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Abstract

Biodiesel production sustainability relates to decision making on blending involving economic and environmental criteria. Several feedstocks candidate in European countries biodiesel industry, namely different vegetable origin oils. The present research aims at assessing the impact of technological constraints allocated to the decision objectives taking into account inherent uncertainty. For this purpose chance-constrained programming is used in order to maintain tolerance towards fuel quality.

Keywords: Biodiesel blends, Uncertainty, Chance constrained programming, Shadow prices, Multiobjective programming, GHG emissions

1 Introduction

According the OECD-FAO outlook 2011-2020, biodiesel use in the European Union (EU) will increase by almost 85% over the projection period. Factors such as chemical composition, supply, cost, storage properties and engine performance determine the selection of vegetable oil feedstocks for biodiesel production (Refaat, 2009). Blending of different feedstocks is useful to achieve technical compliance and reduce costs (Gulsen et al. 2014).

In the last years, controversy has been raised around the use of biodiesel due to the high greenhouse gases (GHG) emissions. Some studies have proved that the use of biofuels may not lead to a better environmental performance in terms of GHG

emissions, mainly because of Land Use Change (LUC) and Indirect Land Use Change (ILUC) due to feedstock cultivation (Soimakallio & Koponen, 2011).

The implementation of policies like the European Community Renewable Energy Directive (RED) and the subsequent substitution for food crops has conducted the biodiesel market to take into account not just the costs but also the GHG emissions associated to biodiesel. Besides cost reduction, blending can also help to manage the GHG emissions characteristics of the biodiesel (Olivetti et al. 2014). A bi-objective mathematical programming model to optimize the blend of virgin oils for biodiesel production, minimizing costs and life-cycle GHG emissions was developed by Caldeira et al. (2014) that focused on the feedstocks biodiesel composition, considering it in a deterministic way. The drawback of this approach is that the biodiesel blending process is subject to uncertainty because the feedstock input composition is a stochastic parameter. The type of feedstock affects the fatty acids (FA) ingredients which shape the final biodiesel blend's physical and chemical properties. Consequently the technological specifications that the final blend should conform to are in reality stochastic constraints for the mathematical model.

The consideration of chemical composition uncertainty in blending processes has been considered by several authors using Chance Constrained Programming (CPP) Kumral (2003), Rong and Lahdelma (2007), Sakalli (2011, 2012). CPP is a stochastic programming technique that was first presented by Charnes and Cooper (1959) on a system feasibility in an uncertain environment focusing in the reliability of the system, which is expressed as a minimum requirement on the probability of satisfying constraints (Sahinidis, 2004). By controlling the probability that a constraint may be violated, it adds to the model the flexibility and reality of the stochastic model under consideration (Kampempe, 2012). The application of CCP to develop a blend optimization model for biodiesel production by Gülsen et al. (2014) and Olivetti et al. (2014) showed that besides potential costs reduction, blending can be used to manage GHG emissions uncertainty characteristics of biodiesel.

Moreover, for the biodiesel multiobjective blending problem it is very interesting to know the sensitivity of the solution to technological specifications that are imposed by the authorities. Although the shadow prices of the model could give this type of information, the multiobjective nature of the problem dictates a special approach for assigning the sensitivity of each different objective to the imposed technological constraints.

Therefore, the main goal of the paper is to conduct the sensitivity analysis in a multiobjective biodiesel blending problem taking under consideration the

uncertainty of the chemical composition of the input feedstocks and the required technical specifications.

We first present the methodology that is composed of the deterministic multiobjective blending problem, the method for breaking the shadow prices to the individual objective and the transformation of the technological specifications to stochastic constraints. Finally we apply the above methodology to a case study for a two-objective biodiesel blending problem in Portugal.

2 Methodology

2.1 The deterministic multiobjective blending problem

The general mathematical formulation for a multiobjective blending problem can be written as:

$$
\begin{aligned}
& min\left\{\sum_{i}(c_{1i}\cdot x_i),\sum_{i}(c_{2i}\cdot x_i),\ldots,\sum_{i}(c_{ki}\cdot x_i)\right\} \\
& \text{st.}\sum_{\mathrm{j}}\sum_{\mathrm{i}}\left(\mathrm{q_{ji}}\cdot x_i\right)\ \leq \mathrm{b_p}\ \ \forall\,\mathrm{p} \\
& x_i \geq 0\ \forall\,\mathrm{i}
\end{aligned}
\qquad (1)
$$

where *k* is the set of objectives with cardinality K, *i* is the set of inputs with cardinality I, *j* is the set of ingredients with cardinality J, *p* is the set of required properties of the final blend (cardinality P) which are functions of its ingredient composition, w_k is the weights of the individual objectives, c_{ki} are the individual objective coefficient, x_i is the input quantity, $\mathrm{q_{ji}}$ is the concentration of j-ingredient to i-input, and $\mathrm{b_p}$ is the limit of p-property.

