathbf{x}_R, \qquad \boldsymbol{\lambda} \geq \mathbf{0} \tag{3}$$

$$\mathbf{A}'\mathbf{y} + \boldsymbol{\lambda} \geq \mathbf{p}, \qquad \mathbf{x} \geq \mathbf{0} \tag{4}$$

$$\mathbf{y} \geq \mathbf{r}, \qquad \boldsymbol{\beta} \geq \mathbf{0} \tag{5}$$

and the associated complementary slackness conditions

(6) $$\mathbf{y}'(\mathbf{b} - \mathbf{A}\mathbf{x} - \boldsymbol{\beta}) = \mathbf{0}$$

(7) $$\boldsymbol{\lambda}'(\mathbf{x}_R - \mathbf{x}) = \mathbf{0}$$

(8) $$\mathbf{x}'(\mathbf{A}'\mathbf{y} + \boldsymbol{\lambda} - \mathbf{p}) = \mathbf{0}$$

(9) $$\boldsymbol{\beta}'(\mathbf{y} - \mathbf{r}) = \mathbf{0}$$

where $\mathbf{b}$ is the vector of available resources, $\mathbf{x}_R$ is the vector of realized output levels, $\mathbf{p}$ is the vector of market output prices, $\mathbf{r}$ is the vector of market prices of resources, $\mathbf{A}$ is the matrix of fixed technical coefficients. The vectors $\mathbf{x}$ and $\boldsymbol{\beta}$ are measured in output and input units, respectively, while the vectors $\mathbf{y}$ and $\boldsymbol{\lambda}$ are measured in monetary terms.

This specification does not imply but neither excludes an explicit optimization assumption about economic behavior. The interpretation of constraints (2) through (5) is as follows: $\mathbf{A}\mathbf{x} \leq \mathbf{b} - \boldsymbol{\beta}$ states the physical quantity equilibrium condition on inputs according to which the demand of limiting resources must be less-than-or-equal to the effective supply of those resources. The quantity $\mathbf{b} - \boldsymbol{\beta}$ is interpreted as the effective supply because, while $\mathbf{b}$ is a vector of fixed resource availability, the vector $\boldsymbol{\beta}$ acts as a buffer parameters between the actual demand and the fixed input availability. This implies that a positive shadow price of the limiting inputs may result even though the demand of limiting resources is strictly less than its available nominal supply, that is $\mathbf{A}\mathbf{x} < \mathbf{b}$. The vector $\boldsymbol{\beta}$ is also the dual variable of constraint (5). When $\boldsymbol{\beta} > \mathbf{0}$, the dual variable $\mathbf{y}$ of limiting resources is equal to the input market price, $\mathbf{r}$. Constraint (4) states the economic equilibrium condition according to which the marginal cost of producing output, $(\mathbf{A}'\mathbf{y} + \boldsymbol{\lambda})$, must be greater-than-or-equal to marginal revenue, $\mathbf{p}$.

The Dynamic Framework

The specification of a dynamic framework based upon the structure of problem (2)-(5) begins with the assumption that the output price expectations of the decision maker are governed by an adaptive process such as:

(10) $$\mathbf{p}_t^* - \mathbf{p}_{t-1}^* = \boldsymbol{\Gamma}(\mathbf{p}_{t-1} - \mathbf{p}_{t-1}^*)$$

where the starred vectors are interpreted as expected output prices and $\boldsymbol{\Gamma}$ is a diagonal matrix of unrestricted elements. In general, the elements of the $\boldsymbol{\Gamma}$ matrix are required to be positive and less than 1 in order to guarantee stability of the difference equation in an infinite series of time periods. The case discussed in this paper, however considers a finite horizon of only a few years and no stability issue is at stake. It is as if we were to model an arbitrarily small time interval of an infinite horizon. Within such a small time interval, the relation expressed by equation (10) can be either convergent or explosive without negating the stability of the infinite process. A further assumption is that the expected output supply function is specified as follows:

(11) $$\mathbf{x}_t = \mathbf{B}\mathbf{p}_t^* + \mathbf{w}_t$$

where $\mathbf{B}$ is a positive diagonal matrix and $\mathbf{w}_t$ is a vector of intercepts. Then, equation (10) can be rearranged as

$$(12) \qquad \boldsymbol{\Gamma}\mathbf{p}_{t-1} = \mathbf{p}_t^* - [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{p}_{t-1}^*$$

while, by lagging one period the supply function, multiplying it by the matrix $[\mathbf{I} - \boldsymbol{\Gamma}]$, and subtracting the result from equation (11), we obtain

$$(13) \qquad \begin{aligned} \mathbf{x}_t - [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{x}_{t-1} &= \mathbf{B}\{\mathbf{p}_t^* - [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{p}_{t-1}^*\} + \mathbf{w}_t - [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{w}_{t-1} \\ &= \mathbf{B}\boldsymbol{\Gamma}\mathbf{p}_{t-1} + \mathbf{v}_t \end{aligned}$$

where $\mathbf{v}_t \equiv \mathbf{w}_t - [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{w}_{t-1}$. Hence, the equation of motion involving annual crops and resulting from the assumption of adaptive expectations for output prices is

$$(14) \qquad \mathbf{x}_t = [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{x}_{t-1} + \mathbf{B}\boldsymbol{\Gamma}\mathbf{p}_{t-1} + \mathbf{v}_t.$$

It is important to emphasize that this equation of motion is different from the more traditional dynamic relation where the state variable is usually interpreted as a stock and the control is under the jurisdiction of the decision maker. The equation of motion (14) emerges from an assumption of adaptive expectations about output prices. Prices are not under the control of the decision maker and, furthermore, the state variable is not a stock but a flow variable as it represents yearly output levels. Nevertheless, relation (14) is a legitimate equation of motion that relates entrepreneur's decisions from year to year.

