Min-max regret versus gross margin maximization in arable sector modeling

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ABSTRACT

A sector model presented in this article, uses about 200 representative French cereal-oriented farms to estimate policy impacts by means of mathematical modeling. Usually, such models suppose that farmers intend to maximize expected gross margin. This rationality hypothesis however seems hardly justifiable, especially these days, when gross margin variability due to European Common Agricultural Policy changes may become significant. Increasing uncertainty introduces bounded rationality to the decision problem so that crop gross margins may be better approximated by interval rather than by expected (precise) values. The initial LP problem is specified as an "Interval Linear Programming (ILP)". We assume that farmers tend to decide upon their surface allocation prudently in order to get through with minimum loss, which is precisely the rationale underlying the minimization of maximum regret decision criterion. Recent advances in operations research, namely Mausser and Laguna algorithms, are exploited to implement the min-max regret criterion to arable agriculture ILP. The validation against observed crop mix proved that as uncertainty increases about 40% of the farmers adopt the min-max regret decision rule instead of the gross margin maximization.

Key words: Interval Linear Programming, Min-Max Regret, Common Agricultural Policy, Arable cropping, France

JEL classification : C61, D81, Q18

Introduction

Since the revision of European Common Agricultural Policy (CAP) in 1992, cereal support prices have gradually decreased, compensated by arable area payments. A proportion of arable land set aside conditions the eligibility for area-aid. The Agenda 2000 reforms to the cereal sector, implemented in year 2002, reinforces area-aid that becomes equal on cereals or oilseeds or land set aside, further reducing the intervention price. Set aside obligation still holds fixed at a minimum 10% of the eligible area. World Trade Organization (WTO) negotiations have triggered the pursuit of the reform to the CAP introducing the idea of production-independent support to cereal farms. The decoupling of subsidies and production along with cereal intervention price of 101.3 €/t has been adopted, ratified by the European Council in June 2003, and is known as the Luxembourg compromise. For cereal-oriented farms, these changes imply an important reduction in prices and in the amount of subsidies allocated for oilseeds, cereals and protein seeds. A prime concern of national governments is, therefore, the evolution of the surfaces allocated to these crops and their welfare implications as well as their impacts to farm income.

Several analyses have considered the welfare implications of CAP, estimating industry supply with short and long-run elasticity and when farmers join the set-aside scheme, as well as impacts of risk-aversion to the farmers' behavior (Froud and Roberts, Roberts, Froud and Fraser). The main disadvantages of the above studies are the single output assumption and the level of aggregation. Wilson, Gibbons and Ramsden point out that at the aggregate level analyses fail to capture how farm-level rotational / cropping mix responses to price and policy signals impact upon supply. This analysis at the farm-level is used to consider the implications at industry level. Even if they empirically predict farm level response to policy and its financial and resource implications using detailed mixed-integer modeling, they only descriptively project farm-level results at the industry level.

The present article illustrates how one can use mathematical programming to adequately represent farmers' response at the industry level by aggregating individual responses and also attempts to improve standard methods in order to take into account of uncertainty. A Linear Programming (LP) model of arable agriculture supply featuring the integration of farm-level decisions with regional aggregates based on a methodology proposed by Sourie et al. is used for this purpose. Mathematical modelling provides a tool to evaluate simultaneous policy interventions in a system, such as arable agriculture, taking into account interrelationships like resource and agronomic constraints as well as synergies and competition among activities. Optimisation models maximising consumer and producer surplus selecting among feasible activity plans have been extensively used in agricultural sector modelling. They allow for a technicoeconomic representation of the sector containing a priori information on technology, fixed production factors, resource and agronomic constraints, production quotas and set aside regulations, along with explicit expression of physical linkages between activities. Assuming rational economic behaviour optimisation results in efficient allocation of production. When the base year optimal crop mix approaches the actual one, then the model can be expected to forecast future changes given specific policy parameters and to reveal impacts of different agricultural policy scenarios on production volume, resource allocation and farm income, eventually evaluating policy efficiency. Moreover, optimisation analysis is theoretically appealing as it generates shadow prices for explicit capacity as well as policy constraints providing valuable information to policy makers. However in most cases it is difficult to replicate actual base year data using the model due to disadvantages inherent to the LP. Those usually mentioned in the literature as cited by Lehtonen are : a) normative optimisation behaviour due to strict neoclassical assumptions, b) aggregation problems, c) ad hoc calibration and validation procedure, d) discontinuous response to changing endogenous conditions, and e) tendency to strong specialisation.

In order to paliate the above deficiencies the arable supply model used is sufficiently detailed to reflect the diversity of arable agriculture, articulating hundreds of farm submodels in a block angular form, that have neither the same productivity nor the same economic efficiency so that the production costs are variable in space. For this reason, ex-post aggregation helps to avoid problems (discontinuous response, overspecialization) arising from the sector representation from a single representative farm, which does not consider heterogeneity phenomena. Consequently, the average cost is not considered equal to the marginal one, as marginal behaviour can be inferred for the sector.

Moreover, the model is calibrated via crop rotation constraints as well as flexibility constraints used to avoid arbitrary and non-explicit adjustments or ad hoc parameter and data manipulations. As Lethonen rightly observes, these manipulations hide the structural deficiencies of the models making them difficult to update, deteriorating the trust of policy makers to model-based economic and policy analysis. Crop rotation constraints applied are specified by agronomic practices proper to the examined cropping system in French conditions. Statistical information on maximum coverage by crop is used to specify the so-called flexibility constraints.

The model contains 216 farms of French representative arable regions (cereal specialized, OTEX 13). The model validation process revealed that in most cases the model could not satisfactorily reproduce the observed behavior. However, even if the fit at the farm level is poor, the aggregate activity levels approach to the actual ones, not reproducing though the observed crop mix. This is mainly due to the penny-switching nature of linear programming that, despite the battery of technical (crop rotation) and flexibility constraints, often results in overspecialization. This can be avoided when non-linear terms are included in the objective function. These terms can express diminishing marginal utility or risk-averse behaviour as in the case of risk-adjusted optimisation models. Utility depends not only on expected gross margin values but on variances of crop prices and yields as well as on some risk aversion coefficients. A review of methods introducing risk in mathematical programming can be found in Hardaker, Huirne and Anderson as well as in Hazell and Norton. One could mention the E-V model as well as its linearised versions such as MOTAD and target-MOTAD and also models based on game theory reasoning such as maximin, minmax, safety-first and other models. For all these models, availability of covariance matrices – that require gross margins of individual crops related to different states of nature or years- is fundamental for efficient diversification among farm activities as a means of hedging against risk. Consequently it is extremely difficult to apply these methods to regional models containing hundreds of farms.

Non-interactive methodologies that attempted to elicit multi-criteria utility functions included at least one risk criterion thus also requiring detailed information at the farm level. As a matter of fact, a survey of these methods state (Gomez-Limon, Riesgo and Arriaza) that the risk criterion ranks second after the gross margin maximisation one in the multi-objective function weighted at around 30%. This is probably the reason that modelling implementations based on the aforementioned methodology are tested at best on a few farms. As our intention is to represent diversity using larger samples we opted for LP with interval objective function coefficients. In other words, the uncertainty element in the objective function (with regard to prices and yields) is taken into account through the introduction of intervals on gross margins per surface unit in the objective function. The normative behavior assumption thus incorporates the bounded rationality element. To specify intervals the sole requirement is a reliable idea of the range of variation of gross margins. These ranges may be assessed at the regional level by crop using FADN or Agricultural Chambers census.

It is proved that interval linear programming (ILP) models are equivalent to a specific class of multi-objective (MO) models with objectives generated by the extreme interval values. By means of experiments, an attempt was successively made to all elementary farm models to check if it is reasonable to represent farmers' behavior using the min-max regret criterion. This decision criterion has been proposed by Savage suggesting that the decision-maker regrets after all about the costs of missing opportunities resulted by their final decision versus other decision actions that could be chosen. It is a regret-averse attitude that could apply to farmers at the moment of deciding on the land allocation to crops for the coming cultivation period. Farm sub-models whose observed behavior is explained better when uncertainty is taken into account in the form of ILP, in other words, for those farmers that minmax regret objective function optimal solution approaches closer to the base year crop mix, we adopt hereafter the ILP specification. When the gross margin maximization rule reproduces satisfactorily reality it is retained as a decision rule and the corresponding farm models remain LP specified. Thus, a hybrid block angular arable sector model is formed with an improved predictive ability that the initial LP. The main drawback is the exponential increase of computing time lapse to solve the ILP as for n interval coefficients the min-max optimization of the ILP requires the solution of 2(n-1) LP and 0-1 models. In this study, farm models contain one-digit objective function terms keeping the model size manageable. This issue is discussed in section 4 where relevant technical information is given.