In order to tackle with the multiplicity of objectives we are using the "weighting method". It is classified as an "a posteriori or generation" method for solving multiobjective optimization problems. In this approach at least a representative set of the efficient solutions are generated and presented to the decision maker in order to choose the "most preferred" one (Hwang and Masud cited by Mavrotas 2009).

For the blending problem presented in (1), the mathematical formulation is as follows:

$$
(2)\qquad
\begin{aligned}
&\min \sum_{k}\left(w_k \cdot \sum_{i}(c_{ki} \cdot x_i)\right) \cdot \\
&\text{st.} \sum_{\mathrm{j}}\sum_{\mathrm{i}}\left(\mathrm{q_{ji}} \cdot x_i\right) \geq \mathrm{b_p} \quad \forall\, \mathrm{p} \\
&x_i \geq 0 \;\forall\, \mathrm{i}
\end{aligned}
$$

So as to obtain the Pareto efficient set of solutions we iteratively assign various weight combinations to the objectives and solve the problem.

2.2 Decomposing Shadow Price for the various objective function components

For single objective Linear Programming problems, the shadow price is the objective value change for a unit change on the Right Hand Side value of a certain constraint. In resource allocation problems the shadow price of a resource constraint can be interpreted as the decision maker's maximum value for obtaining an additional unit of that resource. In blending problems the shadow price represents the improvement in the objective function for relaxing a requirement of the final blend.

In multi-objective problems the interpretation of shadow prices can be useful for decisions in policy and industry. Tehrani et al. (2009) presented a methodology to use the shadow price information for joint cost allocation. There is also an application of CO2 emissions allocation in joint product industries (Tehrani, 2006) and an estimation of reduced revenue from reducing nitrogen pollution (Shaik et al., 2002).

However the calculations turns out to be troublesome since the increase of the composite objective value has to be allocated between the individual objectives. We apply a technique presented by McCarl et al. (1996) to decompose the shadow prices.

We are going to express the weighting from of the problem given in equation (2) in matrix notation

$$
(3)\qquad
\begin{aligned}
&\min\left(w_1 \cdot \overrightarrow{C_1} + w_2 \cdot \overrightarrow{C_2} + \cdots + w_k \cdot \overrightarrow{C_k}\right) \cdot \vec{x} \\
&\mathrm{I_{BxJ}} \cdot Q \cdot \vec{x} \geq \vec{\mathrm{B}} \\
&\vec{x} \geq 0
\end{aligned}
$$

where $\overrightarrow{C_k}$ is a 1xI vector containing the objective coefficients for the k-objective , $\vec{x}$ is the Ix1 vector of the decision variables, $\mathrm{I_{PxJ}}$ is an PxJ unity matrix, Q is a JxI matrix containing the $\mathrm{q_{ji}}$ elements, $\vec{\mathrm{B}}$ is a Px1 vector containing the property limits.

The decomposed form of the objective function of the problem that is given in Equation 3 is equal to

(4) $\min \overrightarrow{\mathrm{C_f}} \cdot \vec{x}$, where $\overrightarrow{\mathrm{C_f}} = w_1 \cdot \overrightarrow{C_1} + w_2 \cdot \overrightarrow{C_2} + \cdots + w_k \cdot \overrightarrow{C_k}$

We know that the shadow prices are given by

(5) $\overrightarrow{U_k} = \overrightarrow{\mathrm{C_{fB}}} \cdot B^{-1}$

where $\overrightarrow{\mathrm{C_{fB}}}$ is the objective functions coefficients for the basic variables of the optimal solution and B^{-1} is the basis inverse. From equation (4) we get that

(6) $\overrightarrow{\mathrm{C_{fB}}} = w_1 \cdot \overrightarrow{C_{1B}} + w_2 \cdot \overrightarrow{C_{2B}} + \cdots + w_k \cdot \overrightarrow{C_{kB}}$

where $\overrightarrow{C_{1B}}, \overrightarrow{C_{2B}}, \ldots, \overrightarrow{C_{kB}}$ are the coefficients of the basic variables in the individual objectives context.