Maximum Entropy Estimation of the Equation of Motion

Before proceeding further in the development of the Dynamic Positive Equilibrium Problem, it is necessary to produce an estimate of the matrices $\mathbf{B}$ and $\boldsymbol{\Gamma}$ that define the equation of motion. We assume that the available information on realized output quantities, $\mathbf{x}_{Rt}$, and output prices, $\mathbf{p}_t$, spans the horizon of T periods. The maximum entropy approach employed in this paper to estimate the equation of motion is a variant of the GME approach proposed by Golan et al. (1996). This special case, introduced by van Akkeren and Judge (1999), specifies the support intervals and the number of discrete points within the sample using only the available sample information. When feasible, this variant of the GME approach eliminates the researcher's subjective selection of the discrete number of probabilities and the support's end points. It removes, therefore, the often contentious aspect associated with the original GME formulation. In particular, let us decompose the equation of motion in two parts: the average relation and an associated equation defined in deviations from the average relation:

(15) $$\bar{\mathbf{x}}_R = [\mathbf{I} - \boldsymbol{\Gamma}]\bar{\mathbf{x}}_{R,-1} + \mathbf{B}\boldsymbol{\Gamma}\bar{\mathbf{p}}_{-1} + \bar{\mathbf{v}}$$

(16) $$(\mathbf{x}_{Rt} - \bar{\mathbf{x}}_R) = [\mathbf{I} - \boldsymbol{\Gamma}](\mathbf{x}_{R,t-1} - \bar{\mathbf{x}}_{R,-1}) + \mathbf{B}\boldsymbol{\Gamma}(\mathbf{p}_{t-1} - \bar{\mathbf{p}}_{-1}) + (\mathbf{v}_t - \bar{\mathbf{v}})$$

where the overhead bar indicates the sample average with respect to the index t. For notational convenience we also define $\mathbf{dx}_t \equiv (\mathbf{x}_{Rt} - \bar{\mathbf{x}}_R), \mathbf{dx}_{t-1} \equiv (\mathbf{x}_{R,t-1} - \bar{\mathbf{x}}_{R,-1}),$ $\mathbf{dp}_{t-1} \equiv (\mathbf{p}_{t-1} - \bar{\mathbf{p}}_{-1}), \mathbf{dv}_t \equiv (\mathbf{v}_t - \bar{\mathbf{v}}).$

Following the GME approach, each of the parameters to be estimated is expressed as the convex linear combination of the elements of a support matrix, say $\mathbf{Z}$, where the number of support points is taken to be equal to the number of sample observations. In our case, the parameters to be estimated are the diagonal matrices $\mathbf{\Gamma}$ and $\mathbf{B}$ and the vector $\mathbf{v}_t$. Hence,

$$\begin{aligned}
&\Gamma_{j,j} = \sum_{s=1}^{T} P_{\Gamma,j,j,s} Z_{\Gamma,j,j,s} \\
(17) \qquad &B_{j,j} = \sum_{s=1}^{T} P_{B,j,j,s} Z_{D,j,j,s} \\
&v_t = \sum_{s=1}^{T} P_{v,t,s} Z_{v,t,s}
\end{aligned}$$

where $P_{F,j,j,s}, P_{B,j,j,s}$ and $P_{v,t,s}$ are nonnegative weights for each of the estimated parameters that must add up to unity. In the context of maximum entropy, these weights are regarded as probabilities.

The variant of the GME methodology consists in using the sample observations to define the support matrices $\mathbf{Z}$ and the number of discrete supports as follows:

$$\begin{aligned}
&Z_{B,j,j,s} = dx_{j,s} dx_{j,s-1} \\
(18) \qquad &Z_{\Gamma,j,j,s} = dx_{j,s} dp_{j,s-1} \\
&Z_{v,j,s} = dx_{j,s}
\end{aligned}$$

The relevant ME specification for estimating the equation of motion can, therefore, be stated as finding positive probabilities $\mathbf{P}_B(j,j,s), \mathbf{P}_\Gamma(j,j,s), \mathbf{P}_v(j,s)$ that

(19) $$\max H(\mathbf{P}_B,\mathbf{P}_\Gamma,\mathbf{P}_v) = -\sum_{j,s}\mathbf{P}_B(j,j,s)\log(\mathbf{P}_B(j,j,s)) - \sum_{j.s}\mathbf{P}_\Gamma(j,j,s)\log(\mathbf{P}_\Gamma(j,j,s))$$
$$-\sum_{j,s}\mathbf{P}_v(j,s)\log(\mathbf{P}_v(j,s))$$

subject to

(20) $$\bar{\mathbf{x}}_R = [\mathbf{I}-\mathbf{\Gamma}]\bar{\mathbf{x}}_{R,-1} + \mathbf{B\Gamma}\bar{\mathbf{p}}_{-1} + \bar{\mathbf{v}}$$

(21) $$(\mathbf{x}_{Rt}-\bar{\mathbf{x}}_R)=[\mathbf{I}-\mathbf{\Gamma}](\mathbf{x}_{R,t-1}-\bar{\mathbf{x}}_{R,-1})+\mathbf{B\Gamma}(\mathbf{p}_{t-1}-\bar{\mathbf{p}}_{-1})+(\mathbf{v}_t-\bar{\mathbf{v}})$$

where the $\mathbf{\Gamma}$ and $\mathbf{B}$ matrices are replaced by their corresponding expressions $\mathbf{\Gamma} = \mathbf{P}_\Gamma\mathbf{Z}_\Gamma$ and $\mathbf{B} = \mathbf{P}_B\mathbf{Z}_B$ together with the adding-up conditions on the probabilities. Similarly, the intercept terms $\mathbf{v}_t$ are replaced by their corresponding expressions $\mathbf{v}_t=\mathbf{P}_{\mathbf{v}t}\mathbf{Z}_{\mathbf{v}t}$. The estimated equation of motion calibrates the sample observations exactly.

Phase 1 of DPEP: Estimation of the Marginal Costs

Phase 1 of the Dynamic Positive Equilibrium Problem begins with a specification of the optimization problem for the entire horizon from $t = 1,...,T$ and the statement of a salvage function. We assume that the economic agent wishes to maximize the discounted stream of profit (or net revenue) over the horizon T. After T periods it is assumed that the objective function consists of the discounted value of profit from period T to infinity, which is realized under a condition of steady state. Analytically, then, the Dynamic Positive Equilibrium Problem takes on the following specification:

(22) $$\max V = \sum_{t=1}^{T} \{\mathbf{p}_t'\mathbf{x}_t - \mathbf{r}_t'(\mathbf{b}_t - \boldsymbol{\beta}_t)\} / (1+\rho)^{(t-1)} + \int_T^{\infty} \{\mathbf{p}'\mathbf{x} - \mathbf{r}'(\mathbf{A}\mathbf{x})\} e^{-\rho\tau} d\tau$$

subject to

(23) $$\mathbf{A}_t\mathbf{x}_t + \boldsymbol{\beta}_t \le \mathbf{b}_t \qquad t = 1,...,T$$

(24) $$\mathbf{x}_t = [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\mathbf{x}_{t-1} + \hat{\mathbf{B}}\hat{\boldsymbol{\Gamma}}\mathbf{p}_{t-1} + \mathbf{v}_t \qquad t = 1,...,T.$$

