INTEGRATING AGRI-ENVIRONMENTAL PROGRAMS INTO REGIONAL PRODUCTION MODELS: AN EXTENSION OF POSITIVE MATHEMATICAL PROGRAMMING

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Positive Mathematical Programming (PMP) has become a popular method for regional production models. The standard approach estimates cost (or production) functions for each land-use activity separately from each other. This means that the same crop grown under two technologies is treated as if it were two separate crops, which may lead to unsatisfying results, for example, if agri-environmental programs are modeled. We present an extended version of PMP that leads to more plausible results than the standard version in such cases. The extended method is applicable to other problems where differences in the elasticity of substitution between activities are important.

Key words: agri-environmental programs, calibration, joint production, regional production models, positive mathematical programming.

Since the 1992 reform of agricultural policy in the European Union (EU), agri-environmental programs have become more important in European agriculture. Agri-environmental policies often aim at changing specific agricultural practices to make them more environmentally sound. Typically, participation is voluntary and financial compensation is provided. The programs are designed by the countries or often even by regions, which lead to considerable diversity in approaches (Deblitz et al., Deblitz and Plankl). In 1998, the EU spent an estimated 1,700 million ECU on these programs (European Commission). In the context of WTO negotiations, an impact assessment of such policies on production quantities and farm income is important. Regional partial equilibrium models concentrating on the supply effects seem to be promising tools for such an analysis. Sectoral models are usually too aggregated to include the details that form the core of the agri-environmental measures and farm-level models present an alternative; however, data requirements are rather high and aggregation to regions is often not straightforward.

A method for regional models that has recently become more and more popular is positive mathematical programming (or PMP) (Howitt). This is mostly applied with a quadratic form of the objective function and then called *positive quadratic programming* (PQP). Among the advantages associated with this approach—compared to conventional linear programming—are an exact representation of the reference situation, lower data requirements, and continuous changes in model results in response to continuous changes in exogenous variables. The method assumes a profit-maximizing equilibrium in the baseline situation and uses the observed level of production activities as a basis for estimating the coefficients of a nonlinear objective function. For an introduction to the method, see Howitt, or simplified Umstätter. When a formal estimation procedure for the coefficients is used, a (positive) econometric element is introduced into the (normative) programming approach, a course that has been further pursued in recent developments (Paris and Howitt). The advantage of the earlier version of PMP is that the production activities and coefficients estimated have a more straightforward agronomic

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interpretation. This is important in our context. We thus start from the original approach.

The purpose of this article is to extend the standard PMP approach. The standard approach estimates cost (or production) functions for each land-use activity separately from each other. This means that the same crop grown under two technologies is treated as if it were two separate crops. We will discuss why this approach poses a particular problem and illustrate the problem with a simple example. Next, we will present an extension of the PQP method that solves this problem. Finally, we will apply the extended PQP method to the example and draw conclusions.

To provide some context to our approach we start with a section that gives some background to the PMP method and provides a link with the European history of thought in agricultural economics.

Background

Positive mathematical programming has been developed as a remedy for some of the problems associated with the linear programming approach. The linear programming approach to regional models grew out of a farm-level approach developed in the 1950s soon after the simplex algorithm had been developed. From the European viewpoint, linear programming was a tool to express quantitatively some of the ideas on farm organization which had been around since Aereboe and Brinkmann. In essence, the ideas of these agricultural economists included a theory of joint production in agriculture: Why are many farms combining a multitude of production activities in their production programs while others are much more specialized?

In a classic article Brinkmann described two groups of factors which influenced the degree of specialization. Brinkmann called the first group "integrating" factors, leading to the integration of many production activities on the farm, and the second group "differentiating" factors, leading to specialization. Among the integrating factors making it favorable to combine production activities are crop rotation effects, matching of feed requirements by the animals held with the fodder supply produced on the farm, and a more even seasonal distribution of labors and risk. The differentiating factors, according to Brinkmann, were location (leading to differing transportation costs which favor some products over others), soil and climate, and the specific capabilities of the entrepreneur.

Building on Brinkmann's ideas, Weinschenck developed the concept of different groups of production activities. Certain production activities compete strongly with each other because they show similar characteristics with respect to the integrating factors. In such cases substitution within such a group would be rather strong when there are exogenous changes to the system, for example, relative product price changes. Elasticity of substitution, on the contrary, would be much smaller if the production activities differ in many aspects of the integrating factors.

The advent of linear programming made it possible to capture parts of the integrating factors in a quantitative model (compare Dabbert). Seasonal availability of labor, for example, could be explicitly modeled and matched with the labor demand of certain production activities in the model. Thus, the competition and synergy between different production activities with respect to labor in a given time span formed the basis for the model's reaction to changes in exogenous variables.

This linear approach included some problems. Especially problematic from the viewpoint of economic theory is the use of constraints, which have no clearly defined economic or technological background to bind the model behavior and to keep the model's reaction within a "reasonable" range. Often such constraints are not justified by hard data. The extreme form of such a constraint is a calibration constraint, which locks the model's result to the observed baseline situation. This problem is at the core of the original development of PQP. This can be easily shown with the help of a simple example (Example I).