Thus the shadow prices are equivalent to

(7) $\overrightarrow{U_k} = \overrightarrow{\mathrm{C_{fB}}} \cdot B^{-1} = w_1 \cdot \overrightarrow{C_{1B}} \cdot B^{-1} + w_2 \cdot \overrightarrow{C_{2B}} \cdot B^{-1} + \cdots + w_k \cdot \overrightarrow{C_{kB}} \cdot B^{-1}$

The $\overrightarrow{C_{kB}} \cdot B^{-1}$ component is the decomposed shadow prices of the k-th objective. It is a 1xP vector, expressing the amount that one unit of increasing the p-th constraint will affect the k-objective's value, considering that we are already at the optimal solution.

So we need to contrive a way to compute each $\overrightarrow{C_{kB}} \cdot B^{-1}$ in order to complete the decomposition of the shadow prices. However as McCarl (1996) states: "linear programming solvers do not generally yield the basis inverse". Furthermore computing the basis inverse from scratch would be computationally equivalent to solving again the LP problem. The algorithm proposed consists of the following steps: Solve the composite problem and save the basis. For each k-th objective set $w_k = 1$ and all other weights equal to zero. Load the saved basis of the composite problem and startup the problem (but make no iterations). The reported shadow prices is the $\overrightarrow{C_{kB}} \cdot B^{-1}$ product. Implementation for the GAMS facility can be found on the aforementioned paper.

This method cannot be directly applied to problems that exhibit degeneracy. Although McCarl et al. (1996) gives a technical solution for computing a consistent decomposition of the shadow prices, degenerate problem are expected to have different positive and negative shadow prices (Gal, 1986) representing a diverse effect on the optimal price of an increase versus a decrease on the right hand side of a constraint. In this case different approaches would be more appropriate as discussed in Ho (2000).

2.3 Introducing uncertainty in the constraints: Chance constraint programming

When the ingredients composition of each input is a stochastic quantity the blending must be altered accordingly.

The chemical composition of each fatty acid for each feedstock is actually a stochastic quantity. In order to deal with this fact we are using a Chance Constraint approach auxiliary to the problem discussed previously. The deterministic constraints are actually replaced be non-deterministic ones, having the following form:

(8) $$P\left\{\sum_{i=1}^{N} a_i\, x_i \le b\right\} \ge 1-\alpha, \qquad x_i \ge 0 \; and \; 0<\alpha<1$$

meaning that the constraint is realized with a minimum probability of $1-\alpha$.

If a_i is normally distributed parameter, $a_i \sim N(\mu_i, \sigma_i^2)$ and all a_i are independent the constraint is converted as follows:

(9) $$P\left\{\frac{\sum_{i=1}^{N} a_i\, x_i - \sum_{i=1}^{N} \mu_i\, x_i}{\sqrt{\sum_{i=1}^{N} \sigma_i^2 x_i^2}} \le \frac{b-\sum_{i=1}^{N} \mu_i\, x_i}{\sqrt{\sum_{i=1}^{N} \sigma_i^2 x_i^2}}\right\} \ge 1-\alpha,$$

Where $\frac{\sum_{i=1}^{N} a_i\, x_i - \sum_{i=1}^{N} \mu_i\, x_i}{\sqrt{\sum_{i=1}^{N} \sigma_i^2 x_i^2}}$ represents a standard normal variate with a mean of zero and a variance of one. Then, the stochastic chance-constraint is transformed into the following inequality:

(10) $$\varphi\left(\frac{b-\sum_{i=1}^{N} \mu_i\, x_i}{\sqrt{\sum_{i=1}^{N} \sigma_i^2 x_i^2}}\right) \ge \varphi\,(K_{1-\alpha})$$

Where $K_{1-\alpha} = 1-\alpha$ and $\varphi\,(\cdot)$ represents the standard normal cumulative distribution function (Sakalli, 2011)

This yields the following nonlinear deterministic constraint:

(11) $$\sum_{i=1}^{N} \mu_i\, x_i + K_{1-\alpha} * \sqrt{\sum_{i=1}^{N} \sigma_i^2 x_i^2} \le b$$

As Seggara et al. demonstrate a linearized, more conservative, equivalent of equation (6) is

$$\sum_{i=1}^{N} \mu_i x_i + K_{1-\alpha} * \sum_{i=1}^{N} \sigma_i x_i \leq b \quad (12)$$

[…]

The existence of more Pareto points in the non-linear form of the problem is an expected outcome. In figure 4 an arbitrary linear programming problem with only linear (a) and only non-linear (b) constraints is drawn.