Constraint (23) expresses the technological requirements for producing the vector of crop activities $\mathbf{x}_t$ given the limiting resource availability $\mathbf{b}_t$. Constraint (24) expresses the price expectations of the economic agent through an equation of motion that renders the objective of producing annual crops a real dynamic problem. The objective function is in two parts. The first component expresses the discounted profit over the horizon *T*. The second component is the salvage function where the absence of any time subscript indicates the steady state stream of profit. The salvage function can be stated more conveniently as $\int_T^{\infty} \{\mathbf{p}'\mathbf{x} - \mathbf{r}'(\mathbf{A}\mathbf{x})\} e^{-\rho\tau} d\tau = \{\mathbf{p}'\mathbf{x} - \mathbf{r}'(\mathbf{A}\mathbf{x})\}\rho e^{-\rho T}$ or, in order to relate it to the discrete time horizon T, $\{\mathbf{p}'\mathbf{x} - \mathbf{r}'(\mathbf{A}\mathbf{x})\}\rho e^{-\rho T} \equiv \{\mathbf{p}_{T+1}'\mathbf{x}_{T+1} - \mathbf{r}_{T+1}'(\mathbf{A}_{T+1}\mathbf{x}_{T+1})\}\rho e^{-\rho T}$. With these stipulations, the Lagrangean function of problem (22)-(24) is stated as

(25) $$L = \sum_{t=1}^{T} \{\mathbf{p}_t'\mathbf{x}_t - \mathbf{r}_t'(\mathbf{b}_t - \boldsymbol{\beta}_t)\} / (1+\rho)^{(t-1)} + \{\mathbf{p}_{T+1}'\mathbf{x}_{T+1} - \mathbf{r}_{T+1}'(\mathbf{A}_{T+1}\mathbf{x}_{T+1})\}\rho e^{-\rho T}$$
$$+ \sum_{t=1}^{T} (\mathbf{b}_t - \boldsymbol{\beta}_t - \mathbf{A}_t\mathbf{x}_t)'\mathbf{y}_t + \sum_{t=1}^{T+1} \{[\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{x}_{t-1} + \mathbf{B}\boldsymbol{\Gamma}\mathbf{p}_{t-1} + \mathbf{v}_t - \mathbf{x}_t\}'\boldsymbol{\lambda}_t.$$

The corresponding KKT conditions are

(26) $$\frac{\partial L}{\partial \mathbf{x}_t} = \mathbf{p}_t / (1+\rho)^{t-1} - \mathbf{A}_t{}' \mathbf{y}_t - \boldsymbol{\lambda}_t + [\mathbf{I} - \boldsymbol{\Gamma}]\boldsymbol{\lambda}_{t+1} \le \mathbf{0}$$

(27) $$\frac{\partial L}{\partial \mathbf{x}_{T+1}} = (\mathbf{p}_{T+1} - \mathbf{A}'_{T+1}\mathbf{r}_{T+1})\rho e^{-\rho T} - \boldsymbol{\lambda}_{T+1} = \mathbf{0}$$

(28) $$\frac{\partial L}{\partial \boldsymbol{\beta}_t} = \mathbf{r}_t / (1+\rho)^{(t-1)} - \mathbf{y}_t \le \mathbf{0}$$

(29) $$\frac{\partial L}{\partial \boldsymbol{\lambda}_t} = [\mathbf{I} - \boldsymbol{\Gamma}]\mathbf{x}_{t-1} + \mathbf{B}\boldsymbol{\Gamma}\mathbf{p}_{t-1} + \mathbf{v}_t - \mathbf{x}_t = \mathbf{0}$$

(30) $$\frac{\partial L}{\partial \mathbf{y}_t} = \mathbf{b}_t - \boldsymbol{\beta}_t - \mathbf{A}_t\mathbf{x}_t \ge \mathbf{0}.$$

This discrete dynamic problem can be solved, year by year, using a backward solution approach on the system of KKT conditions (26)-(30). The key to this strategy is the realization that the equation of motion calibrates exactly the sample information for any year, that is, $\mathbf{x}_{R,t} = [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\mathbf{x}_{R,t-1} + \hat{\mathbf{B}}\hat{\boldsymbol{\Gamma}}\mathbf{p}_{t-1} + \hat{\mathbf{v}}_t$ and, therefore, the left-hand-side quantity $\mathbf{x}_{R,t}$ can replace the corresponding right-hand-side expression. In other words, we can equivalently use the available and contemporaneous information about the economic agent's decisions. Furthermore, the costate variable $\boldsymbol{\lambda}_{T+1}$ for the time period outside the horizon is equal to the derivative of the salvage function, that is $\hat{\boldsymbol{\lambda}}_{T+1} = (\mathbf{p}_{T+1} - \mathbf{A}'_{T+1}\mathbf{r}_{T+1})\rho e^{-\rho T}$. One needs knowledge of the steady state output and input prices and of the technical coefficients at time *T+1*. Then, at time *T*, the equilibrium problem to be solved is composed by the following structural relations

(31) $$\mathbf{A}_T\mathbf{x}_T + \boldsymbol{\beta}_T \le \mathbf{b}_T \qquad\qquad \mathbf{y}_T \ge \mathbf{0}$$

(32) $\mathbf{x}_T \leq \mathbf{x}_{R,T} = [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\mathbf{x}_{R,T-1} + \hat{\mathbf{B}}\hat{\boldsymbol{\Gamma}}\mathbf{p}_{T-1} + \hat{\mathbf{v}}_T$ $\quad\quad \boldsymbol{\lambda}_T \geq \mathbf{0}$

(33) $\mathbf{A}'_T\mathbf{y}_T + \boldsymbol{\lambda}_T \geq \mathbf{p}_T / d^T + [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\hat{\boldsymbol{\lambda}}_{T+1}$ $\quad\quad \mathbf{x}_T \geq \mathbf{0}$

(34) $\mathbf{y}_T \geq \mathbf{r}_T / d^T$ $\quad\quad \boldsymbol{\beta}_T \geq \mathbf{0}$

and by the associated complementary slackness conditions. The symbol $d^t = (1+\rho)^{t-1}$ is the discount factor. Knowledge of the realized levels of output at time t and of the vector of costate variables $\hat{\boldsymbol{\lambda}}_{t+1}$, estimated at time $t+1$, allows the solution of the dynamic problem as a sequence of T equilibrium problems. Hence, the dynamic linkage between successive time periods is realized through the vector of costate variables $\boldsymbol{\lambda}_t$. In this way, the equilibrium problem (31)-(34) can be solved backward to time t=1 without the need to specify initial conditions for the vector of state variables $\mathbf{x}_0$ and the price vector $\mathbf{p}_0$. As we indicated previously, there is the need to specify a terminal condition in the form of a salvage function. This dynamic problem arises exclusively from the assumption of adaptive price expectations. Given the DPEP as stated above, the costate variable $\boldsymbol{\lambda}_t$ does not depend explicitly upon the state variable $\mathbf{x}_t$. This implies that the positive character of the problem, with the concomitant use of the realized levels of activity outputs, avoids the usual two-part solution of a dynamic problem where the backward solution is carried out in order to find the sequence of costate variables $\boldsymbol{\lambda}_t$ and the forward solution is devoted to finding the optimal level of the state variables $\mathbf{x}_t$. In the context specified above, the solution regarding the state variable $\mathbf{x}_t$ is obtained contemporaneously with the solution of the costate variable $\boldsymbol{\lambda}_t$.