The paper is organized as follows: A concise presentation of the mathematical structure of the LP model is given in the next section. Formal aspects of the "Interval Linear Programming (ILP)" approach are presented in section 3. The use of the minmax regret criterion within the ILP framework is explained in section 4. The case study and the results thereof are the focus points of section 5. Finally, conclusions and remarks for further research complete the article.

Modeling the Farmers' Behavior: The mathematical formulation

A multi-agent model has been developed to represent the French arable agriculture using linear programming to cope with decision making within different productive units. These units are independent farms in a context of perfect competition. This sector model is built upon a common sort of structure which arises in multi-plant models, known as a block angular structure. One common row is always the objective row whereas diagonally placed blocks of coefficients denote sub-models, each one corresponding to a representative farm. It is supposed that there are no other common rows (or common constraints), that is there is no question of allocation of scarce resources across farms. Therefore optimizing this model it simply amounts to optimizing each sub-problem with its appropriate portion of the objective that is equivalent to treating each farm as autonomous.

The variables of each elementary model represent surfaces to be allocated to the production of various crops by the corresponding farm. For each farm, the number of variables is variable with the total number of crops. The model determines the surface allocation for each elementary model by maximizing the total gross margin of that farm. Hence, the objective function for the f^{th} elementary model is:

$$z^f = \sum_{i \in I} \left(p_i \cdot y_{i,f} + s_i - ch_{i,f}\right) \cdot x_{i,f}$$

where $i \in I$ is the index for crops, p_i is the price for the i^{th} crop, $y_{i,f}$ is the yield for the i^{th} crop on the f^{th} farm, s_i is the subsidy for the i^{th} crop, $ch_{i,f}$ is the production cost for the i^{th} crop on the f^{th} farm, and finally, $x_{i,f}$ is the surface allocated for the i^{th} crop by the f^{th} farm. The sector optimal solution is simply the aggregate of the optimal solutions of all elementary models: total surface allocated to a crop at the regional level is equal to the sum of the surfaces allocated to this same crop in the optimal solutions of each elementary model.

There exist several types of constraints in this model: Land resource constraints, set aside constraints, quotas on demand, rotation constraints, etc. They can be categorized into the following groups defined in detailed mathematical form in the appendix :

Explicit resource constraints (relationship 2, see appendix). Land resource constraints for each farm limit the total arable land to its observed value, irrigation constraints give the observed upper bounds for the irrigable surface. Their parameters are easily determined by historical data and observation.

Set aside constraints (relationships 3, 4). Obligatory set-aside reaches 10% of arable land according to the Berlin agreement. This fraction of land can be either set aside or cultivated by industrial crops.

Demand or market quota constraints (relationship 5). Peas, potatoes and green beans surfaces are constrained by the demand, whereas sugar-beet is restricted by EU common market organization quotas.

Rotation constraints (relationships 6-8). The typical rotation constraint determines crop succession through time. In the agricultural sector modeling practice through mathematical programming these succession restrictions are transformed to spatial restrictions that make the model to respect the farm rotation as it appears in the space at a particular moment (year).

Flexibility constraints (relationship 9, 10). These constraints give upper bounds on surfaces for crops or groups of crops. They reflect implicit constraints (such as availability of labor, technical and technological means, financial resources, etc.) that are not directly represented in the model due to modeling difficulties. For each farm, these upper bounds are a fraction of the total available land for that farm. These fractions are determined for crops and groups of crops over the whole sample. Hence, the same limit applies to all farms. The initial values for these fractions are determined by observing the historical data (see section 5 case-study). Flexibility constraints substitute for some sort of *calibration procedure*.

Uncertainty and Interval Programming

In mathematical programming models, the coefficient values are often considered known and fixed in a deterministic way. However, in practical situations, these values are frequently unknown or difficult to establish precisely. Interval Programming (IP) has been proposed as a means of avoiding the resulting modelling difficulties, by proceeding only with simple information on the variation range of the coefficients. Since decisions based on models that ignore variability in objective function coefficients can have devastating consequences, models that can deliver plans that will perform well regardless of future outcomes are appealing. More precisely, an ILP model consists of using parameters whose values can vary within some interval, instead of parameters with fixed values, as is the case in conventional mathematical programming.

Many techniques have been proposed to solve the resulting problem. Shaocheng studied the case where all the model parameters are represented by intervals and the decision variables are non negative. Recently, Chinneck and Ramadan generalized their approach to the case where variables are without sign restriction. The case which is of greater interest for our purpose is the one where only the objective function coefficients are represented by intervals. This particular problem is the most frequently considered in ILP literature (Bitran, Inuiguchi and Sakawa, Ishibuchi and Tanaka, Mausser and Laguna (1998, 1999a, 1999b), Rommelfanger , Steuer). We now introduce some definitions and notations and briefly present the formal problem.

Interval Linear Programming Problem

Let us consider a Linear Programming (LP) model with *n* (real and positive) variables and *m* constraints. The objective function is to be maximized. Formally:

$$\max \{cx : c \in \Gamma, x \in S\} \qquad \text{(ILP)}$$

where

$$\Gamma = \{c \in \Re^n : c_i \in [l_i, u_i], \forall i = 1..n\}$$

$$S = \{x \in \Re^n : Ax \le b, x \ge 0, A \in \Re^{m \times n}, b \in \Re^m\}$$

Let $\Pi = \{x \in S : x = \arg\max\{cy : y \in S, c \in \Gamma\}\}$ be the set of potentially optimal solutions. Let Y be the set of all the extreme objective functions: $Y = \{c \in \Gamma : c_i \in \{l_i, u_i\}, \forall i = 1..n\}$.To give insight into what the problem becomes when intervals are introduced, we recall the following theorem (Inuiguishi and Sakawa, Steuer):

Theorem

Let us consider the following multiobjective linear programming problem:

$$\upsilon\text{-}\max\{\mathbf{cx} : \mathbf{x} \in S;\ c \in Y\} \qquad \text{(MOLP)}$$

where the υ-max notation stands for the vector maximization. Then, a solution is a potentially optimal solution to (ILP) problem if, and only if, it is weakly efficient to the (MOLP) problem.

Theoretically, this result enables us to mobilize all the tools and concepts of multi-objective linear programming literature, especially to choose/propose suitable solution concepts for (ILP) problem. In the literature, two distinct attitudes can be observed. The first attitude consists of finding all potentially optimal solutions that the model can return in order to examine the possible evolutions of the system that the model is representing. The methods proposed by Steuer as well as Bitran follow this kind of logic. The second attitude consists of adopting a specific criterion (such as the Hurwicz's criterion, the maxmin gain of Falk, the minmax regret of Savage, etc.) to select a solution among the potentially optimal solutions. Rommelfanger, Ishibuchi and Tanaka, Inuiguchi and Sakawa and also Mausser and Laguna proposed different methods with this second perspective. Following this perspective, the next section introduces the approach that we have selected, namely the minimization of the maximum regret approach, and the procedure we adopted for its implementation.

Minimizing the Maximum Regret

Minimizing the maximum regret consists of finding a solution which will give the decision maker a satisfaction level as close as possible to the optimal situation (which can only be known as a *posteriori*), whatever situation occurs in the future. The farmers are faced with a highly unstable economic situation and know that their decisions will result in uncertain gains. It seems reasonable to suppose that they will decide on their surface allocations *prudently* in order to go through this time of economic instability with minimum loss, while trying to obtain a satisfying profit level. This is precisely the logic underlying the minmax regret criterion; i.e. selection of a *robust* solution that will give a high satisfaction level whatever happens in the future and that will not cause regret (Loomes and Sugden, 1982). Therefore, we make the hypothesis that the farmers of the considered region adopt the min-max regret criterion to make their surface allocation decisions. The mathematical translation of this hypothesis for the arable sector supply model was to implement the minmax regret solution procedure proposed in the literature (Inuiguchi and Sakawa, Mausser and Laguna, 1998, 1999a, 1999b). The presentation of the formal problem and the algorithm of minmax regret are presented in the following paragraphs.