Figure 1 shows marginal gross margin for standard wheat ($DB'\ WW_{LP}$) and rape seed ($DB'\ WR_{LP}$) (data for these activities are from table 1), assuming a region with 10,000 hectares. At the observed situation (4000 hectares of rape seed and 6000 hectares of standard wheat) marginal gross margin between the two differs by λ_{WR}, which is the shadow price of the calibration constraint necessary to force this situation on a linear programming model. The theoretical expectation would be that marginal gross margin for the two crops is equal at the observed situation. The easiest way to achieve this is to make sure that the marginal gross margin of rape seed decreases with increasing area covered. This turns the

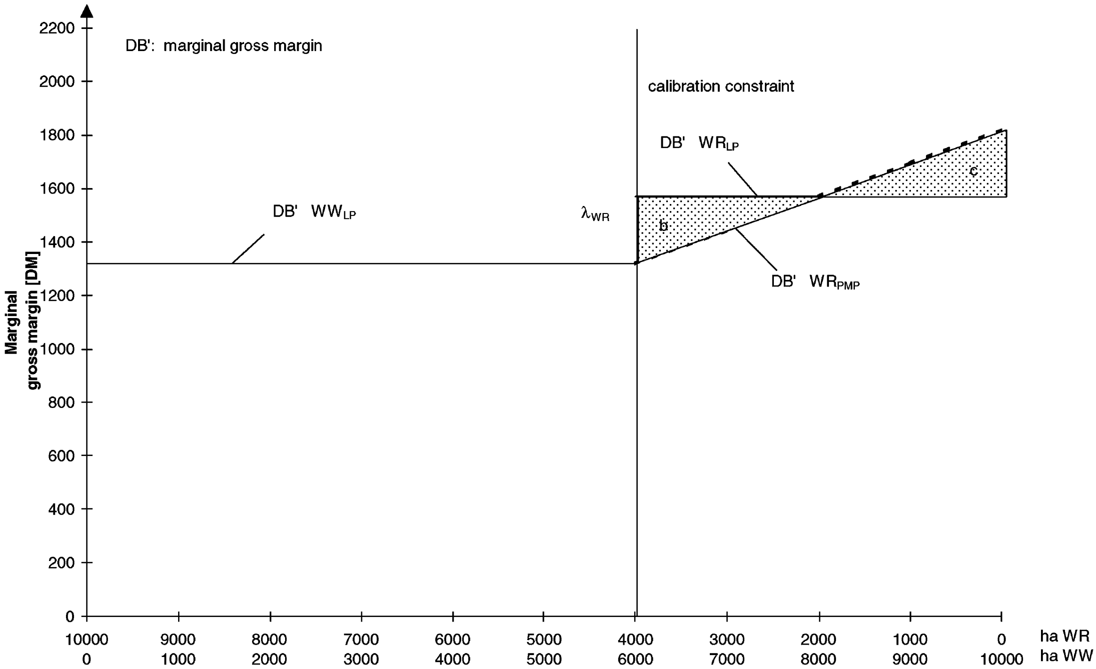

Figure 1. Marginal gross margin for two crops in the linear and the nonlinear model (Example I)

problem into a nonlinear one and we get a new marginal gross margin function for rape seed (DB' WR_{PMP}). This decrease is in line with agronomic expectations. If the marginal gross margin of standard wheat (DB' WW_{LP}) is assumed to be still a constant, then the condition $DB'\ WW_{LP} = DB'\ WR_{PMP}$ must hold at the observed situation. With the second condition that the total gross margin has to stay unchanged (that means triangle b in figure 1 has to be of equal size to triangle c), the slope of the marginal gross margin function can be calculated and in further steps a quadratic objective function can be calculated. The nonlinear model (PQP-Model) will then exactly solve to the observed baseline model without a calibration constraint.

This general idea of PQP has been extended in different ways. Especially important is the question of how to calculate the marginal gross margin for the last hectare of the least profitable crop—because this influences all the slopes of all marginal gross margin functions and thus the behavior of the model. Any version of a PMP model will always exactly calibrate to the baseline situation, but under

Table 1. Baseline Situation: Data for Example Models

	Variable	Unit	Standard Wheat (WW.1)	Wheat AEP[a] (WW.2)	Barley	Rape Seed	Pea for Grain
Average yield	y_i	t/ha	8.0	7.2	4.8	3.6	4.0
Price	p_i	DM/t	240	240	270	450	250
Average variable cost	vk_i	DM/ha	1,200	1,050	1,000	1,250	900
General acreage premium		DM/ha	600	600	600	1,200	680
Agri-environmental premium		DM/ha		200			
Total premium	PR_i	DM/ha	600	800	600	1,200	680
Gross margin	DB_i	DM/ha	1,320	1,478	896	1,570	780
Activity level in baseline situation	$\hat{X}_i$	ha	2,500	4,000	3,000	5,000	500

[a] AEP stands for *agri-environmental program*. The main agronomic difference with the activity standard wheat is that no growth regulator CCC is applied which leads to lower yield and lower variable cost.

different scenarios, model behavior might differ and possibly not be in line with theoretical expectations. This leads us to the specific problem that the modeling of agri-environmental programs can pose to the PMP method.

Outline of the Problem

The standard PQP approach estimates cost (or production) functions for each land-use activity separately from each other. With this approach an attempt is made to implicitly capture the factors that determine the elasticity of substitution between crops on farm and regional levels. The activities may, for example, compete for a fixed labor supply during a given season or for machinery. In general, the more similar these requirements are to fixed factors for two activities, the stronger the substitution between the two activities should be. If two activities are nearly identical, substitution should be very strong. Where the substitution between rather similar activities is an important part of the analysis, the implicit way to approach this problem in the standard PQP approach leads to unsatisfying results if scenarios different from the observed baseline situation are calculated. We show this with the help of an example.