The objective of DPEP during phase 1, therefore, is to solve T equilibrium problems starting from the end point of the time horizon, that is from $t = T, T-1, ..., 2, 1,$ and having the following structure:

(35) $$\min\{\mathbf{v}'_{P1t}\mathbf{y}_t + \mathbf{v}'_{P2t}\boldsymbol{\lambda}_t + \mathbf{v}'_{D1t}\mathbf{x}_t + \mathbf{v}'_{D2t}\boldsymbol{\beta}_t\}$$

subject to

(36) $$\mathbf{A}_t\mathbf{x}_t + \boldsymbol{\beta}_t + \mathbf{v}_{P1t} = \mathbf{b}_t$$

(37) $$\mathbf{x}_t + \mathbf{v}_{P2t} = \mathbf{x}_{R,t}$$

(38) $$\mathbf{A}'_t\mathbf{y}_t + \boldsymbol{\lambda}_t = \mathbf{p}_t / d^t + [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\hat{\boldsymbol{\lambda}}_{t+1} + \mathbf{v}_{D1t}$$

(39) $$\mathbf{y}_t = \mathbf{r}_t / d^t + \mathbf{v}_{D2t}.$$

The objective function is the sum of all the complementary slackness conditions. A solution of the equilibrium problem is achieved when the objective function reaches the zero value. The principal objective of phase 1 is the recovery of the costate variables for the entire horizon and of the dual variables for the structural constraints to serve as information in the estimation of the relevant cost function during the next phase.

The fundamental reason for estimating a cost function to represent the decision process is to relax the fixed-coefficient technology represented by the $\mathbf{A}_t$ matrix and to introduce the possibility of a more direct substitution between products and limiting inputs. In other words, the observation of output and input decisions at time t provides only a single point in the technology and cost spaces. The process of eliciting an estimate of the latent marginal cost levels and the subsequent recovery of a consistent cost function which rationalizes the available data is akin to the process of fitting an isocost through the observed output and input decisions.

Phase 2 of DPEP: Estimation of the Cost Function

By definition, total cost is a function of output levels and input prices. In a dynamic problem, the total cost function is defined period by period as in a static problem and represented as $C(\mathbf{x}_t, \mathbf{y}_t, t) \equiv C_t(\mathbf{x}_t, \mathbf{y}_t)$ (see Stefanou). The properties of a cost function in a dynamic problem follow the same properties specified for a static case: It must be concave and linearly homogeneous in input prices in each time period. The functional form selected to represent the inputs is a Generalized-Leontiev specification with nonnegative and symmetric terms. For the outputs, the functional form is a quadratic specification in order to avoid the imposition of a linear technology. Furthermore, sufficient flexibility must be allowed in order to fit the available empirical data. For this reason, an unrestricted intercept term is added to the specification. Finally, we must guarantee that the cost function is homogeneous of degree one in input prices. All these considerations lead to the following functional form:

(40) $$C_t(\mathbf{x}_t,\mathbf{y}_t) = \mathbf{u}'\mathbf{y}_t(\mathbf{f}_t'\mathbf{x}_t) + \mathbf{u}'\mathbf{y}_t(\mathbf{x}_t'\mathbf{Q}_t\mathbf{x}_t)/2 + \mathbf{y}_t^{1/2\,\prime}\mathbf{S}_t\mathbf{y}_t^{1/2}.$$

where $\mathbf{u}$ is a vector of unit elements. Many different functional forms could be selected in such a way to satisfy the properties of a cost function. The matrix $\mathbf{Q}$ is symmetric positive semidefinite while the $\mathbf{S}$ matrix is symmetric with nonnegative elements on and off the main diagonal.

The marginal cost function at time *t* is the derivative of equation (40) with respect to the output level at time *t*, that is

(46) $$\frac{\partial C_t}{\partial \mathbf{x}_t} = (\mathbf{u}'\mathbf{y}_t)\mathbf{f}_t + (\mathbf{u}'\mathbf{y}_t)\mathbf{Q}_t\mathbf{x}_t = \mathbf{A}_t'\mathbf{y}_t$$

whereas, by Shephard lemma, the limiting input derived demand functions are

(42) $$\frac{\partial C_t}{\partial \mathbf{y}_t} = (\mathbf{f}_t'\mathbf{x}_t)\mathbf{u} + \mathbf{u}(\mathbf{x}_t'\mathbf{Q}_t\mathbf{x}_t)/2 + \Delta_{\mathbf{y}^{-1/2}}\mathbf{S}_t\mathbf{y}_t^{1/2} = \mathbf{A}_t\mathbf{x}_t = \mathbf{b}_t - \boldsymbol{\beta}_t.$$

The matrix $\Delta_{\mathbf{y}^{-1/2}}$ is diagonal with elements of the vector $\mathbf{y}_t^{-1/2}$ on the diagonal.

Notice that there is a significant difference between the marginal cost of the static equilibrium problem and the short-run (period by period) marginal cost of the dynamic equilibrium problem. If one considers the static equilibrium problem formulated in model (2)-(5), the marginal cost is given in relation (4) as $MC(\mathbf{x},\mathbf{y}) \equiv \mathbf{A}'\mathbf{y} + \boldsymbol{\lambda}$. In other words, without a time dimension, the marginal cost is equal to the sum of the marginal cost attributable to the limiting inputs, $\mathbf{A}'\mathbf{y}$, plus the variable marginal cost attributable to the level of outputs, $\boldsymbol{\lambda}$. In the dynamic context elaborated above, the Lagrange multiplier $\boldsymbol{\lambda}_t$ assumes the meaning of a costate variable and signifies the marginal valuation of the state variable $\mathbf{x}_t$ and its dependence on the entire horizon, that is, $\boldsymbol{\lambda}_{T-n} = \sum_{s=0}^{n+1} [\mathbf{I} - \boldsymbol{\Gamma}]^s (\mathbf{p}_{T-n+s} - \mathbf{A}'_{T-n+s}\mathbf{y}_{T-n+s})$, where $n = -1, 0, \ldots T$. In a dynamic context, therefore, the costate variable $\boldsymbol{\lambda}_t$ cannot be used to define the period-by-period marginal cost (as done in a static equilibrium problem where the symbol $\boldsymbol{\lambda}$ is interpreted simply as variable marginal cost) because it incorporates the long-run notion of a trajectory associated with a multi-period horizon. In the dynamic equilibrium problem depicted above, the period-by-period marginal cost is thus defined as $MC_t(\mathbf{x}_t, \mathbf{y}_t) \equiv \mathbf{A}'_t\mathbf{y}_t$, as deduced from relation (33).