The MinMax Regret (MMR) Problem

Suppose that a solution $x \in S$ is selected for a given $c \in \Gamma$. The regret is then:

$R(c,x) = \max_{y \in S}\{cy\} - cx$

The maximum regret is:

$\max_{c \in \Gamma}\{R(c,x)\}$

The *minmax* regret solution $\hat{x}$ is then such that $R_{\max}(\hat{x}) \le R_{\max}(x)$ for all $x \in S$. The corresponding problem to be solved is:

$$\min_{x \in S}\{\max_{c \in \Gamma}\{\max_{y \in S}\{cy\} - cx\}\} \qquad \textit{(MMR)}$$

The main difficulty in solving *MMR* lies into the infinity of objective functions to be considered. Shimizu and Aiyoshi[16] proposed a relaxation procedure to handle this problem. Instead of considering all possible objective functions, they consider only a limited number among them and solve a relaxed problem (hereafter called *MMR'*) to obtain a candidate regret solution. A second problem (called hereafter *CMR*) is then solved to test the global optimality of the generated solution. If the solution is globally optimal, the algorithm terminates. Otherwise, *CMR* generates a constraint which is then integrated into the constraint system of *MMR'* to solve it again for a new candidate solution. This process continues in this manner until a globally optimal solution is obtained. The relaxed *MMR'* problem is:

$$\min_{x \in S}\{\max_{c \in C}\{\max_{y \in S}\{cy\} - cx\}\} \qquad (\textit{MMR'})$$

where $C = \{c^1, c^2, \ldots, c^p\} \subset \Gamma$. This problem is equivalent to:

$\min r$ (*MMR'*)

s.t. $r + c^k x \ge c^k x_{c^k}$, $k = 1, \ldots, p$

$r \ge 0,\ x \in S,\ c^k \in C$

where x_{c^k} is the optimal solution of $\max_{y\in S}\left(c^k y\right)$. A constraint of type $r + c^k x \geq c^k x_{c^k}$ is called a regret cut. Let us denote $\bar{x}$ the optimal solution of *MMR'* and $\bar{r}$ the corresponding regret. Since all possible objective functions are not considered in *MMR'* we cannot be sure that there is no *c* belonging to *Γ \ C* which can cause a greater regret by its realization in the future. Hence, we use the following CMR problem to test the global optimality of $\bar{x}$:

$$\max_{c\in\Gamma}\left\{\max_{y\in S}\{cy\} - c\bar{x}\right\} \qquad \textit{(CMR)}$$

Observe that the objective function value of CMR represents the maximum regret for $\bar{x}$ over Γ, denoted by $R_{\max}(\bar{x})$. If the optimal solution $x_{c^{p+1}} \in S, c^{p+1} \in \Gamma$ of CMR gives $R_{\max}(\bar{x}) > \bar{r}$, it means that c^{p+1} can cause a greater regret than $\bar{r}$ by its realization in the future and that it has to be considered also in C while solving *MMR'*. So, the regret cut $r + c^{p+1}x \geq c^{p+1}x_{c^{p+1}}$ is added to the previous constraint set of the *MMR'* to solve it again and obtain a new candidate. The process is iterated until the generated candidate regret solution is found to be optimal by CMR. This solution procedure idea is summarized by the following algorithm:

The MinMax Regret Algorithm

Step 0: $r^\circ \leftarrow 0, k \leftarrow 0,$ choose an initial candidate $\bar{x}$

Step 1: $k \leftarrow k+1,$ Solve CMR to find c^k and $R_{\max}(\bar{x})$:

If $R_{\max}(\bar{x}) = r^\circ$ then END. $\bar{x}$ minimizes the maximum regret.

Step 2: Add the regret cut $r + c^k x \geq c^k x_{c^k}$ to the constraint set of *MMR′*

Step 3: Solve (*MMR′*) to obtain a new candidate $\bar{x}$ and $\bar{r}$. $r^\circ \leftarrow \bar{r}$. Go to Step 1.

The difficulty in this resolution process lies in the quadratic nature of the CMR problem. Inuiguchi and Sakawa investigated the properties of the minmax regret solution to find a more suitable way to solve CRM. Mausser and Laguna (1998) used their results to formulate a mixed integer linear program equivalent to CMR which is less costly to solve. As Mausser and Laguna (1999a) noticed that the complexity of that mixed integer program severely limits the size of problems to be addressed, therefore they suggested to use heuristics. In the problem studied here, uncertain objective function coefficients are in no farm decision making unit more than 10. Thus, in our experiments we used this equivalent problem mixed-integer formulation.

Let us consider the following ILP model to illustrate how the algorithm works and its underlying logic.

$$\max\ c_1x_1+c_2x_2$$

subject to

$$2.49x_1+2.5x_2\leq 67.43$$

$$3x_1+2x_2\leq 61$$

$$5.5x_1+3x_2\leq 101.5$$

$$1.51x_1+4.5x_2\leq 108$$

$$2.5x_1+0.75x_2\leq 40$$

$$6x_1+x_2\leq 90$$

and $x_1,x_2\geq 0$

where $c_1\in[1,7]$ et $c_1\in[1,3]$.

This problem has a feasible region delimited by the eight vertices (Fig. 1). The set of all the extreme objective functions is Y=$\{(1,1)\ ;(1,3)\ ;(7,1)\ ;(7,3)\}$. The corresponding MOLP problem, by denoting S the feasible region defined by the constraints, is

$$\upsilon\!-\!max\{\ x_1+x_2,\ x_1+3x_2,\ 7x_1+x_2,\ 7x_1+3x_2 : (x_1,x_2)\ \in S\ \}$$

When considered, separately, to each of these objective functions corresponds a different optimal solution (respectively to Y, (7, 20) ; (0, 24) ; (15, 0) ; (13, 10)). Along with the vertices (4.5 , 22.49) ;(10, 15.5) ;(13.75, 7.5), those solutions constitute basic efficient solutions for the MOLP. The set Π of potentially optimal solutions for the ILP (the efficient solutions for the MOLP) is given by convex linear combinations of every adjacent couple of these seven solutions.

Let us apply the algorithm to this problem and discuss the results.

Initialisation Step 0 : $r° \leftarrow 0$, $k = 0$, Let us choose (0,24) as the initial candidate $\bar{x}$.

Iteration 1 Step 1 : $k \leftarrow 1$, Solving CMR leads to $R_{\max}(\bar{x})$ = 81 and c^1 is (7,1) , $R_{\max}(\bar{x}) \geq r°$.

Step 2 :The regret cut $7x_1+x_2 + r \geq (7*15+1*0) = 105$ is then added to the constraint set of the MMR'. In this way, the program will return a new candidate which will try to minimize the potential regret ($105 - 7x_1 - x_2$) that might occur if (15, 10) is not selected as a solution. Notice that this is logical considering since we have selected (0, 24) as the initial candidate solution. The algorithm detects that the objective function for which the other end of the efficient frontier, the point (15, 0), is optimal, may cause an important regret if this turns out to be the real objective function in the future.

Step 3 : (MMR') returns another candidate $\bar{x}$ = (15,0) and $\bar{r}$ = 0. $r° \leftarrow \bar{r}$. Obviously, this solution minimizes the potential regret ($105 - 7x_1 - x_2$)! It will be tested next.

Iteration 2 *Step* 1 : $k \leftarrow 2$, Solving CMR leads to $R_{\max}(\bar{x})$ = 57 and c^2 is (1,3) , $R_{\max}(\bar{x}) \geq r°$.

Step 2 :Following the results of step 1, $x_1+3x_2 + r \geq (1*0+3*24) =72$ is added as the new regret cut to constraint system of the MMR'. As before, the aim is to take into consideration the last regret possibility that CMR has returned. Now, MMR' will try to return a new candidate by considering both potential greatest regrets ($105 - 7x_1 - x_2$) and ($72 - x_1 - 3x_2$).

Step 3 : Under these constraints, MMR' returns $\bar{x}$=(10.42,14.74) and $\bar{r}$=17.32. r°←$\bar{r}$. This time the regret is positive and the corresponding solution is not a vertex (see in figure 1).