Consider a region with 15,000 arable land where the production activities outlined in table 1 are found (Example II). Please note that the crop wheat is grown in two activities: standard production with growth regulator (standard wheat or WW.1 in the following) and a production method for which a premium is paid under an agri-environmental program because no growth regulator is used (wheat AEP or WW.2 in the following). In the following, total wheat (WW) will be called *crop wheat*, while the two methods to grow wheat will be called *variant activities* or *variants*. From an agronomic point of view these variant activities are largely identical: they are sown, fertilized, and harvested at the same time using the same machinery. The only differences are that WW.2 receives one spraying less than WW.1, a somewhat lower amount of fertilizer, and produces a lower yield. The differences between these two variant activities and the other activities listed are much more pronounced.

These data are now used to construct a standard PQP model (assuming decreasing yield and constant variable cost, consistent with Howitt and Umstätter). In addition, a scenario is calculated where the agri-environmental premium of 200 DM/ha is no longer paid, whereas the application of the production method WW.2, which is now less profitable, is still possible. In a real-world situation, such a scenario would be highly relevant in understanding the impact of the agri-environmental program. The results of the calculations are shown in table 2.

Abolition of the premium only results in a rather small decrease in the activity level of WW.2 by 11.1%; contrary to what one would expect, only a small part of that area is taken up by WW.1. This shows that the level of substitution between WW.2 and WW.1 is lower than expected, due to the PMP first-order condition that requires all activities to have equal marginal returns to land. This condition treats the same crop under different technologies

Table 2. The Effect of an Abolition of the Agri-Environmental Premium with the Standard PQP Approach (Example II)

		Observed Baseline Situation, with Agri-Environmental Premium for WW.2	Scenario: No Agri-Environmental Premium for WW.2, All Other Factors are Identical to the Baseline Situation	Differences	
				Absolute	Relative (%)
Total gross margin	1,000 DM	20,140	19,384	−756	−3.8
Wheat standard	ha	2500	2557	+57	+2.3
Wheat AEP	ha	4000	3557	−443	−11.1
Wheat total	ha	6500	6113	−387	−5.9
Barley	ha	3000	3216	+216	+7.2
Rape seed	ha	5000	5081	+81	+1.6
Pea for grain	ha	500	590	+90	+17.9

Table 3. Total Wheat Area in Germany, West Germany, Baden-Württemberg, and Wheat Area Grown Without CCC Under the Agri-Environment Program in Baden-Württemberg

		1990	1992	1994	1996	1998
Germany	1000 ha	2,371.1	2,521.6	2,351.9	2,543.5	2,745.9
West Germany	1000 ha	1,622.1	1,633.3	1,510.4	1,619.6	1,730.9
Baden-Württemberg	1000 ha	203.1	198.2	192.6	208.5	218.7
Wheat without CCC in Baden-Württemberg	1000 ha	n.a.	50.9	89.7	116.9	n.a.
Germany		100.0%	106.3%	99.2%	107.3%	115.8%
West Germany		100.0%	100.7%	93.1%	99.8%	106.7%
Baden-Württemberg		100.0%	97.6%	94.8%	102.7%	107.7%

Note: Wheat without CCC in Baden-Württemberg is included in the total values.
Source: Statistisches Bundesamt (1991, 1993, 1995, 1997, and 1999); Landesamt für Flurneuordnung und Landentwicklung Baden-Württemberg.

as equivalent to different crops. However, in reality the connection between the two wheat activities should be closer because they are similar in respect to the integrating factors mentioned above; for example, crop rotation and labor requirements (in terms of time spans used up), which the standard PQP model does not explicitly account for.

There is empirical evidence that the introduction of wheat without CCC in the agri-environmental program of the German Federal State of Baden-Württemberg has largely led to a substitution of one type of wheat activity for another.

Table 3 shows the development of the wheat production area in Germany, West Germany, and the state of Baden-Württemberg. An agri-environmental program, which offers payments if wheat is grown without CCC, has existed in Baden-Württemberg since 1992. There is no comparable measure offered in other parts of Germany. Even though substantial hectares of the wheat have been grown without the use of the growth regulator since the introduction of the program, the development of the total wheat area in Baden-Württemberg compares closely with the overall development for West Germany. Compared to all of Germany (which includes East Germany with a very different farm structure) there is slightly less similarity but still enough to conclude that the empirical data suggest that wheat grown without CCC in Baden-Württemberg was largely a substitute for wheat grown earlier under a different technology.

Thus, the empirical evidence supports the theoretical reasoning. This means that in order to obtain a more plausible reaction from the model, an approach is needed that ensures a closer dependency between the two wheat activities than between other activities.

