**AUA Working Paper Series No. 2010-4**
**September 2010**

# Positive multi-criteria models in agriculture for energy and environmental policy analysis

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A newer version of this paper has been published on

International Journal of Multicriteria Decision Making, 1(3), pp. 321-337

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Abstract
Environmental consciousness and accompanying actions have been paralleled by the evolution of multi-criteria methods which have provided tools to assist policy makers in discovering compromises in order to muddle through. This paper recalls the development of multi-criteria methods in agriculture, focusing on their contribution to produce input or output functions useful for environmental and/or energy policy. Response curves generated by MC models can more accurately predict farmers' response to market and policy parameters compared with classic profit maximizing behavior. Concrete examples from recent literature illustrate the above statements and ideas for further research are provided.

Keywords: multi-criteria models, interval programming, supply curves, bio-energy, policy analysis

## Introduction

The increasing consciousness of constraints with regard to the environment within which human activities take place has coincided with the development multi-criteria decision making methods. Agricultural activities are by definition involved in the environment on both the input and the output sides. Natural resources used as inputs, such as soil and water, are no longer considered abundant and infinite reserves. On the output side, beside trade-able products that feed human population, there are "by-products" that harm the environment in a systematic way, stressing ecosystems in various degrees from irreversible damage to serious but manageable degradation, in both developed and developing countries. Universal issues to cope with the move from local to the global levels, starting with water shortages and nitrogen leaching in the seventies have been followed by greenhouse gas emissions as the major concern of the last two decades.

Agriculture may be less important than other sectors in terms of its overall contribution to greenhouse gas emissions, but it has a crucial role to play within a strategy for addressing climate change. Opportunities for mitigating GHGs in agriculture fall into three broad categories according to Smith et al (2008):

i. *Reducing emissions* by more efficiently managing the flows of carbon and nitrogen in agricultural ecosystems. For example, practices that deliver added N more efficiently to crops and managing livestock to make most efficient use of feeds thus emitting less methane $CH_4$)

ii. *Enhancing removals*. Any practice that increases the photosynthetic input of C or slows the return of stored C via respiration or fire will increase stored C (carbon sequestration).

iii. *Avoiding (or displacing) emissions*. Agriculture can produce energy from biomass that can displace fossil fuels, the major contributor to greenhouse gas emissions. Crops and residues from agricultural lands can be used as bio-energy feedstock still releasing $CO_2$ upon combustion, but now the C is of recent atmospheric origin (via photosynthesis), rather than from fossil C. The net benefit of this bio-energy feedstock to the atmosphere is equal to the fossil-derived emissions displaced less any emissions from their production, transport and processing.

To determine the most efficient practices at the farm and the sector level in crop, livestock and energy production, multi-criteria decision-making (MCDM) has been extensively used. In a review of MCDM application in agriculture in 1993, Rehman and Romero stated that "the smooth functioning of an agricultural system involves having to balance biological, technical, economic, private, social, political and environmental criteria, and to resolve conflicts inherent therein". Aforementioned pressures operating on agricultural systems, render the need for an MCDM approach for managing such systems even more imperative than before, and "such applications are now beginning to appear".

The intensive practices in agriculture have remarkable effects on society, the economy and the environment. In the early studies applying MCDM, social and environmental objectives are considered along with the economic objectives of the farm. As social and environmental goals in reality do not belong to the family of objectives of any individual farmer, these studies focus on pointing out feasible and satisfactory activity plans that constitute the so-called efficient frontier. A given solution is Pareto-efficient and, therefore, included in the efficient set, if another solution cannot improve upon it without degrading the performance of at least one objective in that efficient solution, thus the concept of an optimum is rendered meaningless so we search for those solutions which are optimal in a Paretian sense instead. These solutions can be determined by Multi-Objective Programming (MOP) techniques with sequential optimization of several objectives, for instance, in the bi-criterion space applying the Non-Inferior-Set-Estimation (NISE) method. NISE was proposed by Cohon et al. (1979), to analyse the conflict between water quality and income in a river basin planning problem. At the farm level, such methods are used to generate the extreme efficient points and bring about trade-offs to determine economic incentives for the farmer to consume inputs in a rational way in the case of nitrate leaching (i.e. Fernandes-Santos et al., 1993), water for irrigation (Louhichi et al., 1999) and soil erosion (Young et al., 1991).

The rationale for financial incentives to implement environmental measures such as soil and water conservation practices is that farmers would not adopt cropping plans (short term) or farming systems (long term) incorporating these practices to the extent desired by society without cost sharing subsidies. Premises underlying this rationale are that profit maximization is the sole criterion for decision making and farming systems incorporating soil and water conservation practices are less profitable than systems that exclude these practices. Validity of the profit maximization model has been questioned by comparing the selection of farming systems based on profit maximization and on multiple criteria decision-making (MCDM) models (criteria may comprise risk, management difficulty, own labour and working capital minimisation, and/or farm income maximization and others). It is proven that MCDM models better reflect the farmers'decisions with subsequent implications for conservation subsidies (Prato & Hajkowicz, 2001). A non-interactive technique to derive farmers' individual utility function, has been developed by Sumpsi, Amador, and Romero (1993, 1997) and extended by Amador, Sumpsi, and Romero (1998). The surrogate utility function estimated, subject to appropriate constraints, is extensively used to explicit response functions of inputs or outputs joined to crop production through parametric optimization. This way, nitrogen and water demand functions are estimated in various countries and circumstances in Europe (i.e. Berbel and Rodriguez, 1998, Gomez-Limon and Berbel, 2000, Arriaza, Gomez-Limon, and Upton, 2002, Manos et al., 2007).