Itération 3 Step 1 : $k \leftarrow 3$, Testing the candidate by CMR leads to $R_{max}(\mathbf{x^*})$ = 17.32 = r° . END.

Thus, $\bar{x}$*****=(10.42,14.74) minimizes the maximum possible regret by $\bar{r}$ = 17.32. It can be noted that the minmax regret solution is a well balanced solution, an efficient solution of the MOLP, which has been obtained by taking into account extreme cases that might prove "fatal" for a decision maker.

Case study

Arable farms have been selected out of a sample of 216 located in the cereal production oriented region of the North-East Ile-de-France. Farm Accounting Data Network (FADN) data (orientations OTEX 13) on number of farms, surfaces cultivated, and land set aside concerning the above farm types have been used in this exercise along with detailed data on inputs of arable crops used by each farm (Sourie et al., 2000). The year 1999 has been chosen as the basis because the percentage of land set aside then fixed by the C.A.P. at 10% of the surface of cereals and oil and protein seeds, equals the one fixed by the Berlin agreement for the period 2000-2002. The horizon 2002 is taken as reference for the reason that CAP reform of 1999, is then totally applied, after two years of transition 2000-2001. Profiles of the group are shown in table 1 (first two columns). This sample adequately represents the diversity of arable cropping farms in the Central and Northern France and has been chosen because of its sensibility to policy changes by reason of:

I. Moderate agricultural returns, (gross margin at 500 €/t in 2000) depending strongly on cereal prices and subsidies; subsidies vary around 360 €/ha, that is about double the per hectare returns
II. Crop mix not diversified, with cereals dominating at 60% and oleaginous crops at 25%; yields are average with 7.3 t/ha for wheat and 3.3 t/ha for rapeseed

III. Obligatory land set aside at 10% of the total arable hectareage, as all crops belong to the SCOP group. The farm size is rather big averaging at 150 ha

"Agenda 2000" implemented in 2002 penalized these farms because of the leveling of subsidies of oleaginous crops to those of cereals at about 120 €/ha of oleaginous crops.

In total, 216 elementary models are articulated in a block angular sector model maximizing the total welfare. Each individual farm model had up to 9 variables. Cereals (wheat, winter and spring barley and maize), oleaginous crops (sunflower and rapeseed) and energy rapeseed allowed to grow in set aside land and fallow land. Second wheat production, an artificial variable difficult to observe actual cultivated surfaces, is also included. Constraints include rotation practices (first winter wheat after maize, rapeseed, sunflower or fallow land), set aside obligation. The so called flexibility constraints reflect statistics regarding the arable farm coverage by specific crops or groups of crops. Thus, cereals, oleaginous crops as a group as well as rapeseed and sunflower are limited to a proportion to total arable land (100%, 50%, 30% and 20% respectively), corn and barley are limited by their own historical records (110% of maximum surface observed the previous 3 years) whereas energy crops coverage in practice does not exceed half of the land set aside. Flexibility constraint right hand side limits can be reconsidered if necessary and after some iterations they can be adjusted respecting statistical trends so that the model results approach as closely as possible to the reality.

The validity of the arable sector model has been checked by comparing optimal activity level outcomes of the model with the actual ones. To evaluate the proximity of the LP solution x_k^{opt} to the observed activity level x_k^{obs} for the crop *k*, we used the following distance measure:

$$M_1^{opt}\left(x^{opt}\right) = \frac{L_1\left(x^{opt}, x^{obs}\right)}{TotalLand} = \frac{\sum_i \left|x_i^{opt} - x_i^{obs}\right|}{\sum_i x_i^{obs}} \qquad (14)$$

As shown in table 1 concerning the cereal oriented region, rape-seed for food and energy as well as sunflower cultivated surfaces are underestimated whereas cereals are overestimated. The difference in absolute value between the observed production levels and the optimized allocations (in other words, the distance between the two solutions using a L_1 metric) is approximately 1.8 million ha. The total arable land considered being 7.8 million ha, the relative distance (the difference between the two solutions in absolute value divided by the total arable land) is about 20%. The fit is usually better at the aggregate level than at the farm level, as compensatory effects across farms counteract, making the model results approach the observed crop mix. As a matter of fact at the elementary farm level, distances become more important: the relative average distance by farm is about 37% with standard deviation of 22% (illustrated by cumulative distribution of the "LP opt" line in figure 2).

Hence, the need for further calibration of the model is clear. In all evidence, such variations can occur for two reasons: an inaccurate specification of the feasible region of the model or an inaccurate specification of the objective functions. In this exercise, we assume that the feasible region of each elementary model adequately represents the allocation possibilities of the farmers. Let us note that the observed solutions for each farm have been verified to be feasible in the corresponding model. Regarding the objective function specification, it is reasonable to suppose that, in a relatively stable environment, farmers will base their decisions on average prices. The LP model is originally designed under this very assumption: objective function coefficients (the gross margins per crop) are calculated based on the 1993-1997 price and yield averages. However, in the present context, with subsequent CAP reforms that downgrade subsidy stability factor in the formation of gross margin, the natural uncertainty about yields combined with an increasing uncertainty about prices enlarge the gross margin variation range. Table 2 illustrates variability of gross margins for crops observed in the sample due to yield and price variations in different policy contexts. Beside the initial situation in year 2000, gross margin variability is estimated for the revised C.A.P. implementing the flat rate subsidy (4-5 columns in table 2), and the mid-term review decoupling measure (6-7 columns in table 2). Calculations are performed using data of the year 2002 in order to facilitate comparisons of impacts. Total uncertainty can be represented by the range determined by $\mu \pm 2\sigma$, where μ is the mean value and σ the standard deviation of the gross margin distribution. Slightly decreased variability for cereals while increasing for oleaginous crops is observed after the implementation according to the Berlin agreement. As expected significant increases are noted throughout all crops when subsidies are decoupled from production. The subsidy bulk inflow does not stabilize any more gross margin at the crop level, nevertheless it is received as a lump sum contributing to the farm income. However, even if a significant part of the farm income is assured, at the moment of decision on which crop to cultivate the farmers cannot ignore such huge variability especially concerning corn, barley and oleaginous crops. Thus, we opted for investigating the problems that may arise because of a possibly inaccurate specification of the objective functions. In other words, an implicit assumption is that the objective function coefficients, which correspond to crop gross margins per hectare, are perceived by farmers as imprecise numbers rather than crisp values of expected gross margins. Therefore, they will be represented in the model by intervals transforming the original LP to an interval linear programming problem.

The interval linear programming approach with the minmax regret criterion objective function has been implemented to investigate if the model's validity can be improved. The GAMS software is used to implement the proposed minmax regret algorithm using the linear and integer programming modules of the CPLEX solver (Brooke et al., 1998). Gross margin intervals have been used in the model for crops that appear in table 2, so that the number s of interval-valued coefficients can be up to 9. For the initial regret candidates to start the algorithm, we used the LP optimal solutions.

Results and discussion

The principal effect of the ILP approach with the minmax regret is: when the difference between the gross margins is relatively small, the minmax regret approach gives more "balanced" solutions, more so when the interval coefficients get larger. In fact, as the intervals get larger, the gross margins for different crops start to overlap or, if they already have an intersection, this increases. It then becomes more difficult for the farmer to anticipate which crop will be more profitable. Hence, the min-max regret approach tends to return more and more balanced solutions as the size of the intervals increase. Figure 3 illustrates that point, in a cereal farm (with $M_1^{opt} = 34\%$) where, at the LP optimal solution, wheat is selected at the expense of spring barley and energy rapeseed. In the minmax regret crop mix, wheat surface is decreased whereas spring barley and rapeseed on set aside appear again approaching drastically to the reality ($M_1^{\min\max} = 14\%$). A detailed discussion on this point is presented by Kazakci and Vanderpooten (2002). Across the sample, the effects of the min-max regret approach on the proximities to the observed crop mix obtained at the microscopic level are considerable: for about 38% of the farms, the relative distance ($M_1\left(x^{\min\max}\right)$) of the minmax regret solution to the corresponding observed solution is smaller than the relative distance of the LP's optimum solution to the observed one. The opposite is true for the 18% while both objective function specifications give identical solutions for the rest of the farms. Concerning the improvement in the proximities to the observed solutions, the worst proximities ($\max\left(M_1^j\right)$) obtained for these 38% of the farms provide an average improvement of 10% with respect to the LP's proximities (see cumulative distribution illustrated by bars in figure 2).