The objective of phase 2 is to estimate the parameters of the cost function given in equation (40), $\mathbf{f}_t, \mathbf{Q}_t$ and $\mathbf{S}_t$. This estimation will be performed using the Kullback-Leibler criterion known also as the cross-entropy formalism. Economic theory requires that the $\mathbf{Q}_t$ matrix be symmetric positive semidefinite. In order to guarantee this condition during the estimation process, two approaches based upon the Cholesky factorization can be used. The following specification

(43) $$\mathbf{Q}_t = \mathbf{L}_t \mathbf{D}_t \mathbf{L}'_t$$

allows the estimation of a semidefinite matrix if the data support this structure. The $\mathbf{L}_t$ matrix is a unit lower triangular matrix and $\mathbf{D}_t$ is a diagonal matrix with nonnegative elements. It can be shown that the $\mathbf{Q}_t$ matrix is positive semidefinite (definite) if and only if the diagonal elements of $\mathbf{D}_t$ are nonnegatine (positive) (see Lau). These diagonal elements are called the Cholesky values. This first specification of the Cholesky factorization is computational intensive. Hence, if a researcher wishes to make the sufficient assumption that the $\mathbf{Q}_t$ matrix is positive definite, a computationally more saving structure of the Cholesky factorization can be implemented.

Following Golan et al., all the parameters to be estimated will be defined as convex combinations of a corresponding set of predetermined support values and where the weights are regarded as probabilities. Hence, it is assumed that for each (j, j') parameter

(44) $$L_{jj',t} = \sum_s Z_{L,t}(j, j', s) P_{L,t}(j, j', s), \qquad j, j' = 1, \ldots, J$$

(45) $$D_{j,j,t} = \sum_s Z_{D,t}(j, j, s) P_{D,t}(j, j, s), \qquad s = 1, \ldots, S$$

where $\mathbf{Z}_{L,t}$ and $\mathbf{Z}_{D,t}$ are the matrices of the known support values for the probability distributions of the $\mathbf{L}_t$ and $\mathbf{D}_t$ matrices, respectively, while $\mathbf{P}_{L,t}$ and $\mathbf{P}_{D,t}$ are the corresponding probability matrices. In matrix notation, equations (44) and (45) correspond to $\mathbf{L}_t = \mathbf{Z}_{L,t}\mathbf{P}_{L,t}$ and $\mathbf{D}_t = \mathbf{Z}_{D,t}\mathbf{P}_{D,t}$, respectively, where the multiplication is performed only with respect to the index s, $s = 1, \ldots, S$. A similar specification involves the vector $\mathbf{f}_t$ and the matrix $\mathbf{S}_t$, that is $\mathbf{f}_t = \mathbf{Z}_{f,t}\mathbf{P}_{f,t}$ and $\mathbf{S}_t = \mathbf{Z}_{S,t}\mathbf{P}_{S_t}$.

The Kullback-Leibler criterion can be stated as the problem of minimizing the distance between two probability distributions where one of the distributions represents conditional or *a priori* information. Suppose, therefore, that $\mathbf{P}_L{}^C(j,j',s),\mathbf{P}_D{}^C(j,j,s),\mathbf{P}_f{}^C(j,s),\mathbf{P}_S{}^C(i,i',s)$ represent conditional (or *a priori*) information about the corresponding parameters. The Kullback-Leibler criterion is then to find positive values of all the posterior probabilities, $\mathbf{P}_L(j,j',s),\mathbf{P}_D(j,j,s),\mathbf{P}_f(j,s),\mathbf{P}_S(i,i',s)$, such that, at time t

(46) $$\min KL(\mathbf{P}_{Lt},\mathbf{P}_{Dt},\mathbf{P}_{ft},\mathbf{P}_{St}) = \sum_{j,j',s}\mathbf{P}_{Lt}(j,j',s)\log(\mathbf{P}_{Lt}(j,j',s)\div\mathbf{P}^C_{L,t-1}(j,j',s))$$
$$+\sum_{j,s}\mathbf{P}_{Dt}(j,j,s)\log(\mathbf{P}_{Dt}(j,j,s)\div\mathbf{P}^C_{D,t-1}(j,j,s))$$
$$+\sum_{j,s}\mathbf{P}_{ft}(j,s)\log(\mathbf{P}_{ft}(j,s)\div\mathbf{P}^C_{f,t-1}(j,s))$$
$$+\sum_{i,i',s}\mathbf{P}_{St}(i,i',s)\log(\mathbf{P}_{St}(i,i',s)\div\mathbf{P}^C_{S,t-1}(i,i',s))$$

subject to

(47) $$\mathbf{A}_t'\hat{\mathbf{y}}_t = (\mathbf{u}'\hat{\mathbf{y}}_t)\mathbf{f}_t + (\mathbf{u}'\hat{\mathbf{y}}_t)\mathbf{Q}_t\mathbf{x}_{R,t}$$
$$= (\mathbf{u}'\hat{\mathbf{y}}_t)\mathbf{Z}_{ft}\mathbf{P}_{ft} + (\mathbf{u}'\hat{\mathbf{y}}_t)(\mathbf{Z}_{Lt}\mathbf{P}_{Lt})(\mathbf{Z}_{Dt}\mathbf{P}_{Dt})(\mathbf{Z}_{Lt}\mathbf{P}_{Lt})'\mathbf{x}_{R,t}$$

(48) $$\mathbf{A}_t\mathbf{x}_{R,t} = (\mathbf{f}_t'\overline{\mathbf{x}}_t)\mathbf{u} + \mathbf{u}(\mathbf{x}_{Rt}'\mathbf{Q}_t\mathbf{x}_{Rt})/2 + \Delta_{\hat{\mathbf{y}}^{-1/2}}\mathbf{S}_t\hat{\mathbf{y}}_t{}^{1/2}$$
$$= ((\mathbf{Z}_{ft}\mathbf{P}_{ft})'\mathbf{x}_{Rt})\mathbf{u} + \mathbf{u}(\mathbf{x}_{Rt}'(\mathbf{Z}_{Lt}\mathbf{P}_{Lt})(\mathbf{Z}_{Dt}\mathbf{P}_{Dt})(\mathbf{Z}_{Lt}\mathbf{P}_{Lt})'\mathbf{x}_{Rt})/2 + \Delta_{\hat{\mathbf{y}}^{-1/2}}(\mathbf{Z}_{St}\mathbf{P}_{St})\hat{\mathbf{y}}_t{}^{1/2}$$

where the symbol $(\div)$ represents an element-by-element ratio. Given the time horizon $t=1,\ldots,T$, the *a priori* probabilities at time $t=1$ can be taken as the uniform distribution.