An Extension to PQP

The extension to PQP starts by denominating the two wheat activities "variant activities" (denoted by subscript v below) and distinguishing them from a total wheat activity which describes the sum of the two variants. Each of these three activities—the two variant wheat activities and the total wheat activity—now needs a separate calibration coefficient. The idea, therefore, is to divide the slope of the marginal gross margin function of each variant activity into two parts. One part depends on the activity level of the variant activity and the other on the activity level of total wheat. The basic linear model, which produces the values that are needed for calibration of the nonlinear function of the PQP model, has the following structure in the extended version

$$\max f(X)$$

where

$$(1) \qquad f(X) = GDB = \sum_i \sum_v (X_{i,v} * DB_{i,v})$$

subject to

$$(2) \qquad \sum_i \sum_v (X_{i,v}) \le \sum_i \sum_v (\hat{X}_{i,v})$$

land constraint (produces λ_{land} in the solution)

$$(3) \qquad \sum_v (X_{i,v}) \le \sum_v (\hat{X}_{i,v}) * (1 + \varepsilon_1)$$

crop constraint (produces λ_i in the solution)

$$(4) \qquad X_{i,v} \le \hat{X}_{i,v} * (1 + \varepsilon_2)$$

constraint that limits activities where there are several activities within one crop (produces $\lambda_{i,v}$ in the solution)

$$(5) \qquad X_{i,v} \geq 0$$

$$(6) \qquad \varepsilon_2 > \varepsilon_1$$

where GDB is the total gross margin, $DB_{i,v}$ is the gross margin of activity i, v in the linear approach, $X_{i,v}$ is the activity level of activity i, where the same crop has more than one variant activity these are distinguished by subscript v, $\varepsilon_2, \varepsilon_1$ are the perturbation coefficients, the small positive numbers (see Howitt), and $\hat{X}_{i,v}$ is the activity level of $X_{i,v}$ actually observed in the baseline situation.

The corresponding dual is

$$\min g(X)$$

where

$$(7) \qquad g(X) = GK = \sum_i \sum_v (X_{i,v}) * \lambda_{\text{land}} + \sum_i \left(\sum_v (X_{i,v}) * \lambda_i \right) + \sum_i \sum_v (X_{i,v} * \lambda_{i,v})$$

subject to

$$(8) \qquad \lambda_{\text{land}} + \lambda_i + \lambda_{i,v} \geq DB_{i,v}$$

$$(9) \qquad \lambda_{\text{land}}, \lambda_i, \lambda_{i,v} \geq 0.$$

A problem with the standard PMP approach is the calibration of the least profitable activity. For consistency it should also be nonlinear but the lack of a shadow price does not allow a calculation of a coefficient for the objective function in a way similar to those of the other activities. When variant activities are added, this problem also appears for the least profitable variant activity. In the primal problem, the number of constraints now exceeds the number of activities. With $\varepsilon_2 > \varepsilon_1$ the constraints on the total crop are more limiting than on the variants. This means that the solution yields a nonzero shadow price on each of the variant constraints except the marginal (of the least profitable variant of each crop). Similarly, the solution yields a shadow price on each of the crop constraints except on the least profitable one. As appendix A shows, an elegant way of dealing with this problem exists. However, as expected, the calibration procedure to the least profitable crop changes all shadow prices of the preceding crops to modified shadow prices (see appendix A).

It is important now to show that a PMP objective function exists which includes the concepts of variants and at the same time fulfills the PMP conditions. It turns out that this task is best achieved by referring to the most generalized form of the PMP objective function[1]

$$(10) \qquad GDB = \sum_i \left[X_i * \left(DB_i + \lambda_i * \left(1 - \frac{X_i}{\hat{X}_i} \right) \right) \right].$$

Based on (10) we suggest this objective function for the extended version

$$(11) \qquad GDB = \sum_i \left(\sum_v \left[DB_{i,v} * X_{i,v} + \lambda_{i,v} * X_{i,v} * \left(1 - \frac{X_{i,v}}{\hat{X}_{i,v}} \right) \right] + \lambda_i * \sum_v X_{i,v} * \left(1 - \frac{\sum_v X_{i,v}}{\sum_v \hat{X}_{i,v}} \right) \right).$$

We now want to show that this function (11) fulfills the two major conditions of PMP: (*a*) the marginal gross margins of each activity are identical in the base line situation and (*b*) the average PMP gross margin in the baseline situation is identical to the average LP gross margin of each activity in the baseline situation.

From equation (11), it can be directly shown that the second condition is fulfilled. In the baseline situation, it is true that $X_{i,v} = \hat{X}_{i,v}$ and hence both $(1 - \frac{X_{i,v}}{\hat{X}_{i,v}})$ and $(1 - \frac{\sum_v X_{i,v}}{\sum_v \hat{X}_{i,v}})$ are equal to zero. The remaining terms in the equation are identical to equation (1).

To show that the second condition also applies, the marginal gross margin function is derived. This is the first partial derivative with respect to any $X_{i,v}$ from equation (11),

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[1] This general form of PMP objective function should be regarded as a neutral form, *neutral* in a sense that it is not explicitly based on increasing cost or declining yields. One obtains this form if one inserts the equations used to calculate the coefficients for the objective function in the respective objective function. For decreasing yields:

$$GDB = \sum_i [(y_i * p_i * (\alpha_i - \gamma_i * X_i) + (PR_i - vk_i)) * X_i]$$

$$\alpha_i = 1 + \frac{\lambda_i}{y_i * p_i}, \gamma_i = \frac{\lambda_i}{y_i * p_i * \hat{X}_i}$$

For increasing cost:

$$GDB = \sum_i [X_i * (y_i * p_i + PR_i - vk_i * (\beta_i + \delta_i * X_i))]$$

$$\delta_i = \frac{\lambda_i}{vk_i * \hat{X}_i}, \beta_i = 1 - \frac{\lambda_i}{vk_i}$$

$$(12) \quad \frac{\partial DB_{i,v}}{\partial X_{i,v}} = DB_{i,v} + \lambda_{i,v} * \left(1 - 2 * \frac{X_{i,v}}{\hat{X}_{i,v}}\right) + \lambda_i * \left(1 - 2 * \frac{\sum_v X_{i,v}}{\sum_v \hat{X}_{i,v}}\right).$$

In the baseline situation $X_{i,v} = \hat{X}_{i,v}$ means that the marginal gross margin for any activity in that baseline situation is

$$(13) \quad \frac{\partial DB_{i,v}}{\partial X_{i,v}} = DB_{i,v} - \lambda_{i,v} - \lambda_i = \lambda_{\text{land}}.$$

This shows that in the baseline situation the marginal gross margins of all activities are equal to the shadow price of the land constraint.