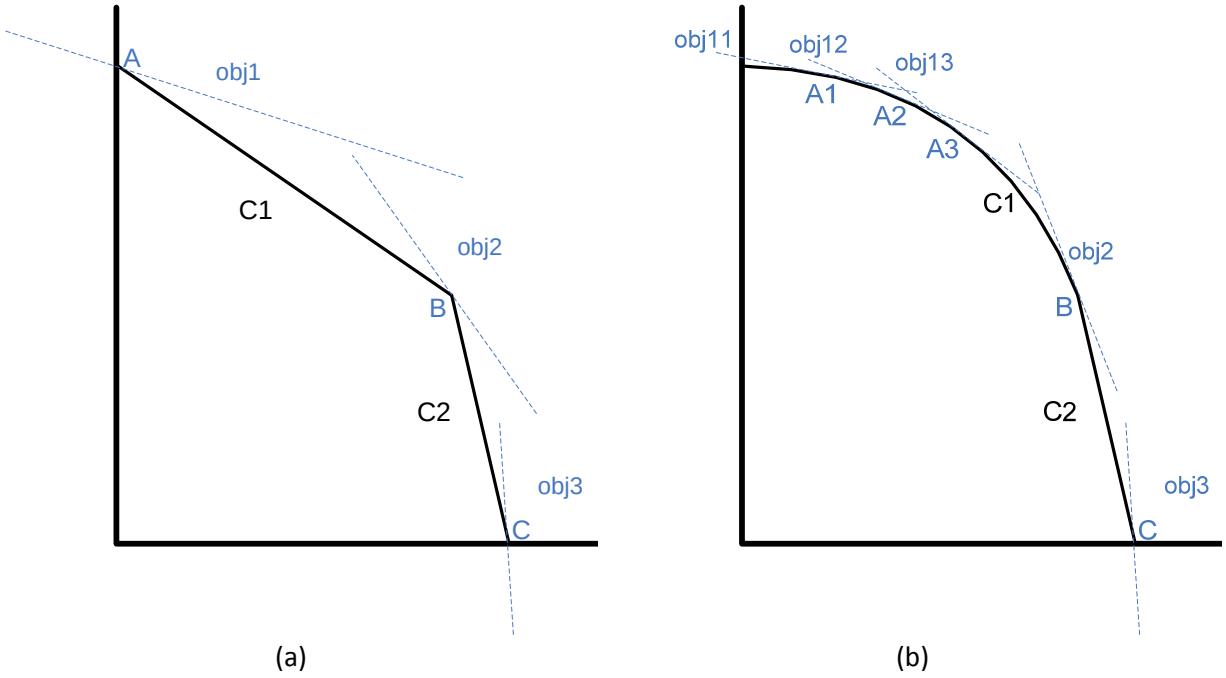

Figure 1, Why a linear constraint model has more Pareto points than its non-linear version

In 4a, the problem has two constraints (C1, C2) and three Pareto points (A,B,C). As the relative weights of the objectives are changing, so is the slope of the objective function (from obj1 to obj2 and finally to obj3). The change of the slope results in the three corners of the feasible region becoming Pareto solutions for different weight combinations. However in 4b, the C1 linear constraints have been converted to a non-linear one and it is obvious why the Pareto points have been multiplied. As the slope of obj1 is changing there is almost a continuous generation of new Pareto points.

3 Case Study

We implement the discussed methodology to the Portuguese case. According to information provided by the Portuguese energy agency (DGEG), in 2012 the main feedstocks used for biodiesel production in Portugal were soya (49%), rapeseed (34%) and palm (14%). For this reason we use those three feedstock in our model. The realization of the model is as follows.

The model involves the minimizing of two objective functions: biodiesel production-feedstock costs and life cycle GHG emissions. Costs are calculated by multiplying the quantity of each three feedstocks (palm, rapeseed and soybean oil) by its market price and GHG emissions by the product of the quantity of each feedstock to its life-cycle emissions per quantity unit.

The prices of the feedstock oils are the average price between November 2008 and November 2013, provided by Index Mundi (2014). For palm and soybean GHG emissions were drawn from Gülşen *et al.* (2012) while for rapeseed from Malça et al. (2014). In order to scale the objectives, the above data was divided by the largest value in each row resulting in the relative price or GHG emissions that is given inside the parentheses in table 1.