This methodological advance marked a turning point in the MC literature, transforming mathematical programming models in agriculture by definition of normative nature to positive models. The term 'positive' implies that, as in econometrics, the parameters of the objective function are derived from an economic behavior assumed to be rational given all the observed conditions that generate the initial activity levels. The main difference with econometrics is that the objective function is vaguely defined, not necessarily obeying to a strict theoretical form. Furthermore, MP models do not require a series of observations to reveal the economic behaviour, which as a drawback deprives them of inference and validation tests. Multi-criteria methods manage to transform the objective function so that optimal solutions include not only crop plans on the vertices of the feasible polyhedron but also points on hyper-plans, enabling the model to approach observed levels of activities outperforming its LP counterparts. For this reason farm level models that incorporate multiple goals can be more effective, assisting policy makers in developing more efficient and targeted policy measures, thereby adjusting the existing policy regime.

Multi-criteria surrogate utility functions include at least one risk criterion, thus requiring detailed information at the farm level. As a matter of fact, a comparative study of various methods proved (Arriaza & Gómez-Limón 2003) that the risk criterion ranks second after the gross margin maximisation, one weighted at around 30% and both often amounting over 90%. Usually non-linear risk-related terms are introduced in the objective function seeking efficient diversification among activities as a means of hedging against risk. To implement such models, availability of covariance matrices – which require gross margin time-series of all candidate crops - is fundamental (Hardaker et al., 2004). Consequently, it is fairly difficult to apply these methods to sector or regional models containing numerous farms, thus relevant publications even though theoretically appealing are applied to a limited number of representative farms (Petsakos et al., 2008) or to limited activities or products (Katranidis & Kotakou, 2008). However, the uncertainty element can be taken into account expressing objective function coefficients (gross margins per surface unit) in intervals rather than in crisp values. To specify intervals the sole requirement is a reliable idea of the range of variation of gross margins.

Interval linear programming (ILP) models are equivalent to a specific class of multi-objective (MO) models with objectives generated by the extreme interval values. Consequently, there is a need to select an appropriate criterion to resolve the problem in order to obtain a compromise solution. A conservative criterion well suited to the risk averse attitude of farmers, namely the min-max regret criterion, has been proposed in the literature, along with efficient algorithms. Optimal plans from these algorithms are rather located on feasible polyhedron hyper-plans than on vertices, thus resulting in cropping plans closer to the observed than those resulting from the optimal solution of similar LP models. For this reason, as it is the case with non-interactive MC models, it is interesting from a policy maker's point of view to use such a model to explicit hidden response functions. In fact, supply response functions regarding biomass for energy production have been generated from an ILP specification of French arable agriculture (Kazakci et al., 2007). Supply curves generated by ILP are presumably better estimates of true supply curves improving results of traditional LP models. As in the case of nitrogen and water inputs, it is valuable not only for policy makers but also for investors to approximate the cost of biomass raw materials, and it may assist in the development of bio-energy chains aiming at attenuating the greenhouse effect.

The aim of this paper is to present response curves generated through parametric optimization from non-interactive MC as well as from ILP models. Specific cases in agriculture are described where these functions are compared with their counterparts generated by respective LP models. Differences and consequences for environmental and energy policy are analyzed, pointing out the contribution of MC logic to cope with major contemporary issues. The paper is organized as follows: A concise presentation of the mathematical structure of the MC model is given in the next section. Formal aspects of the "Interval Linear Programming (ILP)" approach and the min-max regret algorithm are presented in section 3. Section 4 details parametric optimization and the generation of response functions. Selected examples of estimated response functions are the focus points of section 5. Concluding remarks and ideas for further research complete the article.

## Non-interactive multi-criteria methodology

Multi-criteria approaches, mainly goal programming and multi-objective programming, are most common in agricultural studies (Piech & Rehman 1993). In most of these early multi-criteria approaches, the goals incorporated in the model and the weights attached to them are elicited through an interactive process with the farmer (Dyer, 1972). This interaction with the farmer and the self-reporting of goals has limitations, since farmers often find it difficult to define their goals and articulate them, feel uncomfortable when asked about their goals, or are often influenced by the presence of the researcher and adjust their answers to what they feel the researcher wants to hear.