Thus some farmers maximize gross margin while others demonstrate regret-averse attitude. Revealed preferences by the farm by farm scrutiny lead us to attempt to model risk behavior at the aggregate level assuming different preferences among producers. For each individual farm elementary model a simple rule replaces the objective function with that, between gross margin maximization and min-max regret, performing better in terms of proximity of the resulted crop mix to the observed one. When both criteria performance is equivalent, maximization of profit is selected by default. This way we end up with a hybrid model with two possible objective function specifications for each farm. This model has by definition a higher predictive capacity than the initial LP. As a matter of fact the relative distance at the aggregate level decreases from 20% to 16% for the hybrid model, the aggregate crop mix appearing in figure 4.

The hybrid model will be used to evaluate different policy scenarios, more specifically the recent midterm revised CAP against the CAP implemented in 2002 according to the Berlin agreement. The mid-term review of the CAP features the decoupling of subsidies from production aiming at the same time at the redistribution of the expenditure to the benefit of the rural development budget at the expense of the CAP pillar one. In practical terms concerning the arable sector new measures can be summarized in the following propositions:

- No subsidies per crop
- Environmental set aside
- Energy crops in direct competition with food crops
- Subsidy for the greenhouse effect mitigation of 45€/ha exclusively to energy crops

We checked this policy scenario against the base case (Berlin agreement in the year 2002) using the hybrid model in order to assess the impacts to the French arable sector and provide to policy makers with useful intuitions. The results confirm actual trends, with increased acreage of wheat at the expense of oleaginous crops, namely rapeseed and sunflower. Rapeseed surface is increased though in land set aside, with fallow land practically disappearing (CAP 2002 vs. conditions 2000). The simulation of the midterm revision of CAP applying decoupling does not alter drastically the crop mix, except that it reinforces the above trends (mid-term review 2002). Environmental set-aside is restored to 50000 ha whereas rapeseed for energy purposes decreases to half the surfaces cultivated in the period 1999-2000 (figure 5). This is expected as energy crops are now exposed in direct competition with food crops and the carbon subsidy does not compensate for losses due to cultivation on higher opportunity cost land than that set aside under the previous CAP regimes.

Conclusions

The aim of this study was to improve the representative capacity of sector supply model, a linear programming model intended to reproduce the behavior of the farmers with respect to their surface allocations to various cultures and to study the impacts of the policy changes on cultivated surfaces. The uncertainty has been modeled by the introduction of interval valued parameters at the objective function level. The resulting model from this approach is an "Interval Linear Programming Model".

Within this framework, we considered 216 elementary linear programming models corresponding to the farms specializing in cereal production. It was assumed that farmers' behavior could be represented using the min-max regret criterion. To test this hypothesis, the min-max Regret (MMR) algorithm was implemented for each of the elementary models. The aim of the algorithm is to find the solution minimizing the maximum regret for a linear programming model with objective function coefficients in the form of intervals. Experiments with various sets of intervals were performed.

Analysis of the results and the comparison with the optimal solutions of the LP for the elementary models showed that the MMR approach had a character which softened the often abrupt nature of the linear programming, for which the least difference between the unitary margins implies the exclusion of the least profitable crop. In many cases, the MMR approach gave better balanced and distributed solutions, and this more so when the overlapping of the interval profits for various crops increased. We also observed that our hypothesis was only partially true. Although some improvements were achieved, the proximities obtained by the MMR approach were not always satisfactory enough to support that the farmers decide on their surface allocations according to the logic of min-max regret. Thus the profit maximizing attitude is retained in 62% of the farms so we ended up with a hybrid block angular model with two possible objective function specifications for each farm (block). This model has by definition a higher predictive capacity than the initial LP. As a matter of fact the relative distance at the aggregate level decreases from 20% to 16%.

The hybrid model has been used to evaluate different policy scenarios, more specifically the recent midterm revised CAP against the CAP implemented in 2002 according to the Berlin agreement. Results confirm actual trends due to the application of the Berlin agreement CAP and demonstrate cereals increase versus oleaginous crops as well as rapeseed for biodiesel decrease versus fallow set aside.

References

Amador F., J.M.Sumpsi, C.Romero. (1999). A non-interactive methodology to assess farmers' utility functions: An application to large farms in Andalusia. *European Review of Agricultural Economics* 25:92-109

Bitran G, (1980). Linear multiple objective problems with interval coefficients. *Management Sciences* 26:694-705.

Brooke A., D. Kendrick, A. Meeraus, and R. Raman. (1998). *GAMS, A User's Guide*. GAMS Development Co.

Chinneck J. W. and K. Ramadan (2000). Linear programming with interval coefficients. *Journal of Operations Research Society* 51:209-220.

Froud J., and D. Roberts (1993). The welfare effects of the new CAP cereals regime: a note. *Journal of Agricultural Economics*, 44:496-501.

Gomez-Limon J.A., L.Riesgo, and M.Arriaza, (2002). Agricultural risk aversion revisited: A multicriteria decision-making approach, Paper presented at EAAE Xth Congress, Zaragoza.

Hardaker J.B., R.B.M. Huirne, and J.R. Anderson. (1997). *Coping with Risk in Agriculture*, CAB International.

Hazell P. and R.D. Norton. (1986) *Mathematical programming for economic analysis in agriculture*. New York: Macmillan Co.

Inuiguchi M. and M. Sakawa. (1995) Minmax regret solutions to linear programming problems with an interval objective function. *European Journal of Operations Research* 86:526-536.

Ishibuchi H. and H. Tanaka. (1990) Multiobjective programming in the optimization of the interval objective function. *European Journal of Operations Research* 48:219-225.

Kazakçi A. O. (2001). Programmation linéaire par intervalles pour étudier l'offre agricole dans la région Poitou-Charentes, représentation de l'incertitude sous formes des intervalles dans la fonction objectif du modèle MAORIE. Série Mémoires et Thèses 35, INRA, Thiverval-Grignon, FRANCE.

Kazakci A.O., and D. Vanderpooten. (2002). Modelling the uncertainty about crop prices and yields using intervals: The min-max regret approach. In Rozakis & Sourie (eds.), *Options Méditerannéenes*, Special Issue *'Comprehensive modeling of bio-energy systems'*, A-48:9-22.

Lehtonen, H. (2001). Principles, structure and application of dynamic regional sector model of Finnish agriculture. PH. D. Thesis, Agrifood Research Finland Economic Research (MTTL), Publications 98.

Loomes G., R. Sugden. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *Economic Journal* 92:805-824.

Markowitz H.M. (1959). *Portfolio selection: efficient diversification of investments*, New York, John Wiley & Sons Ltd..

Mausser H. E. and M. Laguna. (1998). A new mixed integer formulation for the maximum regret problem. *International Transactions of Operations Research* 5:389-403.

____. (1999). A heuristic to minimax absolute regret for linear programs with interval objective function coefficients. *European Journal of Operations Research* 117:157-174.

____. (1999). Minimizing the maximum relative regret for linear programs with interval objective function coefficients. *Journal of the Operations Research Society* 50:1063-1070.

Roberts D., Froud J., and R.W. Fraser. (1996) Participation in set-aside: what determines the opting in price? *Journal of Agricultural Economics* 47-1:89-98

Rommelfanger H. (1989). Linear programming with fuzzy objectives. *Fuzzy Sets and Systems* 29:31-48.

Shaocheng T. (1994). Interval number and fuzzy number linear programming. *Fuzzy Sets and Systems* 66:301-306.

Shimizu K. and E. Aiyoshi. (1980) Necessary conditions for min max problems and algorithms by a relaxation procedure. *IEEE Transactions on Automatic Control* 25:62-66.

Sourie J.-C., G. Millet, E. Kervegant, and P. Bonnafous. (2001). Incidences de l'agenda 2000 sur l'offre de céréales, d'oléagineux et de protéagineux : applications du modèle MAORIE. *Etudes Economiques* 38, INRA, Thiverval-Grignon, France.

Steuer R. (1981). Algorithms for linear programming problems with interval objective function coefficients. *Mathematics of Operations Research* 6 :333-349.