Table 1, Case study data

	Feedstock Oil		
	Palm	Rapeseed	Soybean
Price (€/tn)	629 (0.761)	826 (1.000)	753 (0.911)
GHG emission (gCO2e/MJ)	67 (1.000)	48 (0.716)	58 (0.856)

Furthermore the model is subject to technical specification constraints that the final properties of the biodiesel blend shall conform to. Each virgin oil feedstock presents a typical Fatty Acid (FA) profile that influence those final properties. It is generally assumed that FA compositional profiles remain unchanged during conversion of the feedstocks to fuels via transesterification. For this reason, the fatty esters properties are directly related to the FA profile. Structural features such as chain length, degree of unsaturation and branching of the chain determine the fuel properties. Table 1 shows the compositional profile for biodiesel (FAME) from palm, rapeseed and soya adopted from Hoekman et al. (2012). To each FA a nomenclature CX:Y is associated, where X is the number of carbon atoms and Y the number of carbon–carbon double bonds in the FA chain.

Table 2.FA compositional profile (%) for palm, rapeseed and soya adapted from Hoekman

et al. 2012.

Fatty Acid		j	Palm		Rapeseed		Soya	
Common Name	**Nom**	**(FA index)**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**
Caprylic	C8:0	1	0.1					
Capric	C10:0	2	0.1		0.6			
Lauric	C12:0	3	0.25	0.1	0.26			
Myristic	C14:0	4	1.01	0.21	0.08	0.03	0.16	0.12
Palmitic	C16:0	5	42.81	2.87	4.3	1.04	11.54	2.0
Palmitoleic	C16:1	6	0.18	0.06	0.19	0.10	0.24	0.22
Heptadecenoic	C17:1	7			0.10			
Stearic	C18:0	8	4.11	0.97	1.55	0.57	3.97	0.81
Oleic	C18:1	9	41.6	2.28	61.74	3.33	23.61	1.39
Linoleic	C18:2	10	9.62	1.35	21.56	1.82	53.5	1.85
Linolenic	C18:3	11	0.28	0.14	8.46	1.36	6.98	1.11
Arachidic	C20:0	12	0.28	0.10	0.39	0.19	0.39	0.27
Gondoic	C20:1	13	0.17	0.06	1.01	0.54	0.26	0.05
Eicosatrienoic	C20:2	14			0.1			
Behenic	C22:0	15	0.1		0.49	0.27	0.33	0.17
Erucic	C22:1	16			0.41	0.19	0.30	
Lignocric	C24:0	17	0.1		0.1		0.11	0.02
Nervonic	C24:1	18			0.15	0.07		

In the literature, prediction models based on the FA composition are used for the following biodiesel proprieties: density (Den), cetane number (CN), cold filter plugging point (CFPP), iodine value (IV) and oxidative stability (OS) (Giakoumis, 2013, EN 14214, Bamgboye & Hansen 2008, Ramos et al. 2009, Park et al, 2008). These predictions were used as technical constraints in this model. These models have been used in Caldeira et al. (2014) and according to the authors the derived results are in agreement with values found in the literature.

Finally, in order to analyze the proportions of each feedstock in the blend, an additional constrained is added; the sum of the feedstocks shall be equal to unity. We implicitly consider that biodiesel production is fully consumed by the oil refinery industry and that the supply of the feedstocks are unlimited.

So the realized deterministic model is expressed in the following way:

$$min\left\{\sum_i (Pr_i \cdot Q_i)\,, \sum_i (GHG_i \cdot Q_i)\right\}$$

$$s.t$$

(13)

$$PropConst_p + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot q_{i,j}\right) \right) \geq PropLTgt_p \quad \forall\, p \in Plb$$

$$PropConst_p + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot q_{i,j}\right) \right) \leq PropGTgt_p \quad \forall\, p \in Pub$$

$$\sum_i \mathrm{Q_i} = 1$$

$$\mathrm{Q_i} \geq 0 \qquad \forall\, \mathrm{i}$$

where

Sets

i ϵ {'soya','rapeseed','palm'}, the various feedstock oils

p ϵ {DenLB, DenUB, IV, CN, OS, CFPP}, set of properties

Plbϵ {DenLB, CN, OS}, set of properties with lower bound

Pub ϵ {DenUB, IV, CFPP}, set of properties with upper bound

j ϵ {1,2,…,18} , Fatty Acids index

Variables

Q_i, The % of feedstock i in the final blend (%)

Data

Pr_i,the ratio of the price of feedstock I to the most expensive feedstock (number)

GHG_i, the ratio of the GHG emission of feedstock i to the feedstock with the highest GHG emissions (number)

$q_{i,j}$,$\forall_i \in$ F ; $\forall_j \in$ A :The % of FA-j in feedstock i (%)

$PropCoef_{p,j}$: Coefficient of FA-j in the prediction model for property k (units)

$PropCoef_k$: Constant in the prediction model for property-k (units)

$PropGTgt_p$: Target for properties with lower bound (units)

$PropLTgt_p$: Target for properties with upper bound (units)