A well-known, non-interactive methodology to derive the utility function of each farmer has been devised by Sumpsi et al. (1996) to overcome inconveniences and complications resulting from interaction. The basic characteristic of this methodology is that the farmer's actual and observed behaviour is used for the determination of the objectives and their relative importance. Assume that:

$x$ = vector of decision variables
$F$ = feasible set
$f_i(x)$ = mathematical expression of the $i^{th}$ objective
$w_i$ = weight measuring relative importance attached to the $i^{th}$ objective
$f^*_i$ = ideal or anchor value achieved by the $i^{th}$ objective
$f_{*i}$ = anti-ideal or nadir value achieved by the $i^{th}$ objective
$f_i$ = observed value achieved by the $i^{th}$ objective
$f_{ij}$ = value achieved by the $i$-th objective when the $j^{th}$ objective is optimized
$n_i$ = negative deviation (underachievement of the $i^{th}$ objective)
$p_i$ = positive deviation (overachievement of the $i^{th}$ objective)

The first step involves the definition of an initial set of objectives $f_{1(x)}, \ldots, f_{i(x)}, \ldots, f_{q(x)}$. The researcher can define this initial set of objectives according to previous research and related literature or through preliminary interviews with the farmers. In the second step, each objective is optimized separately over the feasible set. At each of the optimal solutions the value of each objective is calculated and the pay-off matrix is determined (Sumpsi et al. 1996). Thus, the first entry of the pay-off matrix is obtained by:

$$Max f_1(x), \text{subject to } x \in F \qquad (1)$$

since $f_1^* = f_{11}$. The other entries of the first column of the matrix are obtained by substituting the optimum vector of the decision variables in the remaining *q-1* objectives. In general, the entry $f_{ij}$ is acquired by maximizing the $f_j(x)$ subject to $x \in F$ and substituting the corresponding optimum vector $x^*$ in the objective function $f_i(x)$.

The elements of the pay-off matrix and the observed (actual) values for each objective are then used to build the following system of *q* equations. This system of equations is used to determine the weights attached to each objective:

$$\sum_{j=1}^{q} w_j f_{ij} = f_i \qquad i = 1,2,...,q \qquad (2)$$

$$\sum_{j=1}^{q} w_j = 1$$

The non-negative solution generated by this system of equations represents the set of weights to be attached to the objectives so that the actual behaviour of the farmer can be reproduced ($f_1, f_2, \ldots, f_q$). Usually the above system of equations has no non-negative solution and thus the best solution has to be alternatively approximated.

To minimize the corresponding deviations from the observed values, the entire series of L metrics can be used. The $L_1$ criterion that minimizes of the sum of positive and negative deviational variables assumes a separable and additive form for the utility function. Alternatively, the $L_\infty$ criterion according to which the maximum deviation D is minimized can be used. Both criteria are commonly used in agricultural studies, partly because they can be managed through an LP specification. The $L_\infty$ criterion corresponds to a Tchebycheff utility function that implies a complementary relationship among objectives (Amador et al. 1998).

To solve the minimization problem using $L_1$ criterion in order to determine criteria weights (minimization of the sum of positive and negative deviational variables) a goal programming model is specified as shown below:

$$Min \sum_{i=1}^{q} \frac{(n_i + p_i)}{f_i} \quad \text{subject to:}$$

$$\sum_{j=1}^{q} w_j f_{ij} + n_i - p_i = f_i \qquad i = 1,2,...,q \qquad (3)$$

$$\sum_{j=1}^{q} w_j = 1$$

Weights determined by (3) can be used to the multi-criteria utility function. The generic form of the utility function is shown below:

$$u = \max\left\{\frac{w_i}{k_i}\left[f_i^* - f_i(x)\right]\right\} - \lambda \sum_{i=1}^{q} \frac{w_i}{k_i} f_i(x) \qquad (4)$$

$k_i$ is a normalizing factor (for example: $k_i = f_i^* - f_{i*}$).

After estimating the farmer' individual utility function, we maximize it subject to the constraint set and the results compared to the actual values of the *q* goals. This way the ability of the utility function to accurately reproduce farmers' behaviour is checked and the model is validated. Namely, the following mathematical programming problem is solved:

$$MinD - \lambda \sum_{i=1}^{q} \frac{w_i}{k_i} f_i(x) \quad \text{subject to:}$$

$$\frac{w_i}{k_i}\left[f_i^* - f_{i(x)}\right] \leq D \qquad i = 1,2...,q \tag{5}$$

with $\mathbf{x} \in \mathbf{F}$

The utility function structure of model (5) can be modified by varying values of parameter lamda ($\lambda$). When $\lambda$ equals to zero the model turns to a min-max optimization whereas for higher values the additive form of the utility prevails. Intermediate values of $\lambda$ correspond to the augmented Tchebycheff function. The preference structure which provides the solution closest to the actual situation will be considered the utility function consistent with the preferences revealed by the farmer. In other words model (5) with the "correct" $\lambda$ parameter can be used later to perform parametric optimization for generating response curves.