The maximum entropy probabilities estimated at time *t=1* become the prior (or conditional) probabilities at time $t = 2$, and so on. In this way, the parameters of the cost function in any given year are estimated under the requirement that they differ the least from the estimates of the previous year.

In the above formulation, the total marginal cost $\mathbf{A}_t'\hat{\mathbf{y}}_t$, the limiting input demand $\mathbf{A}_t\mathbf{x}_{R,t}$, the realized level of activities $\bar{\mathbf{x}}_{R,t}$, and the shadow prices of limiting inputs $\hat{\mathbf{y}}_t$, are known elements of the specification. The parameters to be estimated are the probabilities, $\mathbf{P}_{Lt}, \mathbf{P}_{Dt}, \mathbf{P}_{ft}, \mathbf{P}_{St}$. The solution probabilities of problem (46)-(48) allow the recovery of all the parameters of the total cost function, $\mathbf{f}_t, \mathbf{Q}_t$, and $\mathbf{S}_t$.

Phase 3 of DPEP: Calibration and Policy Analysis

The estimated cost function can now be used to replace the marginal cost levels and the demands of inputs in the equilibrium problem of phase 1. This replacement assures the calibration of the model, liberates the specification from a fixed-coefficient technology, and allows direct substitution among outputs and inputs. At this stage, therefore, it is possible to implement policy scenarios based upon the variation of output and input prices.

The structure of the calibration DPEP is given below. With the knowledge of the costate variables, $\hat{\boldsymbol{\lambda}}_t$ and $\hat{\boldsymbol{\lambda}}_{t+1}$ from the solution of the DPEP obtained in phase 1, the following specification calibrates the solution of the output decisions and the input dual variables for any period:

(49) $$\min_{\mathbf{x},\mathbf{y},\boldsymbol{\beta},\mathbf{v}} \{\mathbf{v}'_{P1,t}\mathbf{y}_t + \mathbf{v}'_{D1,t}\mathbf{x}_t + \mathbf{v}'_{D2,t}\boldsymbol{\beta}_t\}$$

subject to

(50) $$(\hat{\mathbf{f}}'_t\mathbf{x}_t)\mathbf{u} + \mathbf{u}(\mathbf{x}'_t\hat{\mathbf{Q}}_t\mathbf{x}_t)/2 + \Delta_{\mathbf{y}^{-1/2}}\hat{\mathbf{S}}_t\mathbf{y}_t^{1/2} + \boldsymbol{\beta}_t + \mathbf{v}_{P1,t} = \mathbf{b}_t$$

(51) $$(\mathbf{u}'\mathbf{y}_t)\hat{\mathbf{f}}_t + (\mathbf{u}'\mathbf{y}_t)\hat{\mathbf{Q}}_t\mathbf{x}_t = \mathbf{p}_t / d^{t-1} + [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\hat{\boldsymbol{\lambda}}_{t+1} - \hat{\boldsymbol{\lambda}}_t + \mathbf{v}_{D1,t}$$

(52) $$\mathbf{y}_t = \mathbf{r}_t / d^{t-1} + \mathbf{v}_{D2,t}.$$

The use of the costate values obtained during phase 1 is required by the necessity of eliminating the constraint on the decision variables, $\mathbf{x}_t \leq \mathbf{x}_{R,t}$, which were used in the phase 1 specification precisely for eliciting the corresponding values of the costates. As observed above, the costate variables are the dynamic link between any two periods and their determination requires a backward solution approach. If we were to require their measurement for a second time during the calibration phase, we would need to add also the equation of motion in its explicit form, since the constraint $\mathbf{x}_t \leq \mathbf{x}_{R,t}$ would no longer be acceptable. In this case the T problems would be all linked together and ought to be solved as a single large-scale model. The calibration phase, therefore, is conditional upon the knowledge of the costate variables obtained during phase 1 and involves the period-by-period determination of the output decisions and dual variables of the limiting inputs.

Given the dynamic structure of the model, a policy scenario becomes a prediction at the end of the T-period horizon . All the model components at period T are known and the researcher wishes to obtain a solution of the Dynamic Positive Equilibrium Problem for the $T+1$ period. The parameters of the cost function are assumed constant and equal to those at time T. The costate variables at times $T+2$, and $T+1$, $\hat{\boldsymbol{\lambda}}_{T+2}$,, $\hat{\boldsymbol{\lambda}}_{T+1}$ are taken to be equal to the steady state marginal values of the salvage function. The remaining parameters, $\mathbf{b}_{T+1}, \mathbf{r}_{T+1}$ and $\mathbf{p}_{T+1}$ will assume the value of interest under the desired policy scenario.

The relevant structure of the dynamic positive equilibrium problem during the policy analysis phase takes on the following specification:

(53) $$\min_{\mathbf{x},\mathbf{y},\boldsymbol{\beta},\mathbf{v}} \{\mathbf{v}'_{P1,T+1}\mathbf{y}_{T+1} + \mathbf{v}'_{D1,T+1}\mathbf{x}_{T+1} + \mathbf{v}'_{D2,T+1}\boldsymbol{\beta}_{T+1}\}$$

subject to

(54) $$(\hat{\mathbf{f}}'_{T+1}\mathbf{x}_{T+1})\mathbf{u} + \mathbf{u}(\mathbf{x}'_{T+1}\hat{\mathbf{Q}}_{T+1}\mathbf{x}_{T+1}) / 2 + \Delta_{\mathbf{y}^{-1/2}}\hat{\mathbf{S}}_{T+1}\mathbf{y}_{T+1}{}^{1/2} + \boldsymbol{\beta}_{T+1} + \mathbf{v}_{P1,T+1} = \mathbf{b}_{T+1}$$

(55) $$(\mathbf{u}'\mathbf{y}_{T+1})\hat{\mathbf{f}}_{T+1} + (\mathbf{u}'\mathbf{y}_{T+1})\hat{\mathbf{Q}}_{T+1}\mathbf{x}_{T+1} = \mathbf{p}_{T+1} / d^T + [\mathbf{I} - \hat{\boldsymbol{\Gamma}}]\hat{\boldsymbol{\lambda}}_{T+2} - \hat{\boldsymbol{\lambda}}_{T+1} + \mathbf{v}_{D1,T+1}$$

(56) $$\mathbf{y}_{T+1} = \mathbf{r}_{T+1} / d^T + \mathbf{v}_{D2,T+1}.$$

Projected policy prices, either on the output or input side, will induce responses in the output and input decisions which are consistent with the process of output price expectations articulated in the previous sections.