Williams P.H. *Model Building in Mathematical Programming*, 4th ed., John Wiley & Sons Ltd., 1999.

Wilson P., J. Gibbons and S. Ramsden. (2003). The impact of cereal prices and policy on crop rotations and supply response. *Journal of Agricultural Economics* 54-2 :313-323.

APPENDIX. Agricultural sector model specification

Indices

$u \in U$	crop index, (c=1 for wheat, 2: wheat monoculture, 3: wheat after peas, 4: wheat in set aside, 5: barley, 6: winter barley, 7: corn, 8: fresh peas, 9: rape-seed, 10: sunflower, 11: peas, 12: potatoes, 13: sugar beets, 14: green beans, 15: sugar beet-ethanol, 16: wheat-ethanol, 17: rapeseed-ester, 18: land set aside)
$c \in C \subset U$	index for the subset of food crops, $C=\{1, \ldots, 14\}$
$d \in D \subset U$	index for the subset of energy crops, $D=\{15, 16, 17\}$ ($\lvert D \rvert = m$)
$i \in I \subset U$	index for the subset of food crops based upon which land set aside is calculated, $I=\{1, \ldots, 11\}$
$h \in H \subset U$	index for crops demand quota, H= $\{8, 12, 13, 14\}$
$t \in T \subset U$	index for crops preceding wheat, $T=\{7, 8, 9, 10, 12, 13, 14, 15, 17, 18\}$
$g_1 \in G_1$	index for crops that belong to group 1, $G_1= (\{1\text{-}6\}, \{9, 10\}, \{13, 15\}, \{9\}, \{10\})$
$g_2 \in G_2 \subset U$	index for crops that belong to group 2, $G_2= (\{8\}, \{11\}, \{5\})$
$f \in F$	index for farms
$k \in K$	index for agronomic constraints

Parameters

$g_{c,f}$	gross margin for food crop c grown on farm f (€/ha)
p_d	price at the farm gate for energy crop d (€/t)
$y_{d,f}$	yield of energy crop d grown on farm f (t/ha)
s_d	subsidy paid to farmers for energy crop d (€/ha)
$c_{d,f}$	production cost for energy crop d on farm f (€/ha)
γ	subsidy to land set aside (€/ha)
w_f	multiplier used to scale up arable land of farm f to the national level
σ_f	total arable land available on farm f (ha)
$\sigma_{1,f}$	land available on farm f for sugar-beet for sugar production (ha)
θ	fraction of arable land that must be set aside (for 1998: 10 % of total land with cereal, oil and protein seeds)
π_k	maximum fraction of land permitted for crops included in agronomic constraint k

Decision Variables

$x_{c,f}$	area allocated to food crop c on farm f (ha)
$x_{d,f}$	area allocated to energy crop d on farm f (ha)

$$\max \sum_{f \in F} \sum_{c \in C} g_{c,f} x_{c,f} + \sum_{f \in F} \sum_{d \in D} \left(p_d y_{d,f} + s_d - c_{d,f} \right) x_{d,f} + \gamma x_{18,f} \qquad (1)$$

subject to:

Land resource constraints
$$\sum_{u \in U} x_{u,f} \le w_f \sigma_f \qquad \forall f \in F \qquad (2)$$

Set aside constraints:
$$\sum_{d \in D} x_{d,f} + x_{18,f} \ge \theta_{\min} w_f \left(\sigma_f - x_{i,f} \right) \qquad \forall f \in F \qquad (3)$$

$$\sum_{d \in D} x_{d,f} + x_{18,f} \le \theta_{\max} w_f \sigma_f \qquad \forall f \in F \qquad (4)$$

Quotas on demand
$$x_{h,f} \le \bar{x}_{h,f} \qquad \forall h \in H, \ \forall f \in F \qquad (5)$$

wheat set-aside
$$x_{4,f} \le x_{15,f} \qquad \forall f \in F \qquad (6)$$

Rotation wheat
$$x_{1,f} \le \sum_{t \in T} x_{t,f} \qquad \forall f \in F \qquad (7)$$

Rotation wheat-peas
$$x_{3,f} \le x_{11,f} \qquad \forall f \in F \qquad (8)$$

Agronomic limits 1
$$\sum_{u \in G1} x_{u,f} \le \pi_{g1} w_f \sigma_f \qquad \forall g1 \in G1, f \in F \qquad (9)$$

Agronomic limits 2
$$x_{g2,f} \le \pi_{g2} w_f \sigma_f \qquad \forall g2 \in G2, f \in F \qquad (10)$$

Non-negativity constraints:
$$x_{u,f} \ge 0 \qquad \forall u \in U, f \in F \qquad (11)$$

Figure 1. Decision Space in the Example and Regret Cuts.