**Uncertainty and Interval Programming**

In mathematical programming models, the coefficient values are often considered known and fixed in a deterministic way. However, in practical situations, these values are frequently unknown or difficult to establish precisely. These days it seems necessary to relax the *certainty assumption* in farm based models incorporating risk considerations of the decision makers, in this case farmers, for two important reasons. Firstly, under the last CAP reform, price and yield variations directly influence gross margins, as no crop specific subsidies exist anymore. Secondly, and more importantly, the sky-rocketed cereal prices of 2007 followed by their collapse in 2008 boosted price volatility. This situation obliges modellers to pay special attention to uncertainty of prices, which combined with the vagaries of nature and the new institutional environment, make farmers very cautious. Interval Programming (IP) has been proposed as a means of introducing uncertainty avoiding data greedy variance-covariance matrices, by proceeding only with simple information on the variation range of the objective function coefficients represented by intervals. We now introduce some definitions and notations and briefly present the formal problem.

*Interval Linear Programming Problem*

Let us consider a Linear Programming (LP) model with *n* (real and positive) variables and *m* constraints: max {cx : c ∈ Γ, x ∈ S} (ILP)

where $\Gamma = \{c \in \Re^n : c_i \in [l_i, u_i], \forall i = 1..n\}$

$$S = \{x \in \Re^n : Ax \le b, x \ge 0, A \in \Re^{m \times n}, b \in \Re^m\}$$

Let $\Pi = \{x \in S : x = \arg\max\{cy : y \in S, c \in \Gamma\}\}$ be the set of potentially optimal solutions and Y be the set of all the extreme objective functions: $\mathrm{Y} = \{c \in \Gamma : c_i \in \{l_i, u_i\}, \forall i = 1..n\}$. In the literature, two distinct attitudes can be observed. The first attitude consists of finding all potentially optimal solutions that the model can return in order to examine the possible evolutions of the system that the model is representing. The methods proposed by Steuer (1981) follow this kind of logic. The second attitude consists of adopting a specific criterion (such as the Hurwicz's criterion, the maxmin gain of Falk, the minmax regret of Savage, etc.) to select a solution among the potentially optimal solutions. Ishibuchi and Tanaka, Inuiguchi and Sakawa and also Mausser and Laguna (1998) proposed different methods with this second perspective. Following this perspective, the next section introduces the selected approach, namely the minimization of the maximum regret approach, and the procedure adopted for its implementation.

*Minimizing the Maximum Regret*

Minimizing the maximum regret consists of finding a solution which will give the decision maker a satisfaction level as close as possible to the optimal situation (which can only be known as a *posteriori*), whatever situation occurs in the future. The farmers are faced with a highly unstable economic situation and know that their decisions will result in uncertain gains. It seems reasonable to suppose that they will decide on their surface allocations *prudently* in order to go through this time of economic instability with minimum loss, while trying to obtain a satisfying profit level. The min-max regret solution procedure is implemented here as proposed in the literature (Inuiguchi and Sakawa, Mausser and Laguna, 1998, 1999). The mathematical translation of this hypothesis that is, the presentation of the formal problem and the algorithm of min-max regret are presented in the following paragraphs.

Suppose that a solution $x \in S$ is selected for a given $c \in \Gamma$. The regret is then:

$R(c,x) = \max_{y\in S}\{cy\} - cx$ and the maximum regret is: $\max_{c\in\Gamma}\{R(c,x)\}$

The *minmax* regret solution $\hat{x}$ is then such that $R_{\max}(\hat{x}) \le R_{\max}(x)$ for all $x \in S$. The corresponding problem to be solved is: $\min_{x\in S}\{\max_{c\in\Gamma}\{\max_{y\in S}\{cy\} - cx\}\}$ *(MMR)*

The main difficulty in solving *MMR* lies into the infinity of objective functions to be considered. Shimizu and Aiyoshi proposed a relaxation procedure to handle this problem. Instead of considering all possible objective functions, they consider only a limited number among them and solve a relaxed problem (hereafter called *MMR'*) to obtain a candidate regret solution. The relaxed *MMR'* problem is:

$\min_{x\in S}\{\max_{c\in C}\{\max_{y\in S}\{cy\} - cx\}\}$ (*MMR'*)

where $C = \{c^1, c^2, ..., c^p\} \subset \Gamma$.

This problem is equivalent to: $\min r$ (*MMR'*)

s.t. $r + c^k x \ge c^k x_{c^k}$, $k = 1, \ldots, p$

$r \ge 0,\ x \in S,\ c^k \in C$

where $x_{c^k}$ is the optimal solution of $\max_{y\in S}(c^k y)$. A constraint of type $r + c^k x \ge c^k x_{c^k}$ is called a regret cut. Let us denote $\bar{x}$ the optimal solution of *MMR'* and $\bar{r}$ the corresponding regret. Since all possible objective functions are not considered in *MMR'* we cannot be sure that there is no $c$ belonging to $\Gamma \setminus C$ which can cause a greater regret by its realization in the future. Hence, we use the following CMR problem to test the global optimality of $\bar{x}$: $\max_{c\in\Gamma}\{\max_{y\in S}\{cy\} - c\bar{x}\}$ *(CMR)*

Observe that the objective function value of CMR represents the maximum regret for $\bar{x}$ over Γ, denoted by $R_{\max}(\bar{x})$. If the optimal solution $x_{c^{p+1}} \in S, c^{p+1} \in \Gamma$ of CMR gives $R_{\max}(\bar{x}) > \bar{r}$, it means that $c^{p+1}$ can cause a greater regret than $\bar{r}$ by its realization in the future and that it has to be considered also in C while solving *MMR'*. So, the regret cut