Because the function does not clearly refer to decreasing marginal yields, nor to increasing marginal cost, equation (11) has a disadvantage. For these reasons further transformations of the function are useful in order to come up with a form that is actually operational. These transformations can be found in appendix B. Here we just present the results. Equation (14) is derived from equation (11) and is equivalent to the decreasing yield approach of Howitt

$$(14) \quad GDB = \sum_i \sum_v [X_{i,v} * (y_{i,v} * p_{i,v} * (\alpha_{i,v} - \beta_{i,v} * X_{i,v} - \gamma_{i,v} * \bar{X}_i) + PR_{i,v} - vk_{i,v})]$$

with

$$(15) \quad \bar{X}_i = \sum_v X_{i,v}.$$

Equation (14) is the final form of the extended PQP objective function. From equation (B.11) in appendix B it can be seen how the coefficients $\alpha_{i,v}$, $\beta_{i,v}$, $\gamma_{i,v}$ are calculated

$$(16) \quad \alpha_{i,v} = 1 + \frac{\lambda_i + \lambda_{i,v}}{y_{i,v} * p_{i,v}}$$

coefficient axis intercept

$$(17) \quad \beta_{i,v} = \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v} * \hat{X}_{i,v}}$$

slope coefficient of variant activity level

$$(18) \quad \gamma_{i,v} = \frac{\lambda_i}{y_{i,v} * p_{i,v} * \sum_v \hat{X}_{i,v}}$$

slope coefficient of total crop activity level.

Coefficient $\beta_{i,v}$ in equation (17) is not defined if $\hat{X}_{i,v} = 0$. This problem can be solved by only allowing calculation for nonzero values in the model code.

Results

The results presented from the example model illustrate the potential extension of the method. Table 4 shows the results of the extended model in comparison to the original version. Empirically relevant results that have been derived with the help of this method have been presented elsewhere (Röhm and Dabbert).

The results show that in both versions abolishing the agri-environmental program that pays a premium for the variant wheat AEP leads to a decrease. However, in the extended version this decrease is now much greater than in the original version. Moreover, much of the area set free by the decline of wheat AEP (WW.2) activity is used up by standard wheat (WW.1), as intended with the development of the method. The method produces results that are in line with the empirical evidence and thus performs, in certain cases, better than the original method.

Conclusions

We have presented an extension to the PQP method that has been developed for the practical task of implementing European-style agri-environmental programs into regional models of agricultural production. The method has already been applied in more extended empirical models (Röhm and Dabbert, Röhm). Although the mathematical derivation appears rather complicated, the implementation into a GAMS code simplifies the application considerably because coefficient calculations can become a routine, fully automated procedure. The arguments for applying the extended method are mainly based on more plausible results from an agronomic viewpoint. Thus, we believe that the method could also be useful in other cases, where agronomic considerations play a role, especially if one thinks that a more closely related exchange between two crops (e.g., because they are from the same plant family and susceptible to the same pests) is desired than between these and other crops. We present the extension using the example of agri-environmental programs (or, more

Table 4. Comparison of the Results of the Original Method and the Extended Method for a Scenario Where the Agri-Environmental Program, Present in the Baseline Situation, is now Abolished ($\kappa_K = \kappa_V = 0.10$), Example II

		Baseline Situation	Scenario: Abolishment of the Agri-Environmental Program (It is Still Feasible to Grow the Variant WW.2, But no Premium is Paid)					
			Original Method			Extended Version of PMP		
				Differences			Differences	
				Absolute	Relative (%)		Absolute	Relative (%)
Total gross margin (1,000 DM)		20,140	19,384	−756	−3.8	19,480	−660	−3.3
Standard wheat (WW.1)	(ha)	2500	2,557	+57	+2.3	3,692	1,192	47.7
Wheat AEP (WW.2)	(ha)	4000	3,557	−443	−11.1	2,599	−1,401	−35.0
Wheat total	(ha)	6500	6,113	−387	−5.9	6,291	−209	−3.2
Barley	(ha)	3000	3,216	+216	+7.2	3,117	117	3.9
Rape	(ha)	5000	5,081	+81	+1.6	5,044	44	0.9
Pea for grain	(ha)	500	590	+90	+17.9	549	49	9.7
Shadow price of land	(DM/ha)	702.0	674.0	−28	−4.0	686.9	−15	−2.2

precisely, a specific type with rather high importance). However, similar problems might also occur in other areas of policy and technological development analysis, for instance, in analyzing the supply-side effects of genetically modified crops.