However, because the FA composition for each feedstock oil ($q_{i,j}$) is actually a stochastic quantity, we will convert the deterministic technical constraints to equivalent stochastic through the chance constraint method. So the constraints are transformed as follows:

(14)

$$P\left\{PropConst_k + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot q_{i,j}\right) \right) \geq PropLTgt_p \right\} \geq 1-\alpha \quad \forall\, p \in Plb$$

$$P\left\{PropConst_k + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot q_{i,j}\right) \right) \leq PropGTgt_p \right\} \geq 1-\alpha \quad \forall\, p \in Pub$$

Where $P(f) \geq 1-\alpha$ is the probability of constraint f to be realized, with a minimum probability of $1-\alpha$. Following the Chance Constrained methodology already discussed, the above constraints are converted back to deterministic ones and are now formed as

(15)

$$PropConst_p + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot \overline{q_{i,j}}\right) \right) - Zvalue \cdot \left(\sqrt{\sum_j \left({PropCoef_{p,j}}^2 \cdot \sum_i \left({Q_i}^2 \cdot {std_{i,j}}^2 \right) \right)} \right) \geq PropLTgt_p, \quad \forall\, p \in Plb$$

$$PropConst_p + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot \overline{q_{i,j}}\right) \right) + Zvalue \cdot \left(\sqrt{\sum_j \left({PropCoef_{p,j}}^2 \cdot \sum_i \left({Q_i}^2 \cdot {std_{i,j}}^2 \right) \right)} \right) \leq PropGTgt_p, \quad \forall\, p \in Pub$$

where the $Zvalue$ corresponds to the test coefficient for Gaussian distribution, $\overline{q_{i,j}}$ is the mean value of $q_{i,j}$ and ${std_{i,j}}^2$ is the standard deviation for $q_{i,j}$. This parameter allows the control of the constraint level and reflects the risk preferences of the user. The linearized version of the above non-linear chance constraints (eq. 12) are:

(16)

$$PropConst_p + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot \overline{q_{i,j}}\right) \right) - Zvalue \cdot \left(\sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot std_{i,j}\right) \right) \right) \geq PropLTgt_p, \quad \forall\, p \in Plb$$

$$PropConst_p + \sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot \overline{q_{i,j}} \right) \right) + Zvalue$$
$$\cdot \left(\sum_j \left(PropCoef_{p,j} \cdot \sum_i \left(Q_i \cdot std_{i,j} \right) \right) \right) \leq PropGTgt_p,$$
$$\forall\, p \in Pub$$

4 Results and Discussion

As already discussed we used the weighting method to explore the multiobjective model. We calculated all the weight combinations for the two objectives in 0.01 steps, a total of 101 points. To make this more clear we give the first and last 5 (Cost , GHG) weight combinations: {0.00, 1.00 / 0.01, 0.99 / 0.02, 0.98 / 0.03, 0.97 / 0.04, 0.96 / 0.05, 0.95 / ……. / 0.95, 0.05 / 0.96, 0.04 / 0.97, 0.03 / 0.98, 0.02 / 0.99. 0.01 / 1.00, 0.00}.

in order to be able to use the shadow price decomposition we proceed with the linearized version of the chance constraint model. In the 95% confidence level (z=1.96) the model was infeasible for all of the weight combinations. This can be attributed to the fact that the linearized model is a much more restricting transformation of the probabilistic constraint, since $\sqrt{\sum_{i=1}^{N} \sigma_i^2 x_i^2} < \sigma_i x_i$. For this reason we reduced the confidence interval to 90% (z value=1.644853).

So, for the linearized version, the Pareto front was as follows:

Table 3. Pareto fron of linearized chance constraint model

Weights range	Cost	GHG
Cost weight∈ {0.00, 0.54}	795.44	50.94
Cost weight ∈ {0.55, 1.00}	776.24	52.79

The blend composition for the mentioned weight combinations is shown in figure 1. The X-axis gives the weight combination and the Y-axis the proportion of each input feedstock in the final blend.