$r + c^{p+1}x \geq c^{p+1}x_{c^{p+1}}$ is added to the previous constraint set of the *MMR'* to solve it again and obtain a new candidate. The process is iterated until the generated candidate regret solution is found to be optimal by CMR. The difficulty in this resolution process lies in the quadratic nature of the CMR problem. Mausser and Laguna (1998) used their results to formulate a mixed integer linear program equivalent to CMR which is less costly to solve. Thus, in this exercise the equivalent problem mixed-integer formulation is used. This solution procedure idea is summarized by the following algorithm:

*The MinMax Regret Algorithm*

Step 0: $r^\circ \leftarrow 0, k \leftarrow 0,$ choose an initial candidate $\bar{x}$. For the initial regret candidates to start the algorithm, the LP optimal solutions may be used.

Step 1: $k \leftarrow k+1$, Solve CMR to find $c^k$ and $R_{\max}(\bar{x})$ :

If $R_{\max}(\bar{x}) = r^\circ$ then END. $\bar{x}$ minimizes the maximum regret.

Step 2: Add the regret cut $r + c^k x \geq c^k x_{c^k}$ to the constraint set of *MMR′*

Step 3: Solve (*MMR′*) to obtain a new candidate $\bar{x}$ and $\bar{r}$. $r^\circ \leftarrow \bar{r}$. Go to Step 1.

The difficulty in this resolution process lies in the quadratic nature of the CMR problem. Inuiguchi and Sakawa investigated the properties of the minmax regret solution to find a more suitable way to solve CRM. Mausser and Laguna (1998) used their results to formulate a mixed integer linear program equivalent to CMR which is less complex to solve. As Mausser and Laguna (1999) noticed that the complexity of that mixed integer program severely limits the size of problems to be addressed, therefore they suggested to use heuristics. In the problem studied here, uncertain objective function coefficients are in no farm decision making unit more than five. Thus, in our experiments we used this equivalent problem mixed-integer formulation.

Consider the following ILP model to illustrate how the algorithm works and its underlying logic.

| | | |
|---|---|---|
| max | *total gross margin* $c_1x_1+c_2x_2$ | subject to |
| | $x_1 + x_2 \leq 60$ | *land availability* |
| | $70\,x_1 + 25\,x_2 \leq 2000$ | *own labour availability* |
| | $12\,x_1 + 2.5\,x_2 \leq 300$ | *working capital* |

and $x_1, x_2 \geq 0$

where $c_1 \in [7.2, 10.4]$ and $c_2 \in [3, 5.5]$.

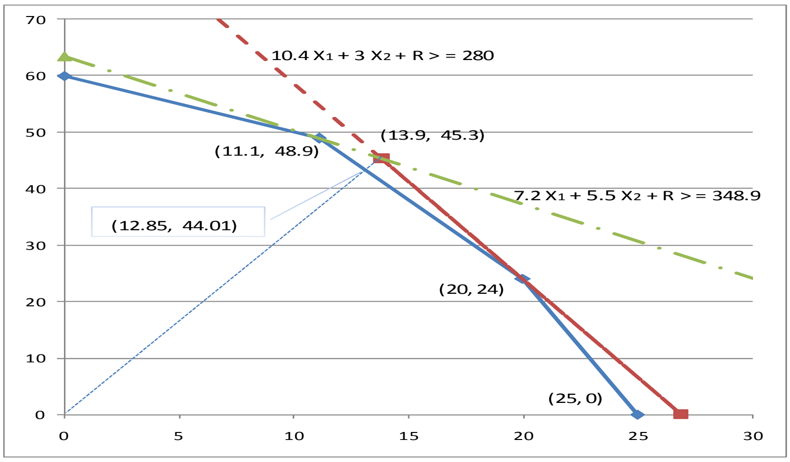


**Figure 1.** Variable space, feasible area and regret cuts.

This problem has a feasible region delimited by the five vertices (Fig. 1). The set of all the extreme objective functions is Y=*{(7.2, 3) ;(7.2, 5.5) ;(10.4, 3) ;(10.4, 5.5)}*. The corresponding MOLP problem, by denoting S the feasible region defined by the constraints, is

$$\upsilon\text{-}max\{\ 7.2\, x_1 + 3\, x_2,\ 7.2\ \ x_1 + 5.5\, x_2,\ 10.4\, x_1 + 3\, x_2,\ 10.4\, x_1 + 5.5\, x_2 : (x_1, x_2) \in S\,\}$$

When considered, separately, each of these objective functions corresponds to a different optimal solution (respectively to Y, (11.1, 48.9) ; (11.1, 48.9); (20, 24) ; (11.1, 48.9)). Along with the vertices (25, 0) ; (60, 0), those solutions constitute basic efficient solutions for the MOLP. The set $\Pi$ of potentially optimal solutions for the ILP (the efficient solutions for the MOLP) is given by convex linear combinations of every adjacent couple of these four solutions.

Let us apply the algorithm to this problem and discuss the results.