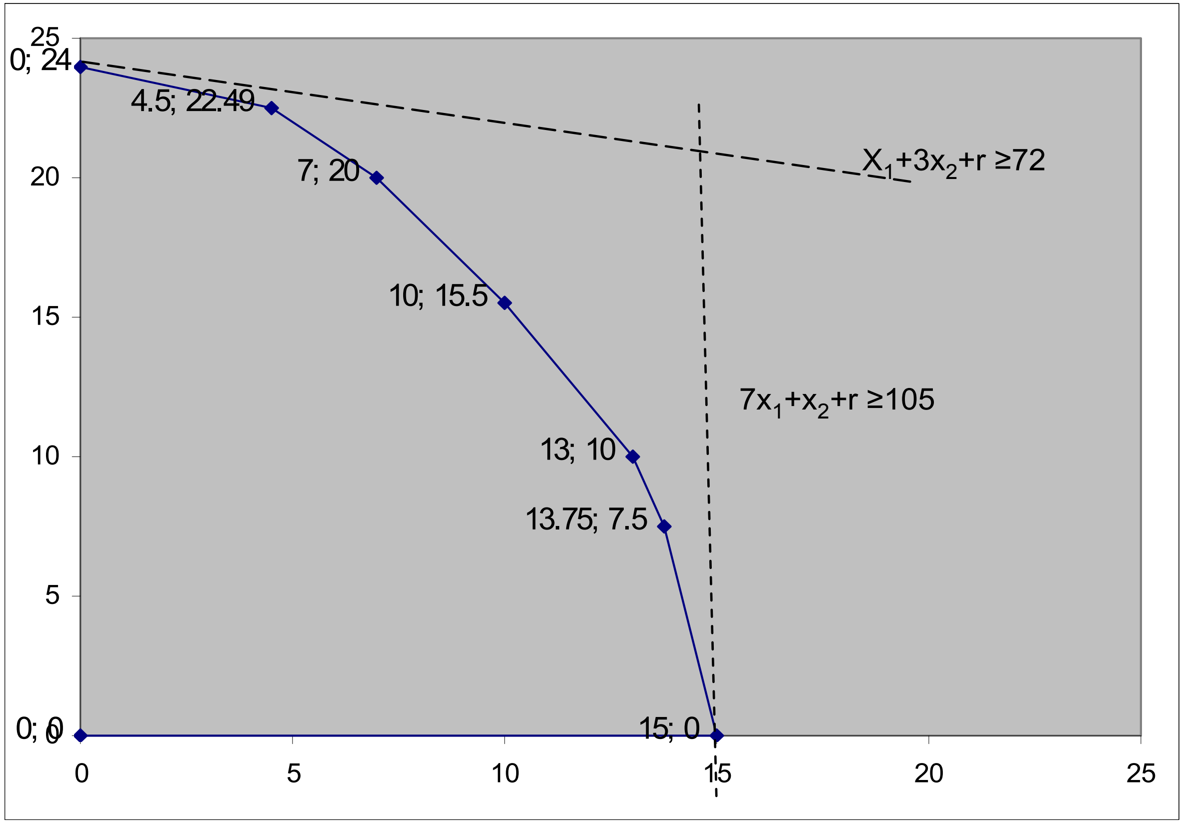

Figure 2. Cumulative Distribution of Relative Distances

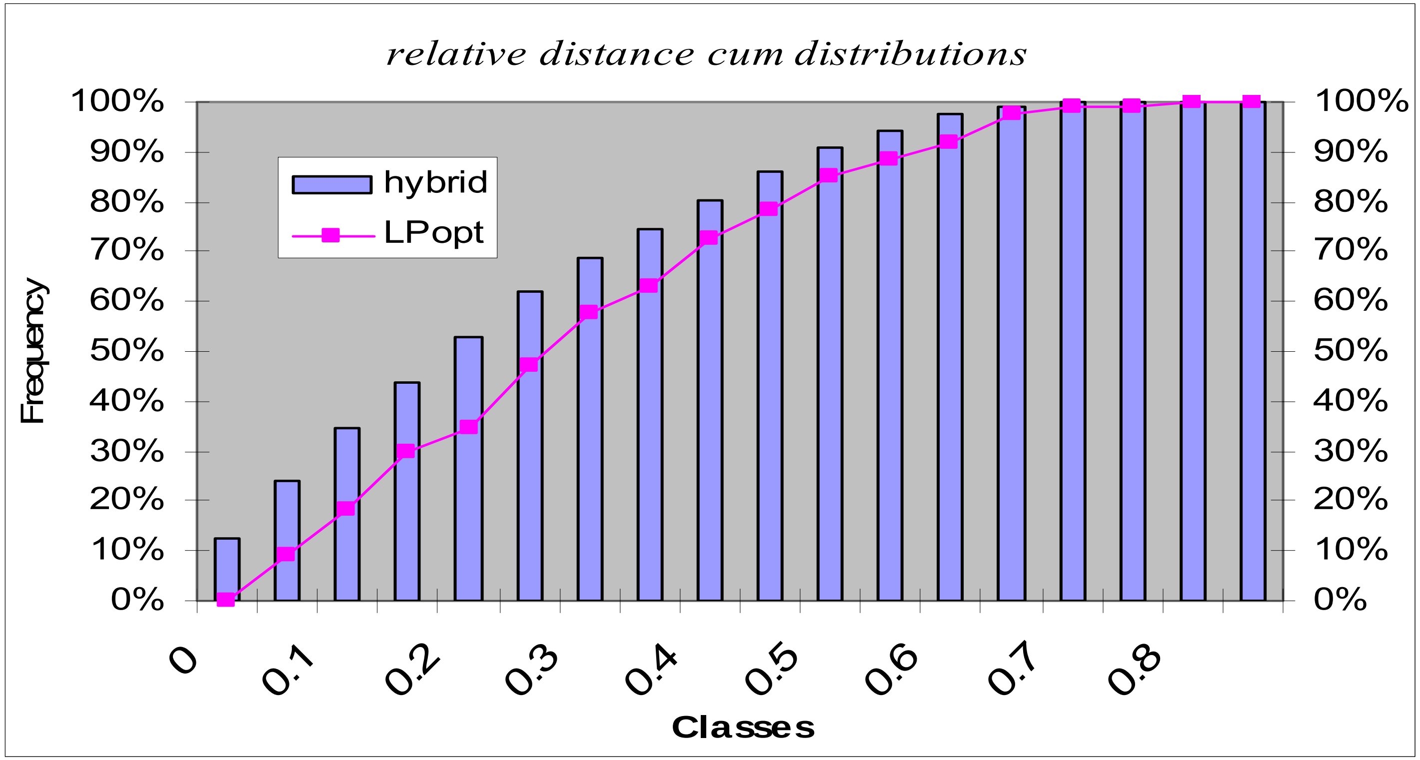

Figure 3. Comparison of the Minmax Regret with the Observed and the LP Solutions for a Typical Farm (surfaces in ha).