It has to be pointed out, however, that the method can only be applied if the microeconomic optimum conditions can be assumed to be a reasonable approximation of reality. In some instances in the implementation of agri-environmental programs, there are binding budgetary constraints that limit the possibility of the farmers receiving payments. Such situations are not suitable as a baseline situation for calibration using the method demonstrated here. However, there are many real-world instances in which the optimum conditions can be reasonably assumed and where the method can be applied. For the example given in this article, the PQP method is much more suitable as a tool for analysis than a linear programming approach would be. The linear programming model does not allow for a gradual reaction of the model—typically the agri-environment variant stays either in the solution or drops out totally. If a linear programming model delivers solutions between these extremes it is usually due to a deliberate (and very likely arbitrary) decision of the model builder to bound the flexibility of the model in a certain way. This shows that for the type of problem discussed in this article PQP can be a useful method with a performance superior to that of an alternative linear programming approach.

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Appendix A

For the least profitable crop, it can be argued that information on its true shadow price is needed for calibration. This true shadow price should be lower than the shadow price on land because the latter is determined from the average gross margin of the least profitable crop in the linear model and not from the marginal gross margin of the least profitable last hectare of the least profitable crop. If there is heterogeneity within the last crop (e.g., due to different soils) then some information on this heterogeneity is needed, which might be available from soil survey maps or from differing regional land prices. Assuming that this information is available, calibration of the least profitable crop can be elegantly computed by introducing the coefficient κ_K, which is defined as

$$\text{(A.1)} \quad \kappa_K = \frac{\lambda_{\text{land}} - \lambda_{AF}}{\lambda_{\text{land}}}$$

with λ_{AF} being the true shadow price of land and κ_K can also be interpreted as a factor that indicates the maximum variation in the marginal crop around its average gross margin. Using κ_K leads to modified shadow prices for all crops

$$\text{(A.2)} \quad \lambda^*_{i(\text{mod})} = \lambda_i + \kappa_K * \lambda_{\text{land}}.$$

However, this is just a preparatory step, which is indicated by the asterisk. Similarly, we can, in addition, introduce a coefficient κ_V, which indicates the maximum variation of the marginal variant activity around its average gross margin. The smaller the coefficient, the more likely is a flexible exchange between the variant activities of one crop. The introduction of these two factors adds additional data requirements to the method and makes the calculation of modified shadow prices more complex

$$\text{(A.3)} \quad \lambda_{i,v(\text{mod})} = \lambda_{i,v} + \kappa_V * \lambda_{i(\text{mod})}.$$

The shadow price of a variant activity is increased by the amount by which the shadow price of the total crop is decreased because part of the latter is used up for calibration of the least profitable variant.[2] This counteraction can be seen from the last term on the right side of the following equation

$$\text{(A.4)} \quad \lambda_{i(\text{mod})} = \lambda_i + \kappa_K * \lambda_{\text{land}} - \kappa_V * \lambda_{i(\text{mod})}.$$

The shadow price of the total crop is increased by the amount of the shadow price that has been used for calibrating the marginal crop ($\kappa_K * \lambda_{\text{land}}$). Rewriting (22) gives

$$\text{(A.5)} \quad \lambda_{i(\text{mod})} = \frac{\lambda_i + \kappa_K * \lambda_{\text{land}}}{1 + \kappa_V}$$

$$\text{(A.6)} \quad \lambda_{\text{land(mod)}} = \lambda_{\text{land}} - \kappa_K * \lambda_{\text{land}}$$

Consequently, the original shadow price of land is decreased by the amount that has been used calibrating the marginal crop.

If this approach is applied to the problem outlined in the body of the text, resultant shadow prices are shown in table A.1 and figure A.1.

From table A.1 it becomes obvious that the condition derived from the dual formulation in equation (8), namely that the sum of the shadow prices must equal the gross margin in the linear model (for nonzero activities), is fulfilled in the solution. The graphical representation of the shadow prices and the modified shadow prices makes it easier now to understand equations (A.3) to (A.6).

[2] Here, instead of λ_i the variable $\lambda_{i(\text{mod})}$ is used to avoid the last term on the right side of equation (B.10) becoming zero for variants of the marginal crop.

Table A.1. Modified and Original Shadow Prices Resulting From the Extended Linear Model in the Baseline Situation, Example II

Gross Margin	[DM/ha]	Wheat Standard (WW.1) 1,320	Wheat AEP (WW.2) 1,478	Barley 896	Rape Seed 1,570	Pea for Grain 780
Original shadow prices [DM/ha]						
Constraint on variant activity	$\lambda_{i,v}$	–	158	–	–	–
Crop constraint	λ_i	–	540	116	790	–
Resource constraint	λ_{land}	–	–	780	–	–
Sum of shadow prices	$\lambda_{i,v+}\ \lambda_{i+}\lambda_{land}$	1,320	1,478	896	1,570	780
Modified shadow prices [DM/ha] $\kappa_K = \kappa_V = 0,10$						
Constraint on variant activity	$\lambda_{i,v(mod)}$	56.2	214.2	–	–	–
Crop constraint	$\lambda_{i(mod)}$	–	561.8	194[a]	868[a]	78[a]
Resource constraint	$\lambda_{land(mod)}$	–	–	702	–	–
Sum of shadow prices	$\lambda_{i,v(mod)+}\ \lambda_{i(mod)+}\lambda_{land(mod)}$	1,320	1,478	896	1,570	780

[a]Formally, there is a separate value for the variant and the crop. However, because there is only one variant per crop in these cases the total shadow price for the crop is shown.