*Initialisation* Step 0 : $r^\circ \leftarrow 0$, $k = 0$, Let us choose (11.1, 48.9) as the initial candidate $\bar{x}$.

*Iteration 1* Step 1 : $k \leftarrow 1$, Solving CMR leads to $R_{\max}(\bar{x})$ = 17.78 and $c^1$ is (10.4, 3), $R_{\max}(\bar{x}) \geq r^\circ$ .

Step 2 : The regret cut 10.4 $x_1$+ 3 $x_2$ + r ≥ (10.4*20+3*24) = 280 is then added to the constraint set of the MMR'. In this way, the program will return a new candidate which will try to minimize the potential regret (280 – 10.4 $x_1$

- 3 $x_2$) that might occur if (20, 24) is not selected as a solution. Notice that this is logical considering since we have selected (11.1, 48.9) as the initial candidate solution. The algorithm detects that the objective function for which the other end of the efficient frontier, the point (20, 24), is optimal, may cause an important regret if this turns out to be the real objective function in the future.

Step 3 : (MMR') returns another candidate $\bar{x}$= (20, 24) and $\bar{r}$ = 0. $r° \leftarrow \bar{r}$ . Obviously, this solution minimizes the potential regret (280 – 10.4 $x_1$ - 3 $x_2$) ! It will be tested next.

*Iteration 2* *Step* 1 : $k \leftarrow 2$ , Solving CMR leads to $R_{\max}(\bar{x})$ = 72.89 and $c^2$ is (7.2, 5.5), $R_{\max}(\bar{x}) \geq r°$ .

Step 2: Following the results of step 1, 7.2 $x_1$+ 5.5 $x_2$ + r ≥ (7.2*11.1+5.5*48.9) =348.9 is added as the new regret cut to constraint system of the MMR'. As before, the aim is to take into consideration the last regret possibility that CMR has returned. Now, MMR' will try to return a new candidate by considering both potential greatest regrets (280 – 10.4 $x_1$ - 3 $x_2$) and (348.9 – 7.2 $x_1$ – 5.5 $x_2$).

Step 3: Under these constraints, MMR' returns $\bar{x}$= (12.85, 44.01) and $\bar{r}$ =14.29. r°←$\bar{r}$ . This time the regret is positive and the corresponding solution is not a vertex (see in figure 1).

*Itération 3* Step 1 : $k \leftarrow 3$, Testing the candidate by CMR leads to $R_{max}(\mathbf{x^*})$ = 14.29 = r° . END.

Thus, $\bar{x}$*= (12.85, 44.01) minimizes the maximum possible regret by $\bar{r}$ = 14.29. Graphically this regret equals to the minimum distance between the intersection of regret cut lines (figure 1) juxtaposed by the CMR auxiliary models in the variable space until finding the minimum regret and the feasible frontier. Then the task undertaken by the MMR model basically corresponds to the projection of the regret-cuts-intersection-point to the feasible area. The ILP solution corresponds to the projection of the intersection point to the frontier direction towards point (0,0) in the variable space. It can also be noted that the min-max regret solution is a well balanced solution, an efficient solution of the MOLP, which has been obtained by taking into account extreme cases that might prove “fatal” for a decision maker. The regret optimal solution most likely lies on a side of the feasible area as in figure 1.

**Parametric optimization to estimate response functions**

The typical farm model structure is based upon statements about the short-run physical restrictions to production (resource availability limits), decision rules (profit max, risk aversion etc) and the economic environment within which the farmer operates (imports or quotas, tariffs on certain levels, competitive or monopolistic price formation or guaranteed prices, etc). In mathematical programming models response functions for output or input variables are implicit (Kutcher and Norton, 1979). They can be numerically determined by means of conducting several solutions of the model under variations of market or policy parameters. Response elasticity and associated changes in activity levels, income, employment or environmental pollution may be calculated ex post.

As previously mentioned response functions may concern input demand and/or output supply curves. As far as we know MC response curves have been compared with the traditional LP ones only with regard to output supply. Two case-studies are presented below where response curves of MC models have been measured against the ones generated by LP counterparts, the first concerning non-interactive MC and milk supply (Rozakis et al., 2009) and the second estimating biomass-to-energy supply through Interval Programming minimising max regret (Kazakci et al., 2007). These cases have demonstrated that alternatives to the LP models achieved a higher predictive capacity thus response curves derived better approximate producer behaviour and are fairly suitable for policy analysis.

*Milk supply response functions derived by MC utility function vs. max profit*

Sector or regional response curves generated through parametric optimization are in fact aggregates of individual response curves from decision making units obeying various rationales. In the case of non-interactive MC, values of weights in the surrogate utility function indicate the importance of criteria, thus some individual farms may be driven exclusively by one objective (for instance maximizing profit when its weight value is 100%), others by two objectives (for instance when weight values attributed to gross margin and risk sum up to 100%) and so on. As Herbert Simon noticed (1979) "..empirical data do confirm that supply curves generally have positive slopes... but positively sloped supply curves could result from a wide range of behaviours satisfying the assumptions of bounded rationality rather than those of utility maximisation".