An Application of DPEP to California Agriculture

California's agriculture is divided in twenty one regions. We have selected a region of the Central Valley that produces seven annual crops: rice, fodder crops, field crops, grains, tomatoes, sugar beets, and truck crops. Eight years of reporting are available from 1985 to 1992. Three limiting inputs are also recorded: land, water and "other" inputs. The available information is given in table 1 and deals with total availability of limiting inputs, their prices, total realized production and the associated prices. A technical coefficient matrix was defined in terms of input per unit of output.

The first step of the DPEP procedure requires the estimation of the diagonal matrix of output price expectations, $\mathbf{\Gamma}$. The estimation was performed by means of a maximum entropy procedure with "endogenous" probability supports as described above. The estimate of $\mathbf{\Gamma}$ is given in Table 2. Some of the coefficients are negative but, as explained above, they are admissible estimates of a sufficiently short horizon of eight periods. No stability properties are violated within the context of this example.

The second step of the DPEP approach requires the estimation of the dynamic equilibrium problem (35)-(39) using a backward solution as explained above. For implementing this phase, a discount rate of 3 percent was selected. We have experimented also with discount rates that varied from 1 to 9 percent without detecting any significant shift in the estimation results.

The parameters of the cost function ($\mathbf{Q}_t, \mathbf{S}_t$, and $\mathbf{f}_t$) were estimated by the Kullback-Leibler cross-entropy approach for the years 1985-1991. The year 1992 was kept in reserve in order to measure the prediction error associated with the assumption of no structural change between the year 1991 and 1992. Notice that the parameters of the cost function were estimated for each year. In this case, therefore, the specification of the support values could not follow the suggestion of van Akkeren and Judge because it does not apply when only one observation is available. Hence, the original GME approach was adopted for the support matrices $\mathbf{Z}_D, \mathbf{Z}_L, \mathbf{Z}_S$ and $\mathbf{Z}_f$. In particular, the supports for the $\mathbf{Z}_D$ matrix were defined as the product of a vector of weights $\mathbf{w}_1^1 = (0, 1.5, 3)$ and a parameter $par(j) = mc(j) / x_{Rj}$, where *mc(j)* is marginal cost. The supports for the $\mathbf{Z}_L$ matrix were defined as the product of a vector of weights $\mathbf{w}_2^1 = (-2.5, 0, 2.5)$ and the same parameter $par(j)$. The supports of the matrix $\mathbf{Z}_S$ were defined as the product of the weights $\mathbf{w}_1^1$ and the parameter $b(i)$, the *i*-th input availability. Finally, the supports of the matrix $\mathbf{Z}_f$ were defined as the weights $\mathbf{w}_2^1$.

The estimate of the $\mathbf{Q}$ matrix for 1991 in the cost function is given also in table 2. The off diagonal coefficients of this matrix are rather small relative to their diagonal counterparts indicating a limited degree of substitutability between pairs of the seven crops. It is possible to compute the associated matrix of supply elasticities by inverting the implicit supply function given by the marginal cost function. The estimated matrix turned out to be positive definite although the approach allowed the estimation of a positive semidefinite matrix.

Table 3 presents the estimate of the matrix **S** of limiting inputs for 1991 in the cost function. The associated matrix of derived demand elasticities indicates that land is marginally elastic in this agricultural region. Allen and Morishima elasticities of substitution are also given in table 3. The matrix **S** is positive definite.

Table 4 presents the **S** matrices from 1985 to 1991. It is remarkable to notice how similar these matrices are, indicating that during the period 1985-1991 there were no significant structural changes. In order to aid in the evaluation of these matrices the corresponding eigenvalues are given along with the associated condition numbers. The eigenvalues show a robust stability of these matrices as do their condition numbers (the ratio of the largest to the smallest eigenvalues).

The **Q** matrices are of dimension (7 by 7) and their reporting for the seven years would require several pages. In order to avoid such a visual chaos we present their eigenvalues and condition numbers which, again, show a remarkable stability throughout the time interval under study. We wish to recall that the estimation of these matrices (**Q** and **S**) was performed by means of the Kullback-Leibler cross-entropy ratio which allows to minimize the deviation between two probability distributions. In this case, the minimum deviation translates into a minimum distance between two successive pairs of matrices, as illustrated by the results of table 4.

The empirical results from the estimation of the cost function have indicated only small variations from year to year of the $\mathbf{Q}_t$ and $\mathbf{S}_t$ matrices. Thus, an alternative estimation procedure (not currently pursued in this version of the paper) would assume that no technical change occurred during the 8-year horizon. This assumption implies the constancy of the **Q** and **S** matrices. In other words, only one set of **Q** and **S** matrices would be estimated and the endogenous variant of the GME approach could be implemented.

The results of phase 3 contain two parts. The first part deals with the verification that indeed the DPEP methodology calibrates the output decisions and land allocations within the time period of 1985-1991. The second part consist in a prediction of output decisions and land allocation for 1992 using the cost function of 1991. Table 5 and table 6 show that the calibration goal is achieved within very precise limits. The prediction exhibits an error that varies from 2 to 20 percent in the case of the output decisions and from zero to 29 percent in the case of the shadow prices of limiting inputs. The average percent error for the prediction of the output decisions is 12.13 while the average percent error for the shadow input prices is 9.66. It may be possible that a different functional form of the cost function could lead to a smaller prediction error.

Finally, table 6 presents the estimates of the costate variables associated with the equation of motion involving price expectations. Their values tend to increase as we move toward the beginning of the time period because their structure is given by the following expression: $\boldsymbol{\lambda}_{T-n} = \sum_{s=0}^{n+1} [\mathbf{I} - \boldsymbol{\Gamma}]^s (\mathbf{p}_{T-n+s} - \mathbf{A}'_{T-n+s}\mathbf{y}_{T-n+s})$. One costate value in each year is equal to zero by virtue of the degeneracy built into the primal equilibrium problem (see relations (36) and (37)).

Sensitivity Analysis Involving the Support Intervals

A well-known limitation of the original GME approach is given by its dependence upon the researcher's subjective specification of the support intervals of the corresponding probability distributions. As a consequence, the parameter estimates depend crucially upon the pre-selected support values. Furthermore, given the structure of the maximum entropy, there is no possibility of making general statements about the direction of response of the estimated parameters in relation to either an enlargement or a shrinkage of the support intervals (see Caputo and Paris). It remains, therefore, to judge the appropriateness of the supports' choice case by case. Finally, we must recall that a quantitative analysis is always characterized by two stages: estimation and prediction. A sensitivity analysis involving some or all the support intervals may have a differential impact upon the parameter estimates and the predictions.