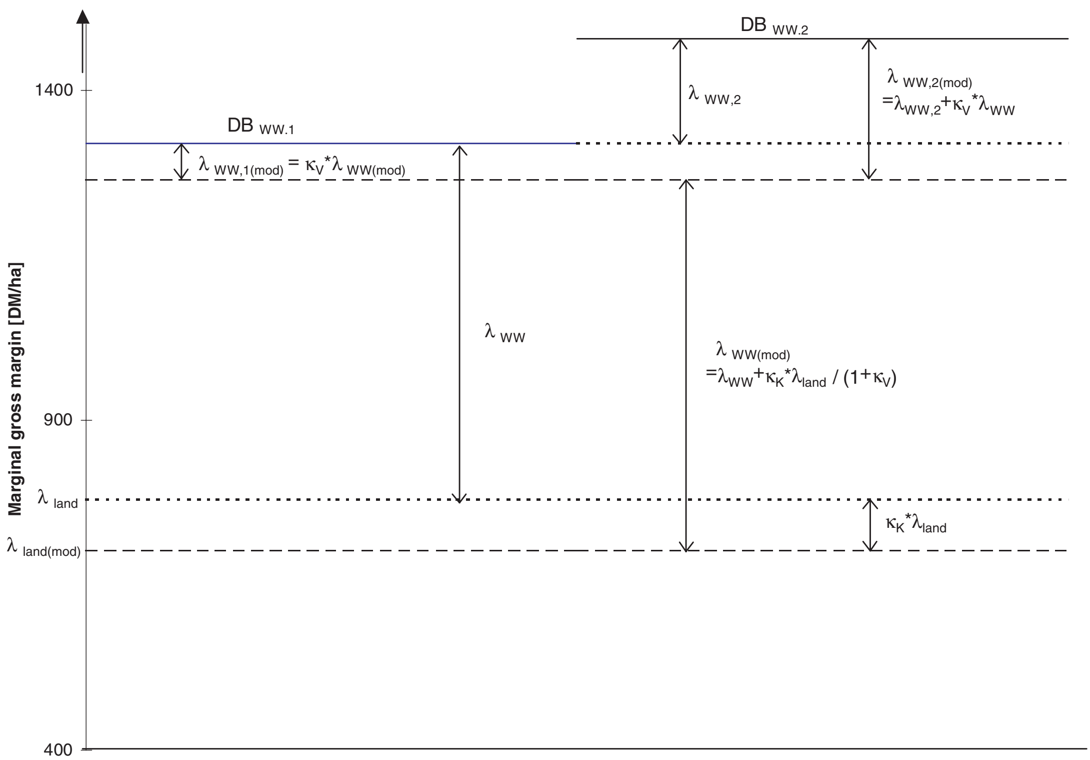

Figure A.1. Calibration of the marginal crop and the marginal variant, process of calculating the modified shadow prices, Example II

Appendix B

The following transformations—starting from equation (11)—aim at developing a function that is equivalent to the decreasing yield approach of Howitt (1995) but takes into account the necessities of agri-environmental programs. For ease of presentation we substitute

$$(15)\quad \sum_v X_{i,v} = \bar{X}_i$$

$$(\text{B.1})\quad \sum_v \hat{X}_{i,v} = \hat{\bar{X}}_i.$$

With these substitutions equation (11) reads

$$(\text{B.2})\quad GDB = \sum_i \left(\sum_v \left[DB_{i,v} * X_{i,v} + \lambda_{i,v} * X_{i,v} * \left(1 - \frac{X_{i,v}}{\hat{X}_{i,v}}\right)\right] + \lambda_i * \bar{X}_i * \left(1 - \frac{\bar{X}_i}{\hat{\bar{X}}_i}\right)\right).$$

In equation (B.2), the summation of the PMP gross margin is done as a first step over all variants v of crop i. Average gross margins of each variant differ from the gross margins $DB_{i,v}$ by the term $+\lambda_{i,v} * X_{i,v} * (1 - \frac{X_{i,v}}{\hat{X}_{i,v}})$ in the linear case. Within the cornered brackets the variants of one crop will all have the same marginal gross margin in the optimal solution. These marginal gross margins will differ from λ_{land} because the second term of the equation has not been included. For the calculation of the total gross margin, after having summed up the variants of one crop, the crop-specific term $+\lambda_i * \bar{X}_i * (1 - \frac{\bar{X}_i}{\hat{\bar{X}}_i})$ is added.

For further transformation the gross margin is written in the long version and the terms within the brackets are transformed as well

$$(\text{B.3})\quad GDB = \sum_i \left(\sum_v \left[(y_{i,v} * p_{i,v}) * X_{i,v} + (PR_{i,v} - vk_{i,v}) * X_{i,v} + \lambda_{i,v} * X_{i,v} - \lambda_{i,v} * X_{i,v} * \frac{X_{i,v}}{\hat{X}_{i,v}}\right] + \lambda_i * \bar{X}_i - \lambda_i * \bar{X}_i * \frac{\bar{X}_i}{\hat{\bar{X}}_i}\right).$$

In the calibration procedure, it is assumed that marginal yields are decreasing and premiums and variable costs are assumed to be linear. They can thus be isolated and put at the end of the equation

$$(\text{B.4})\quad GDB = \sum_i \left(\sum_v \left[(y_{i,v} * p_{i,v}) * X_{i,v} + \lambda_{i,v} * X_{i,v} - \lambda_{i,v} * X_{i,v} * \frac{X_{i,v}}{\hat{X}_{i,v}}\right] + \lambda_i * \bar{X}_i - \lambda_i * \bar{X}_i * \frac{\bar{X}_i}{\hat{\bar{X}}_i}\right) + \sum_i \sum_v [(PR_{i,v} - vk_{i,v}) * X_{i,v}]$$