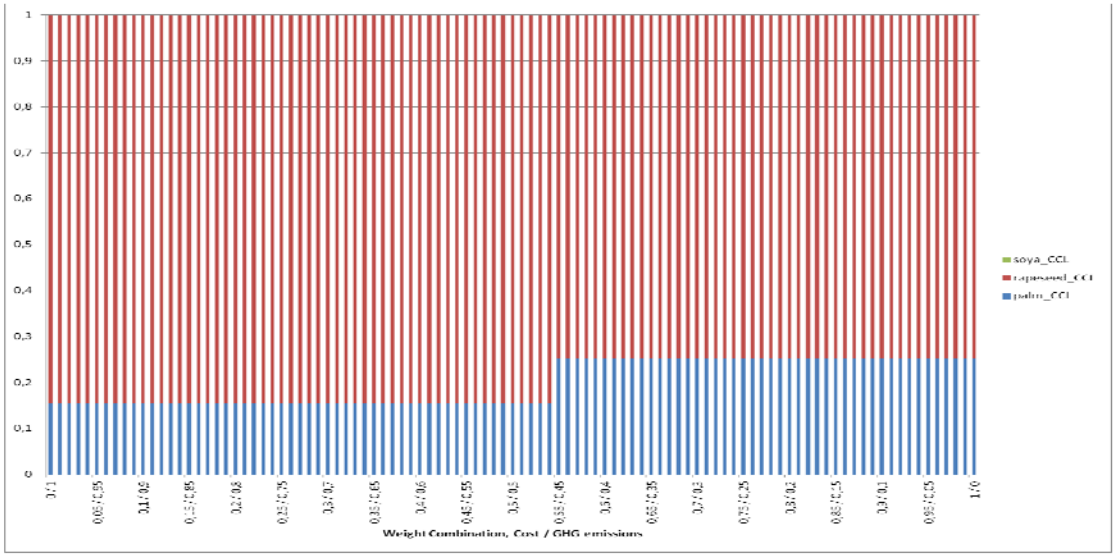

Figure 2, Blend composition for all weight combinations of linearized chance constraint model

We see that only rapeseed and palm is used for making the biodiesel blend.

Afterwards we examined the differences between the linearized CC model and the original non-linear constrained version (non-linearity relationships in the constraints). In figure 2 we show the Pareto front points of the non linear Chance Constraint problem (blue points) along with those of the linearized version (red points). In figure 3 the blend composition of the non-linear model is given.