In the area of Etoloakarnania, Western Greece, the aggregate milk supply is estimated by the weighted addition of the individual supply functions. The alternative supply function indicates a lower milk supply at all price levels compared with the LP based curve (fig.2). Using the traditional model to estimate the regional supply would lead to a serious and unrealistic overestimation. Furthermore, the alternative supply function is less elastic than the traditional one in the prevailing price range (0,8-1€/kgr), but more elastic in low price levels. This means that the inclusion of multiple goals in our model smooths the reaction of farmers to price changes, since their behavior is also influenced by other motives.

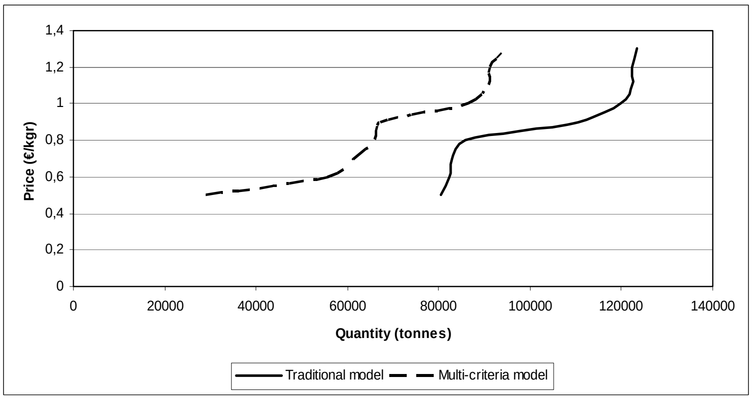


**Figure 2.** Aggregate milk supply (Source: Rozakis et al., 2009)

To conclude on the suitability of the estimated supply function, the authors compared the estimated supply with the actual observed value of milk supply of the Etoloakarnania prefecture. In 2004 the milk supply of the area was 48575 tonnes, while the price of milk was about €0.80 to €0.85 per kgr. The estimated supply function indicates that the supply should be 36% higher. This overestimation is mainly due to the high milk yield of the small farm used in the analysis (120 kgr/ewe) compared to the average milk yield (about 20% higher). If the milk yield was closer to the average then the estimation would be more accurate. On the other hand the supply function estimated using the traditional model yields a supply 75% higher than the actual one which is quite unrealistic.

*Biomass supply response function derived by min-max regret ILP vs. max profit*

Past experience in Europe shows that the biomass raw material cost, defined at the farm level, forms a significant part of the bio-energy cost. Due to an important spatial dispersion of biomass in many productive units (farms) and competition between agricultural activities for the use of production factors (land in particular), strongly dependent on the CAP, the cost estimates of these raw materials raise specific problems. Although it is important that this cost be estimated correctly, three principal difficulties are faced (Sourie, 2002): (a) *the scattering of the resource*, (b) *the competition existing between agricultural activities and non-food crops at the farm level,* and finally (c) *the dependence of raw material costs on agricultural policy measures*.

A bottom-up approach is adopted to reflect the diversity of arable agriculture articulating numerous of farm sub-models in a block angular form, that have neither the same productivity nor the same economic efficiency so that the production costs are variable. Thus, ex-post aggregation helps to relax the *proportionality hypothesis* of LP (Leontief technology) and to avoid problems such as discontinuous response and overspecialization arising in single representative farm models. Similar methodology has been elaborated by Sourie (2002) for the estimation of the supply of energy biomass in the French arable sector. It is postulated that in the short term the farmers choose between food crops $X_c$ and non-food crops $X_d$ so each producer $f$ maximizes gross margin ($g$). Variables $X$ take their values in a limited feasible area defined by a system of institutional, technical and agronomic constraints. When the market price or subsidies linked to price are zero, no surfaces are cultivated by energy crops. From a certain threshold of price value a minimal quantity $q$ of a crop $X_d$ will be produced that is equivalent to the opportunity cost or shadow price by setting down the constraint $y_d\, x_d > q$, where $y_d$ represents the yield of the energy crop $d$. The opportunity cost will vary according to the produced quantities $q$, within each farm but also across farms when the constraint applies to all farms ($\overline{Q_d}$ non-negative quantities of non-food resources). Thus competition with other non-food as well as food crops is taken into account. The relation $p_d^* = J_d(q_d)$ is a (inverse) supply curve of the resource $d$. This approach also leads to an estimate of the agricultural producers' surplus, which is an item of the cost-benefit balance in the bio energy industry (Sourie and Rozakis, 2001).