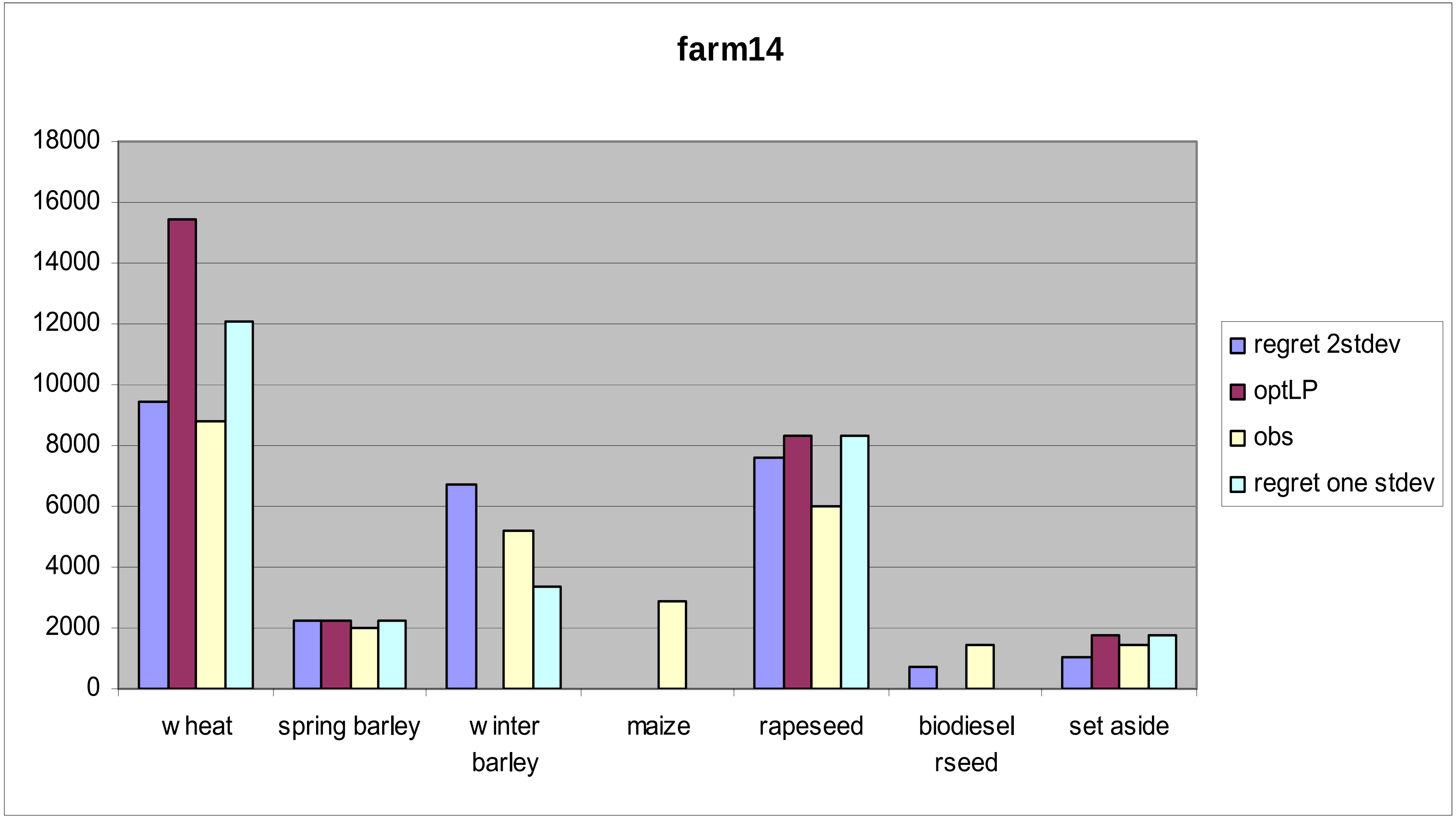

Figure 4. Aggregate Crop Mix (in ha) under Different Model Specifications

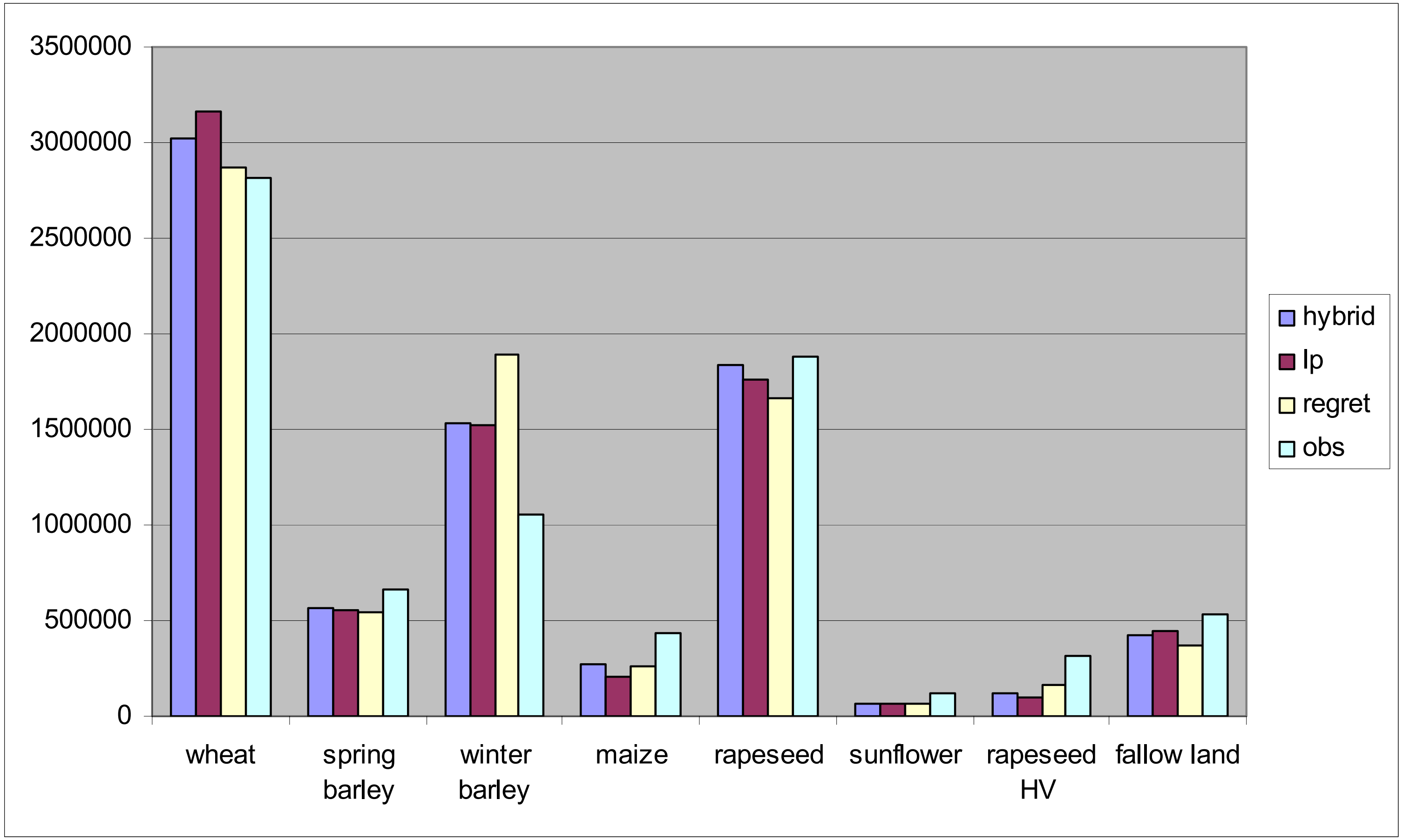

Figure 5. Policy Impacts on the Crop Mix in Cereal Farms

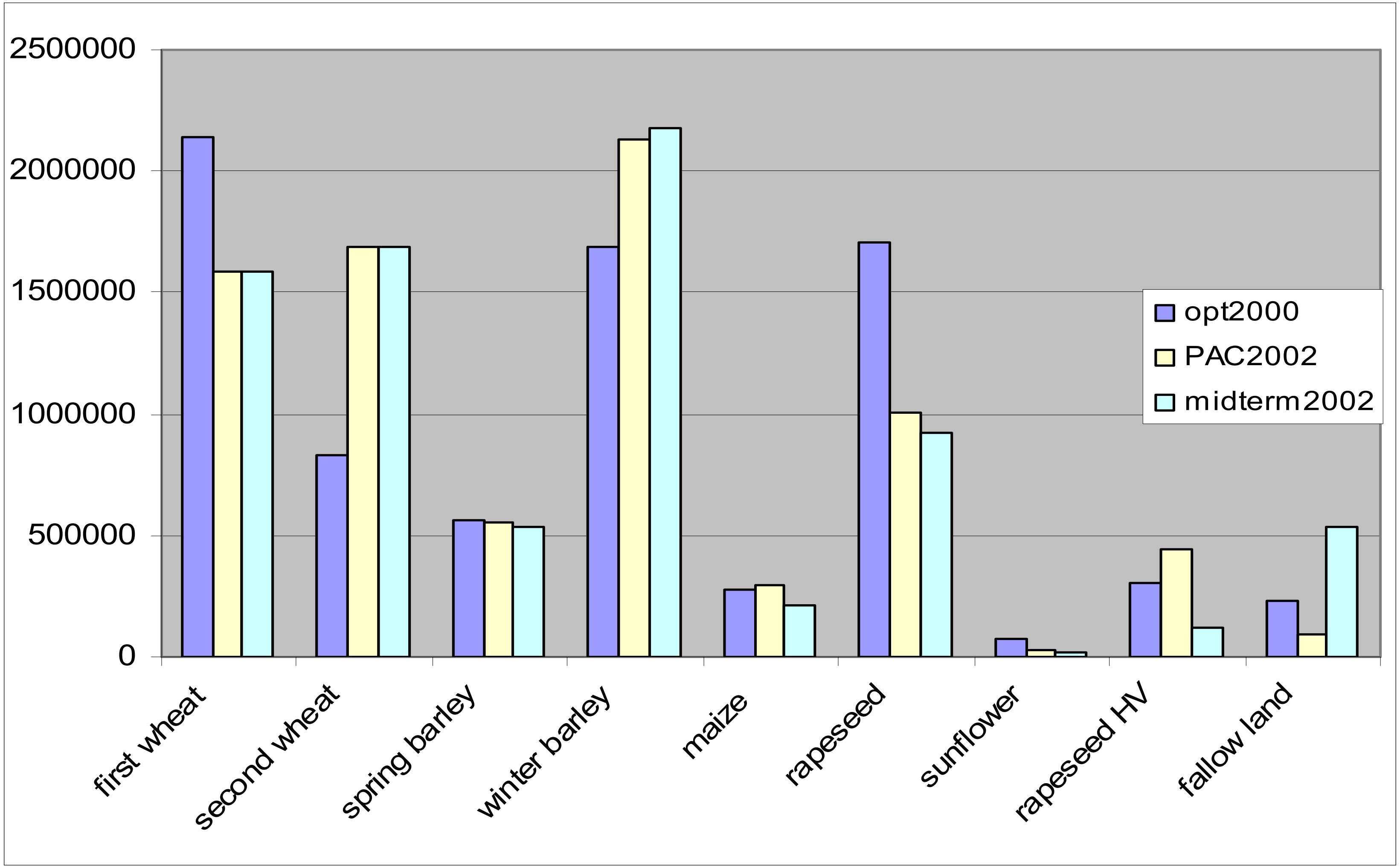

Table 1. Aggregate Results of the LP Model vs. Observed Values

	Observed surfaces in ha	Observed crop mix	LP optimal crop mix	Relative percentage to total surface
Wheat	2 813 264	36.1%	40.5%	4.54%
Barley	661 293	8.5%	7.1%	1.31%
Spring barley	1 052 023	13.5%	19.4%	5.96%
Maize	436 967	5.6%	2.7%	2.91%
Rape seed	1 875 315	24.0%	22.5%	1.50%
Sunflower	117 546	1.5%	0.9%	0.65%
Rapseed ME	310 229	4.0%	1.2%	2.75%
Set aside	536 704	6.9%	5.7%	1.21%
	7 803 341	100.0%	100.0%	20.84%

Table 2. Variability of Arable Crop Gross Margin (€/ha) for Different Policy Regimes

	data 2000			2002 (flat rate subsidy=358 €)		2002 : decoupling	
	mean gm	stdev %gm	subs %gm	mean gm	stdev gm	mean gm	stdev gm
wheat	558	14%	56%	617	13%	259	30%
cont. Wheat	486	16%	64%	546	14%	189	41%
spring barley	572	16%	54%	615	15%	257	35%
winter barley	469	23%	66%	538	20%	182	57%
maize	477	32%	65%	532	28%	174	100%
rapeseed	564	18%	85%	516	21%	158	68%
sunflower	420	19%	114%	386	23%	28	311%
Wheat-to-ethanol	525	14%	73%	543	14%	230	33%
Rapeseed-to-ester	419	22%	91%	426	22%	114	83%
set aside	310	9%	123%	287	10%	- 71	39%