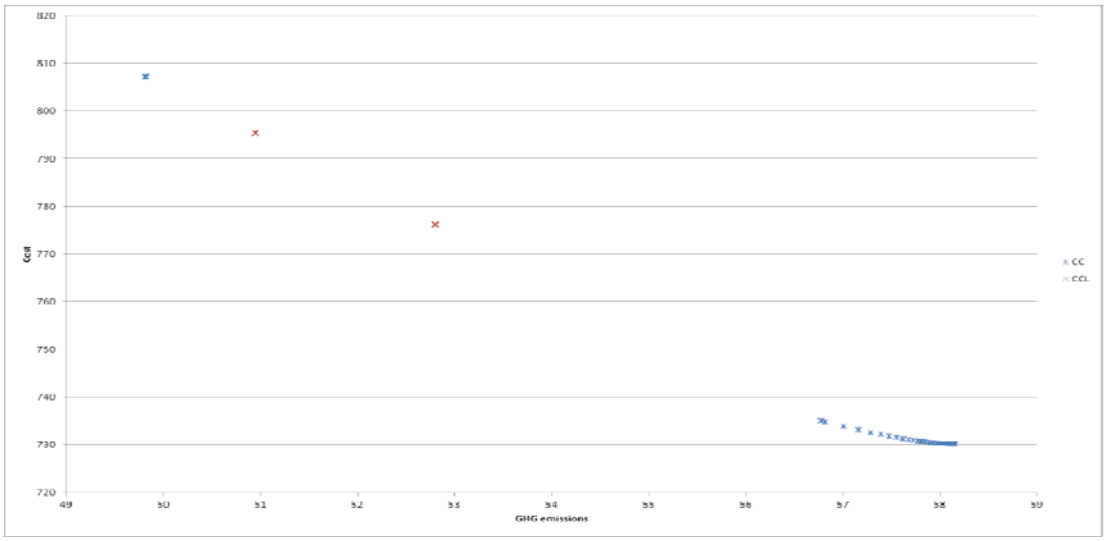

Figure 3, Pareto curves of non-linear and linear chance constraint models

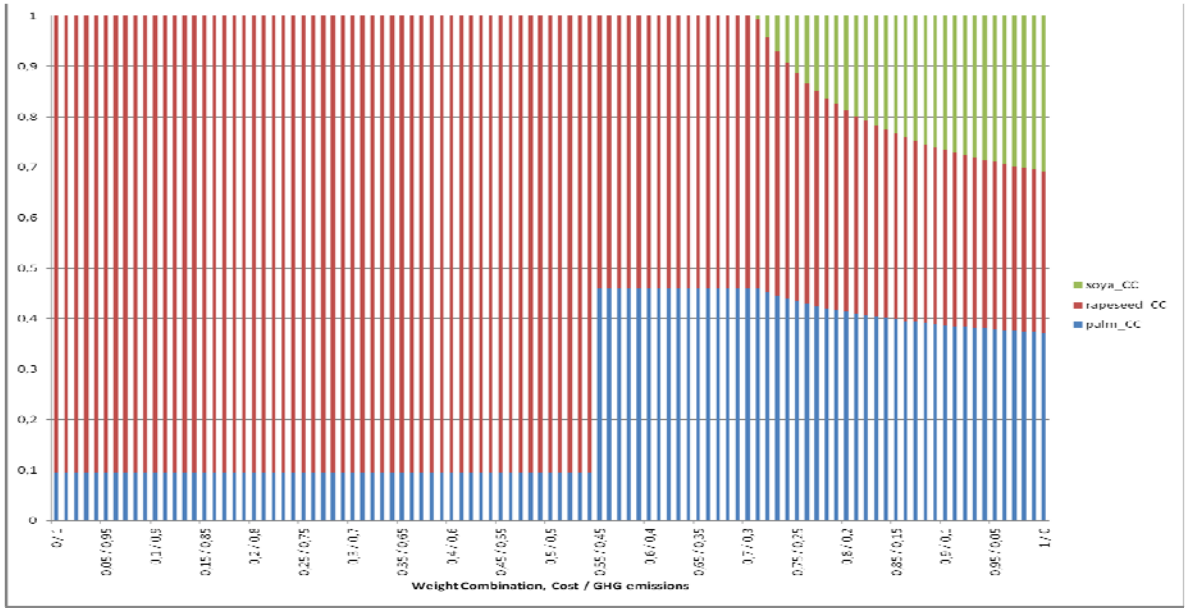

Figure 4, Blend composition for all weight combinations of the original non-linear chance constraint model

Consequently, the linearized version of the CC model retains enough information for the decision maker, since the continuous Pareto front is very fine detailed but the big picture of the Pareto map is not changing dramatically.

Finally we apply the shadow price decomposition discussed in the methodology section and the results are presented in table 4.

Table 4. Shadow prices decomposition

Weight combination	Cost weight∈ {0.00, 0.54}	Upper Bound of Iodine Value ($IV \leq IV_0$)	Cost	+0.00344
			GHG	-0.041
	Cost weight ∈ {0.55, 1.00}	Upper Bound of Cold Filter Plugging Point ($CFPP \leq CFPP_0$)	Cost	-0.0179
			GHG	+0.02128

When cost is more important than GHG emissions (Cost weight ∈ {0.55, 1.00}, bottom two lines) then an increase in the upper bound of CFPP ($CFPP_0$) would result in a decrease (-0.0179) in the Cost component, which has a positive context since the target is to minimize cost, and in an increase in GHG emissions (+0.02128). In the case where GHG emissions are evaluated as more important (Cost weight∈ {0.00, 0.54}), the IV constraint replace CFPP as the binding constraint. If the upper bound of IV (IV_0) was to be increased by one unit then the Cost component of the objective value would increase (+0.00344) while the GHG emissions would decrease by 0.041.

The implications of the above findings are quite important since they can be a guideline for evaluating the efficiency of technical specifications relatively to the cost

and GHG emissions of the biodiesel production process. That could be the case when a new technology that has the potential to alter technological specifications of the input biodiesel oil is under consideration.

5 Conclusions

In the last years, controversy has been raised around the use of biodiesel due to the high greenhouse gases (GHG). The implementation of policies like the European Community Renewable Energy Directive (RED) and the subsequent substitution for food crops has conducted the biodiesel market to take into account not just the costs but also the GHG emissions associated to biodiesel.

Furthermore, the production process is subject to uncertainty because the feedstock input composition is a stochastic parameter. Consequently the technological specifications that the final blend should conform to are in reality stochastic constraints.

Moreover, for the biodiesel multiobjective blending problem it is very interesting to know the sensitivity of the solution to technological specifications that are imposed by the authorities. Although the shadow prices of the model could give this type of information, the multiobjective nature of the problem dictates a special approach for assigning the sensitivity of each different objective to the imposed technological constraints.

In this paper we present the methodology to perform a sensitivity analysis in a multiobjective Chance Constrained blending problem and an application to the case of Portuguese biodiesel blending market.

We find that in almost the whole Pareto front only rapeseed and palm oil feedstocks are used. Regarding the technological constraints, we discover that CFPP (cold filter plugging point) is the limiting factor for cost effectiveness and IV (Iodine Value) for reducing GHG emissions.

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