In the case of Interval Programming the validity of the arable sector model can be checked by comparing optimal activity level outcomes of the LP model with the actual ones in the base year. Then interval linear programming approach using the min-max regret criterion is implemented to investigate if the model's validity can be improved. To evaluate the proximity of the optimal solution $x_k^{opt}$ to the observed activity level $x_k^{obs}$ for the crop $k$, several indicators are suggested in the literature such as the sum of absolute distances of individual crops in the plan, the mean absolute distance, the Theil index, the "similarity" Finger-Kreinin index and others. Thus some farmers maximize gross margin while others demonstrate regret-averse attitude. Revealed preferences by the farm by farm scrutiny lead us to attempt to model arable agriculture assuming different preferences among producers. For each individual farm elementary model a simple rule replaces the objective function with that, between gross margin maximization and min-max regret, performing better in terms of proximity of the resulted crop mix to the observed one. Farm sub-models whose observed behavior is explained better when uncertainty is taken into account in the form of ILP then minimizing maximum regret we adopt hereafter the ILP specification. When the gross margin maximization rule reproduces satisfactorily reality, it is retained as a decision rule and the corresponding farm models remain LP specified. Thus, a hybrid block angular arable sector model is formed with presumably improved predictive ability than the initial LP. This model built by Kazakci et al. (2007) comprises about two hundred cereal oriented farms in a French region, the regional supply curves are determined by aggregating the two hundred individual ones.

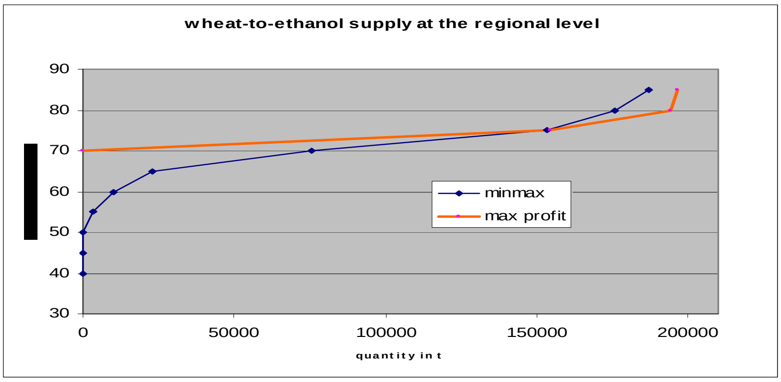


**Figure 3**. Supply curves generated by max profit vs. min-max regret objectives

Different factors affect the relative position of the hybrid min-max regret against classical LP generated supply curves. Not only because the objective function value in terms of total farm gross margin at the min-max regret optimum is lower than the LP optimal value (which results in lower opportunity cost), but also because the energy crop giving relatively stable gross margin is appreciated in the farm compared with other crops with high variability (higher opportunity cost). Depending on the above factors, as well as the interaction with the constraint structure, the min-max supply curves are located to the right of the LP curve up to a certain quantity level. Quantities used in the biofuel industry float in this range, thus we consider that the min-max criterion adoption results in lower opportunity costs of biomass raw material for the biofuel industry. The difference between biofuel estimated cost (agricultural biomass cost plus transport and conversion-to-biofuel expenses) and its market value indicates the minimal subsidy (equivalent to the excise tax exemption) necessary to make biofuels financially viable. Biofuel costs calculated using min-max regret objective functions are 5% lower than respective LP estimated costs broadening the room for tax credit adjustment.

## Concluding remarks

Mathematical programming constitutes a valuable tool for policy making in agriculture especially in the current period of abrupt institutional changes. Agriculture is closely linked to the environment, a cornerstone element in the recent European policy developments. After the achievement of food security that constituted the initial goal of the European Common Agricultural Policy consecutive adjustments tend to compensate the social and environmental positive externalities provided by farmers (subsidies efficiently targeted at public goods that promote agriculture's services to society), gradually eliminating any other form of support. Multi-criteria models have been traditionally used to search compromise plans for farmers to simultaneously satisfy private interests and environmental and social priorities. In this paper the positive function of multi-criteria models is highlighted, demonstrating the superiority of alternative MC models to profit maximisation ones with regard to their predictive capacity. Two categories of Mc models have been presented, namely multi-criteria utility objective function formulated in a non-interactive manner, and secondly models consisting of objective function containing interval coefficients. A thorough presentation of the second category brings about the multi-criteria nature of the ILP when adopting a min-max criterion, in this case minimising the maximum regret caused by embedded uncertainty.

Parametric optimisation assists in rendering explicit, response functions from the above MC modelling structures and the analysis focuses on output supply curves. A comparison of MC derived curves to those generated by classic LPs is done using two case-studies drilled from the recent literature. Advantages for environmental and energy policy analysis are pointed out, especially in the case of biomass raw material for bio-energy production. A good approximation of biomass cost conveys a clear idea of quantities offered by the farmer at given price levels. This information is valuable for predicting the reaction to price changes in different groups of farms, helping policy makers to design more affective and targeted measures. Similarly, the agro-industry is assisted in better estimating its input cost and subsequent profitability, and when supply curves are aggregated for different farm types to design price-discriminating policies.

Further study is required to reveal structural similarities of multi-criteria models with other modelling structures that take risk into account. Additionally we intend to generate response functions based on the same large sample using the aforementioned methods in order to detect differences and finally estimate reliable demand and/or supply curves. In particular, concerning water for irrigation as EU member states are obliged to implement the Water framework, there is urgent need to study the effect of pricing water at a volumetric basis instead of the current flat rate area fee. In Greece, where there are only a handful of basin authorities that already use the volumetric scheme, it is very important to estimate willingness to pay for irrigation water in order to determine price values.

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