With these considerations in mind, we re-estimated the empirical equilibrium problem using two additional sets of supports. The first of these sets shrinks the support intervals by reducing the weights from $\mathbf{w}_1^1 = (0, 1.5, 3)$ to $\mathbf{w}_1^2 = (0, 1, 2)$ and from $\mathbf{w}_2^1 = (-2.5, 0, 2.5)$ to $\mathbf{w}_2^2 = (-1, 0, 1)$. The results of this shrinkage of the support intervals are presented succinctly in Table 7.

Let us consider the $\mathbf{S}_{91}$ matrix first. Only nonnegative weights $\mathbf{w}_1^2 = (0, 1, 2)$ were involved in the estimation of the elements of this matrix. We notice that to a 33 percent reduction in the support intervals there corresponds a reduction of parameter estimates of about 30 percent (except for the element land-land). The eigenvalues of the $\mathbf{S}_t$ matrices are about 30 percent smaller than the corresponding eigenvalues of the original matrices in table 4. The new condition numbers, however, are slightly larger but more uniform.

A similar pattern is uncovered for the $\mathbf{Q}_{91}$ matrix in table 7. The diagonal elements have shrunk by about 30 percent with a 33 percent reduction of the support intervals. The off-diagonal elements show a wider range of changes, perhaps as a consequence of their miniscule original values. The eigenvalues of the $\mathbf{Q}_t$ matrices in table 7 exhibit a 40 percent reduction in comparison to the original counterparts in table 4. The condition numbers, however, are only about 15 percent smaller.

There is no doubt that a variation of between 30 to 50 percent in the selection of support values has produced a significant variation in the parameter estimates, as expected. The same variation of support intervals reveals an interesting pattern of changes in the predictions. First of all, all the crops in table 7 exhibit a prediction error that lies within 20 percent of the original prediction errors in table 5. Some of the errors increase and some decrease, in absolute value. As an overall measure of the prediction error we have computed the average absolute percent error. Such an error is about the same in the two sets of predictions, with an average error of 12,13 for the original predictions in table 5 and an average error of 11.62 for the prediction corresponding to the shrunk set of support values. The average absolute percent error of the input shadow prices shows a reduction of about 20 percent between the two scenarios.

The second set of additional supports enlarges the original weights $\mathbf{w}_1^1$ to $\mathbf{w}_1^3 = (0,3,6)$ and the weights $\mathbf{w}_2^1$ to $\mathbf{w}_2^3 = (-5,0,5)$. The results are presented in table 8.

The matrix $\mathbf{S}$ for 1991 in table 8 (compared with a similar matrix in table 3) indicates that an increase of 100 percent in the support intervals has induced a variation of up to about 90 percent in the coefficients estimates. The eigenvalues of the matrices from 1986 to 1991 follow a similar pattern. The condition numbers of the $\mathbf{S}$ matrix in table 8, however, are very close to those of the original $\mathbf{S}$ matrix in table 3.

A comparison of the $\mathbf{Q}$ matrix in table 8 with the $\mathbf{Q}$ matrix in table 3 indicates that a 100 percent enlargement of the support intervals has induced an increase in the eigenvalues by about the same percentage, except for the largest eigenvalue which exhibits an increase of about 130 percent. For this reason, the condition numbers of the $\mathbf{Q}$ matrix in table 8 are larger (by 3 points) than the eigenvalues of the $\mathbf{Q}$ matrix in table 3. The new eigenvalues exhibit the same pattern as the original ones.

Although, on the estimation side, the enlargement of the support intervals has induced an increase of the estimated parameters of about 100 percent, on the prediction side, the same enlargement has much less dramatic effects. Table 8 shows that the percent difference in output decisions for 1992 is rather close to the same differences in table 5. As a more compact measure, the average absolute percent prediction error for the output decisions in table 8 is 13.02 while the comparison measure in table 5 is 12.13. The average absolute percent prediction error for the input shadow prices is 10.72 in table 8 and 9.66 in table 5. This result is similar to that encountered previously with a 50 percent reduction of the support intervals.

As in any quantitative analysis the prediction aspect is more relevant than the individual parameter estimates, the above sensitivity results lend some comfort to the notion that the subjective choice of support intervals in the GME approach may not be so crucial as some authors have claimed.

Conclusion

The main goal of DPEP is to make a rational and consistent use of the available scarce information regarding economic decisions. In this paper, the routine statistical information produced by the agricultural reporting service of the State of California was analyzed in a dynamic model for annual crops under the assumption that the economic agents form their output price expectations according to a Nerlove-type adaptive process. The DPEP methodology was developed along the framework of the static Symmetric Positive Equilibrium Problem (SPEP) which requires a three-phase development. Phase 1 recovers the latent marginal costs of the limiting inputs and of the realized output levels. Phase 2 estimates a consistent cost function that replaces the fixed coefficient technology and introduces more direct sustitutability between inputs and outputs. Phase 3 verifies the calibration process and allows the analysis of various policy scenarios. In the DPEP approach, however the three-phase scheme proper of SPEP must be preceded by the estimation of the relevant equation of motion that confers the dynamic character to the model. Furthermore, a significant difference between the original SPEP and the DPEP consists in the definition of marginal cost. In a SPEP static model, the marginal cost is the sum of the fixed marginal cost due to limiting inputs plus the variable marginal cost associated with the output levels. In the DPEP model, on the contrary, the period-to-period marginal cost is simply the marginal cost associated with limiting inputs. The marginal cost associated with the output levels becomes the costate variable that assumes an inter-temporal significance.

The estimation of the parameters of the equation of motion was performed using a maximum entropy approach with "endogenous" specification of the support values of the corresponding probability distribution. In other words, the sample data were used to specify both the levels and the number of discrete supports. This procedure, suggested by van Akkeren and Judge, eliminates the need for the researcher to select subjective levels of support.

The estimation of the cost function, with parameters dated by each time period, could not use the “endogenous” specification of the support values because the van Akkeren’s suggestion requires more than one observation. The original GME specification was, therefore, adopted and a sensitivity analysis was performed in order to gauge the effects of the subjective choice of support intervals. It turns out that the parameter estimates are substantially more affected by this choice than are the prediction errors. As the predictions are more important than the parameter estimates, it may be possible to rely with some confidence on the empirical results.

The parameter estimates of the cost function indicate a remarkable stability of the technology implied by the economic agents’ input and output decisions. Hence, it is possible to revise the specification of phase 2 and assume that the matrices $\mathbf{Q}$ and $\mathbf{S}$ of the cost function remain constant throughout the analyzed horizon. In this case, the estimation could be performed using the sample information in order to specify the support values, thus removing an important item of contention among researchers.

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