In the next step, the expression $y_{i,v} * p_{i,v} * X_{i,v}$ is isolated from the summand within the cornered brackets

$$(\text{B.5})\quad GDB = \sum_i \left(\sum_v \left[(y_{i,v} * p_{i,v} * X_{i,v}) * \left(1 + \frac{\lambda_{i,v} * X_{i,v}}{y_{i,v} * p_{i,v} * X_{i,v}} - \frac{\lambda_{i,v} * X_{i,v} * X_{i,v}}{y_{i,v} * p_{i,v} * X_{i,v} * \hat{X}_{i,v}}\right)\right] + \lambda_i * \bar{X}_i - \lambda_i * \bar{X}_i * \frac{\bar{X}_i}{\hat{\bar{X}}_i}\right) + \sum_i \sum_v [(PR_{i,v} - vk_{i,v}) * X_{i,v}].$$

Simplification yields

$$(\text{B.6})\quad GDB = \sum_i \left(\sum_v \left[(y_{i,v} * p_{i,v} * X_{i,v}) * \left(1 + \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v}} - \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v} * \hat{X}_{i,v}} * X_{i,v}\right)\right] + \lambda_i * \bar{X}_i - \lambda_i * \bar{X}_i * \frac{\bar{X}_i}{\hat{\bar{X}}_i}\right) + \sum_i \sum_v [(PR_{i,v} - vk_{i,v}) * X_{i,v}].$$

Now we include the (crop-specific) term $+\lambda_i * \bar{X}_i - \lambda_i * \bar{X}_i * \frac{\bar{X}_i}{\hat{\bar{X}}_i}$ into the cornered brackets. This means that the term will be placed beneath the summation under index v and is thus treated as having variants. This step is possible because the following equation is valid

$$(\text{B.7})\quad \lambda_i * \bar{X}_i - \lambda_i * \bar{X}_i * \frac{\bar{X}_i}{\hat{\bar{X}}_i} = \sum_v \left(\lambda_i * X_{i,v} - \lambda_i * X_{i,v} * \frac{\bar{X}_i}{\hat{\bar{X}}_i}\right)$$

$$(\text{B.8})\quad GDB = \sum_i \sum_v \left[(y_{i,v} * p_{i,v} * X_{i,v}) * \left(1 + \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v}} - \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v} * \hat{X}_{i,v}} * X_{i,v}\right)\right.$$

$$+\lambda_i * X_{i,v} - \lambda_i * X_{i,v} * \frac{\bar{X}_i}{\hat{\bar{X}}_i}\Bigg]$$
$$+\sum_i \sum_v [(PR_{i,v} - vk_{i,v}) * X_{i,v}].$$

Further transformation yields

$$\text{(B.9)} \quad GDB = \sum_i \sum_v \Bigg[(y_{i,v} * p_{i,v} * X_{i,v}) * \Bigg(1 + \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v}} - \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v} * \hat{X}_{i,v}} * X_{i,v} + \frac{\lambda_i * X_{i,v}}{y_{i,v} * p_{i,v} * X_{i,v}} - \frac{\lambda_i * X_{i,v} * \bar{X}_i}{y_{i,v} * p_{i,v} * X_{i,v} * \hat{\bar{X}}_i}\Bigg)\Bigg] + \sum_i \sum_v [(PR_{i,v} - vk_{i,v}) * X_{i,v}]$$

and

$$\text{(B.10)} \quad GDB = \sum_i \sum_v \Bigg[(y_{i,v} * p_{i,v} * X_{i,v}) * \Bigg(1 + \frac{\lambda_i + \lambda_{i,v}}{y_{i,v} * p_{i,v}} - \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v} * \hat{X}_{i,v}} * X_{i,v} - \frac{\lambda_i}{y_{i,v} * p_{i,v} * \hat{\bar{X}}_i} * \bar{X}_i\Bigg)\Bigg] + \sum_i \sum_v [(PR_{i,v} - vk_{i,v}) * X_{i,v}].$$

Finally, the transformations result in a function of the desired form

$$\text{(B.11)} \quad GDB = \sum_i \sum_v \Bigg[X_{i,v} * \Bigg(y_{i,v} * p_{i,v} * \Bigg(1 + \frac{\lambda_i + \lambda_{i,v}}{y_{i,v} * p_{i,v}} - \frac{\lambda_{i,v}}{y_{i,v} * p_{i,v} * \hat{X}_{i,v}} * X_{i,v} - \frac{\lambda_i}{y_{i,v} * p_{i,v} * \hat{\bar{X}}_i} * \bar{X}_i\Bigg) + PR_{i,v} - vk_{i,v}\Bigg)\Bigg].$$

If we substitute the terms here that have been underlined twice by coefficients $\alpha_{i,v}$, $\beta_{i,v}$, and $\gamma_{i,v}$ the equation simplifies to

$$\text{(14)} \quad GDB = \sum_i \sum_v [X_{i,v} * (y_{i,v} * p_{i,v} * (\alpha_{i,v} - \beta_{i,v} * X_{i,v} - \gamma_{i,v} * \bar{X}_i) + PR_{i,v} - vk_{i,v})].$$

Equation (14) is now the final form of the extended PQP objective function.